Lesson 1: What are Scaled Copies?

Goals

- Describe (orally) characteristics of scaled and non-scaled copies.
- Identify scaled copies of a figure and justify (orally and in writing) that the copy is a scaled copy.

Learning Targets

- I can describe some characteristics of a scaled copy.
- I can tell whether or not a figure is a scaled copy of another figure.

Lesson Narrative

This lesson introduces students to the idea of a **scaled copy** of a picture or a figure. Students learn to distinguish scaled copies from those that are not—first informally, and later, with increasing precision. They may start by saying that scaled copies have the same shape as the original figure, or that they do not appear to be distorted in any way, though they may have a different size. Next, they notice that the lengths of segments in a scaled copy vary from the lengths in the original figure in a uniform way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their original counterparts. Students work toward articulating the characteristics of scaled copies quantitatively (e.g., "all the segments are twice as long," "all the lengths have shrunk by one third," or "all the segments are one-fourth the size of the segments in the original"), articulating the relationships carefully (MP6) along the way.

The lesson is designed to be accessible to all students regardless of prior knowledge, and to encourage students to make sense of problems and persevere in solving them (MP1) from the very beginning of the course.

Alignments

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

You will need the Pairs of Scaled Polygons blackline master for this lesson. Print and cut slips A–J for the Pairs of Scaled Polygons activity. Prepare 1 copy for every 2 students. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.

Student Learning Goals

Let's explore scaled copies.

1.1 Printing Portraits

Warm Up: 10 minutes (there is a digital version of this activity)
This opening task introduces the term **scaled copy**. It prompts students to observe several copies of a picture, visually distinguish scaled and unscaled copies, and articulate the differences in their own words. Besides allowing students to have a mathematical conversation about properties of figures, it provides an accessible entry into the concept and gives an opportunity to hear the language and ideas students associate with scaled figures.

Students are likely to have some intuition about the term "to scale," either from previous work in grade 6 (e.g., scaling a recipe, or scaling a quantity up or down on a double number line) or from outside the classroom. This intuition can help them identify scaled copies.

Expect them to use adjectives such as "stretched," "squished," "skewed," "reduced," etc., in imprecise ways. This is fine, as students' intuitive definition of scaled copies will be refined over the course of the lesson. As students discuss, note the range of descriptions used. Monitor for students whose descriptions are particularly supportive of the idea that lengths in a scaled copy are found by multiplying the original lengths by the same value. Invite them to share their responses later.

Addressing

- 7.G.A.1

Instructional Routines

- Think Pair Share

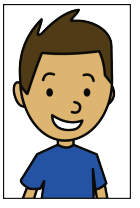
Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and a minute to share their response with their partner.

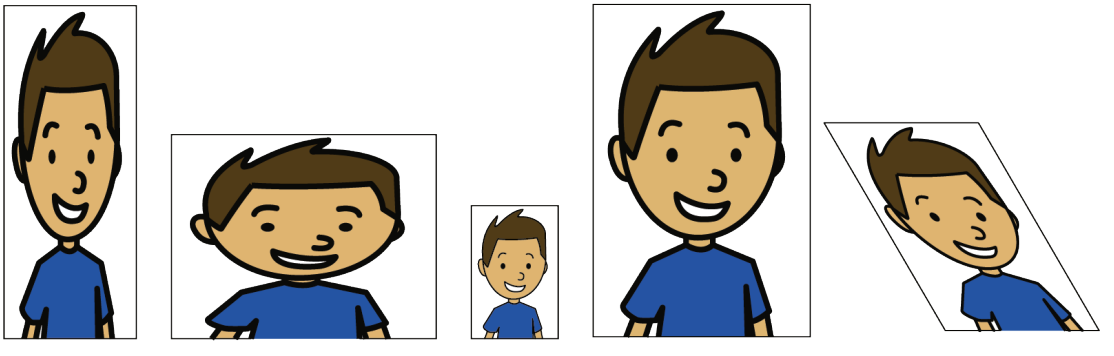
If using the digital activity, have students work in groups of 2–3 to complete the activity. They should have quiet time in addition to share time, while solving the problem and developing language to describe scaling.

