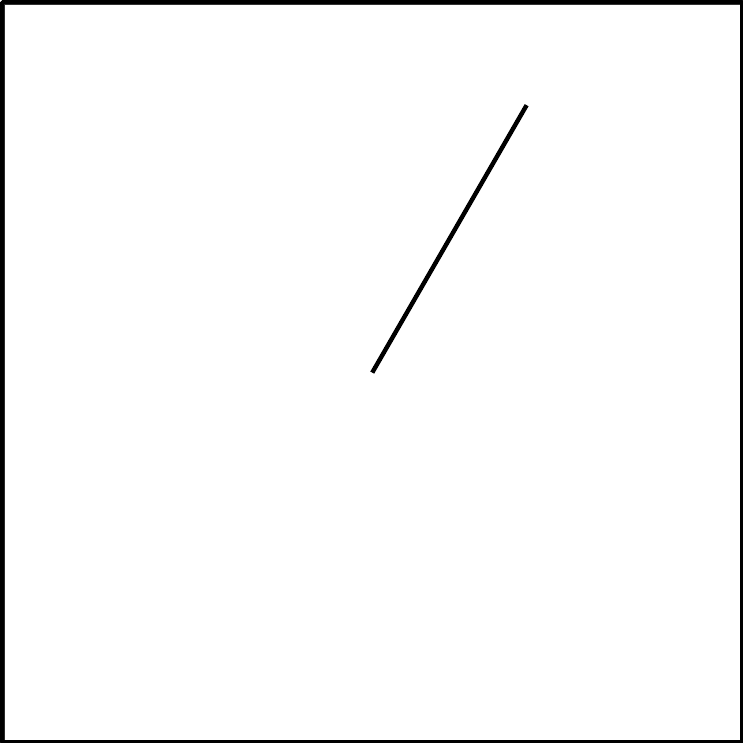
## Lesson 5: The Pythagorean Identity (Part 1)

* Let’s learn more about cosine and sine.

### 5.1: Circle Equations

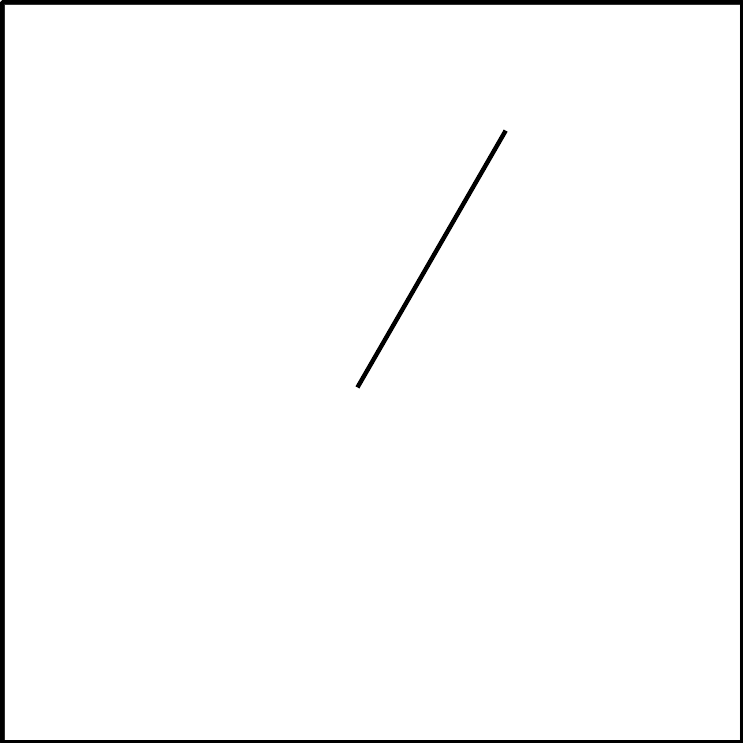
Here is a circle centered at with a radius of 1 unit.

What are the exact coordinates of if is rotated counterclockwise radians from the point ? Explain or show your reasoning.



### 5.2: Cosine, Sine, and the Unit Circle

What are the exact coordinates of point if it is rotated radians counterclockwise from the point ? Explain or show your reasoning.



### 5.3: A New Identity

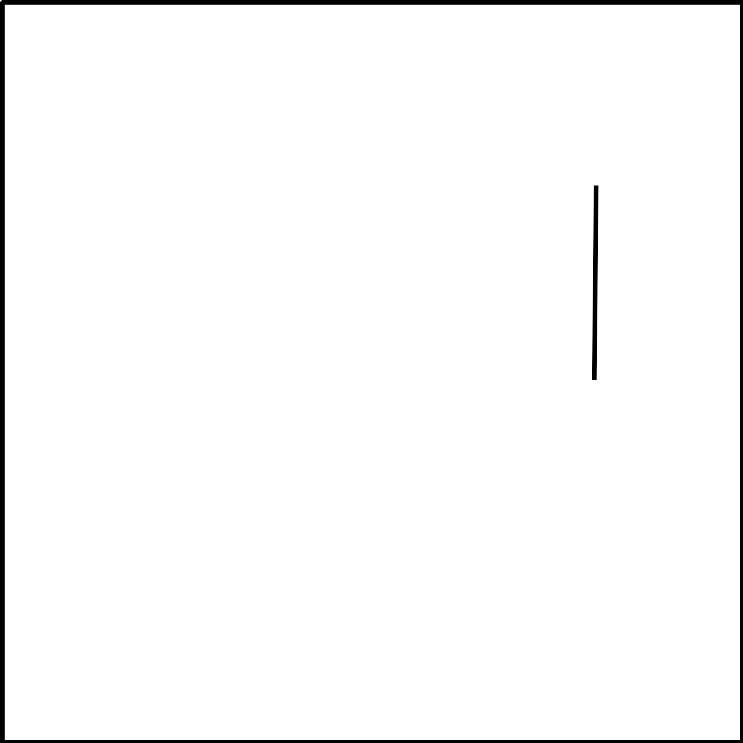
1. Is the point on the unit circle? Explain or show your reasoning.
2. Is the point on the unit circle? Explain or show your reasoning.
3. Suppose that and that is in quadrant 4. What is the exact value of ? Explain or show your reasoning.

#### Are you ready for more?

Show that if is an angle between 0 and and neither nor , then it is impossible for the sum of and to be equal to 1.

### Lesson 5 Summary

Let’s say we have a point with coordinates on the unit circle, like the one shown here:



Using the Pythagorean Theorem, we know that . We also know this is true using the equation for a circle with radius 1 unit, , which is true for the point since it is on the circle.

Another way to write the coordinates of is using the angle , which gives the location of on the unit circle relative to the point . Thinking of this way, its coordinates are . Since and , we can return to the Pythagorean Theorem and say that  is also true.

What if were a different angle and wasn’t in quadrant 1? It turns out that no matter the quadrant, the coordinates of any point on the unit circle given by an angle are . In fact, the definitions of and are the - and -coordinates of the point on the unit circle  radians counterclockwise from . Up until today, we’ve only been using the quadrant 1 values for cosine and sine to find side lengths of right triangles, which are always positive.

This revised definition of cosine and sine means that is true for all values of defined on the unit circle and is known as the **Pythagorean Identity**.



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