Student Task Statement

Here is a portrait of a student.



1. Look at Portraits A–E. How is each one the same as or different from the original portrait of the student?



A B C D E

2. Some of the Portraits A–E are **scaled copies** of the original portrait. Which ones do you think are scaled copies? Explain your reasoning.

3. What do you think "scaled copy" means?

Student Response

1. Answers vary. Sample response:

- Similarities: Pictures A–E are all based on the same original portrait. They all show the same boy wearing a blue shirt and brown hair. They all have the same white background.
- Differences: They all have different sizes; some have different shapes. Pictures A, B, and E have been stretched or somehow distorted. C and D are not stretched or distorted but are each of a different size than the original.

1. C and D are scaled copies. Sample explanation:

- A, B, and E are not scaled copies because they have changed in shape compared to the original portrait. Portrait A is stretched vertically, so the vertical side is now much longer than the horizontal side. B is stretched out sideways, so the horizontal sides are now longer than the vertical. E seems to have its upper left and lower right corners stretched out in opposite directions. The portrait is no longer a rectangle.
- C is a smaller copy and D is a larger copy of the original, but their shapes remain the same.

1. Answers vary. Sample definitions:

- A scaled copy is a copy of a picture that changes in size but does not change in shape.
- A scaled copy is a duplicate of a picture with no parts of it distorted, though it could be larger, smaller, or the same size.
- A scaled copy is a copy of a picture that has been enlarged or reduced in size but nothing else changes.

Activity Synthesis

Select a few students to share their observations. Record and display students' explanations for the second question. Consider organizing the observations in terms of how certain pictures are or are not distorted. For example, students may say that C and D are scaled copies because each is a larger or smaller version of the picture, but the face (or the sleeve, or the outline of the picture) has not changed in shape. They may say that A, B, and E are not scaled copies because something other than size has changed. If not already mentioned in the discussion, guide students in seeing features of C and D that distinguish them from A, B, and E.

Invite a couple of students to share their working definition of scaled copies. Some of the students' descriptions may not be completely accurate. That is appropriate for this lesson, as the goal is to build on and refine this language over the course of the next few lessons until students have a more precise notion of what it means for a picture or figure to be a scaled copy.

1.2 Scaling F

10 minutes (there is a digital version of this activity)
This task enables students to describe more precisely the characteristics of scaled copies and to refine the meaning of the term. Students observe copies of a line drawing on a grid and notice how the lengths of line segments and the angles formed by them compare to those in the original drawing.

Students engage in MP7 in multiple ways in this task. Identifying distinguishing features of the scaled copies means finding similarities and differences in the shapes. In addition, the fact that corresponding parts increase by the *same* scale factor is a vital structural property of scaled copies.

For the first question, expect students to explain their choices of scaled copies in intuitive, qualitative terms. For the second question, students should begin to distinguish scaled and unscaled copies in more specific and quantifiable ways. If it does not occur to students to look at lengths of segments, suggest they do so.

As students work, monitor for students who notice the following aspects of the figures. Students are not expected to use these mathematical terms at this point, however.

- The original drawing of the letter F and its scaled copies have equivalent width-to-height ratios.

- We can use a scale factor (or a multiplier) to compare the lengths of different figures and see if they are scaled copies of the original.
- The original figure and scaled copies have corresponding angles that have the same measure.

Addressing

- 7.G.A.1

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Keep students in the same groups. Give them 3–4 minutes of quiet work time, and then 1–2 minutes to share their responses with their partner. Tell students that how they decide whether each of the seven drawings is a scaled copy may be very different than how their partner decides. Encourage students to listen carefully to each other's approach and to be prepared to share their strategies. Use gestures to elicit from students the words "horizontal" and "vertical" and ask groups to agree internally on common terms to refer to the parts of the F (e.g., "horizontal stems").

Access for Students with Disabilities

Engagement: Internalize Self Regulation. Display sentence frames to support small group discussion. For example, "That could/couldn't be true because...," "We can agree that...," and "Is there another way to say/do...?"
Supports accessibility for: Social-emotional skills; Organization; Language

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Access for English Language Learners

Speaking: Math Language Routine 1 Stronger and Clearer Each Time. This is the first time Math Language Routine 1 is suggested as a support in this course. In this routine, students are given a thought-provoking question or prompt and asked to create a first draft response in writing. Students meet with 2–3 partners to share and refine their response through conversation. While meeting, listeners ask questions such as, "What did you mean by . . .?" and "Can you say that another way?" Finally, students write a second draft of their response reflecting ideas from partners, and improvements on their initial ideas. The purpose of this routine is to provide a structured and interactive opportunity for students to revise and refine their ideas through verbal and written means.
Design Principle(s): Optimize output (for explanation)

How It Happens:

1. Use this routine to provide students a structured opportunity to refine their explanations for the first question: "Identify all the drawings that are scaled copies of the original letter F drawing. Explain how you know." Allow students 2–3 minutes to individually create first draft responses in writing.

2. Invite students to meet with 2–3 other partners for feedback.

   Instruct the speaker to begin by sharing their ideas without looking at their written draft, if possible. Provide the listener with these prompts for feedback that will help their partner strengthen their ideas and clarify their language: "What do you mean when you say....?", "Can you describe that another way?", "How do you know that _ is a scaled copy?", "Could you justify that differently?" Be sure to have the partners switch roles. Allow 1–2 minutes to discuss.

3. Signal for students to move on to their next partner and repeat this structured meeting.

4. Close the partner conversations and invite students to revise and refine their writing in a second draft.

   Provide these sentence frames to help students organize their thoughts in a clear, precise way: "Drawing _ is a scaled copy of the original, and I know this because....", "When I look at the lengths, I notice that....", and "When I look at the angles, I notice that...."

   Here is an example of a second draft:

   "Drawing 7 is a scaled copy of the original, and I know this because it is enlarged evenly in both the horizontal and vertical directions. It does not seem lopsided or stretched differently in one direction. When I look at the length of the top segment, it is 3 times as large as the original one, and the other segments do the same thing. Also, when I look at the angles, I notice that they are all right angles in both the original and scaled copy."

5. If time allows, have students compare their first and second drafts. If not, have the students move on by working on the following problems.

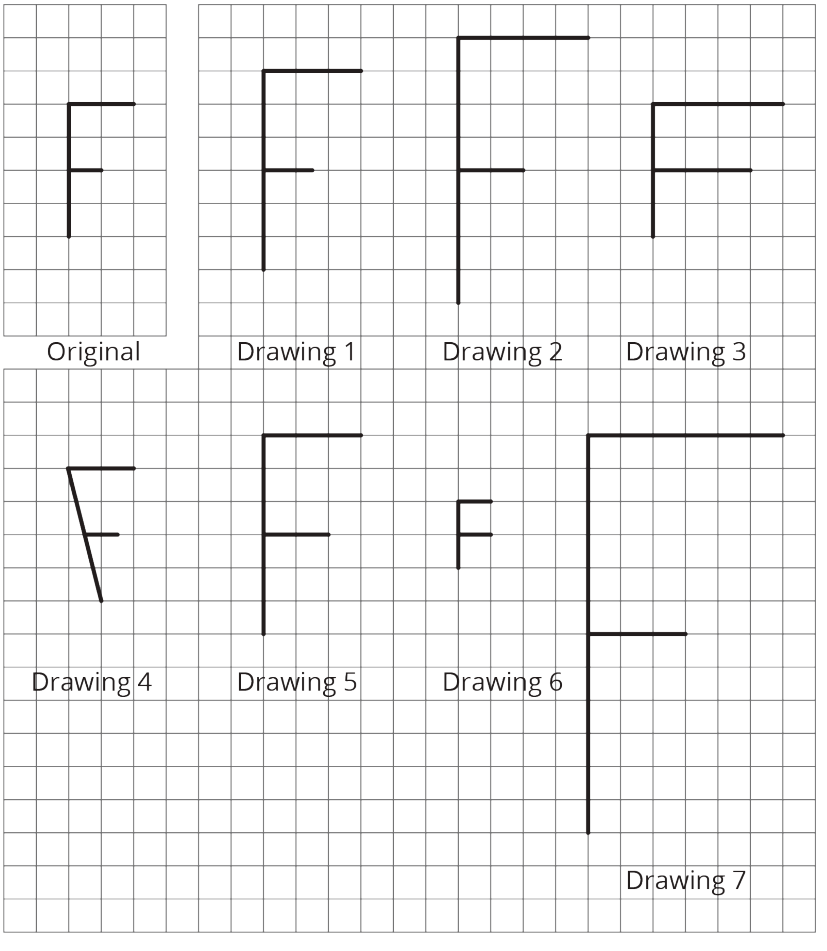
Anticipated Misconceptions

Students may make decisions by "eyeballing" rather than observing side lengths and angles. Encourage them to look for quantifiable evidence and notice lengths and angles.

Some may think vertices must land at intersections of grid lines (e.g., they may say Drawing 4 is not a scaled copy because the endpoints of the shorter horizontal segment are not on grid crossings). Address this during the whole-class discussion, after students have a chance to share their observations about segment lengths.

Student Task Statement

Here is an original drawing of the letter F and some other drawings.



1. Identify **all** the drawings that are scaled copies of the original letter F. Explain how you know.

2. Examine all the scaled copies more closely, specifically the lengths of each part of the letter F. How do they compare to the original? What do you notice?

3. On the grid, draw a different scaled copy of the original letter F.

Student Response

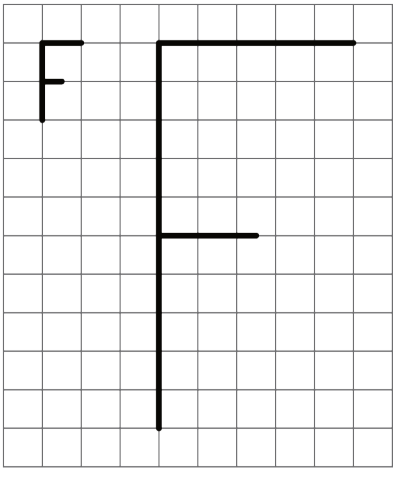
1. Drawings 1, 2, and 7 are scaled copies of the original drawing. Explanations vary. Sample explanation: I know because they are not stretched differently in one direction. They are enlarged evenly in both vertical and horizontal directions.

2. Answers vary. Sample responses:

    - In the scaled copies, every segment is the same number of times as long as the matching segment in the original drawing.

    - In the scaled copies, all segments keep the same relationships as in the original. The original drawing of F is 4 units tall. Its top horizontal segment is 2 units wide and the shorter horizontal segment is 1 unit. In Drawing 1, the F is 6 units tall and 3 units wide; in Drawing 2, it is 8 units tall and 4 units wide, and in Drawing 7, it is 8 units tall and 4 units wide. In each scaled copy, the width is half of the height, just as in the original drawing of F, and the shorter horizontal segment is half of the longer one.

3. Drawings vary. Sample response:



Activity Synthesis

Display the seven copies of the letter F for all to see. For each copy, ask students to indicate whether they think each one is a scaled copy of the original F. Record and display the results for all to see. For contested drawings, ask 1–2 students to briefly say why they ruled these out.

Discuss the identified scaled and unscaled copies.

- What features do the scaled copies have in common? (Be sure to invite students who were thinking along the lines of scale factors and angle measures to share.)
- How do the other copies fail to show these features? (Sometimes lengths of sides in the copy use different multipliers for different sides. Sometimes the angles in the copy do not match the angles in the original.)

If there is a misconception that scaled copies must have vertices on intersections of grid lines, use Drawing 1 (or a relevant drawing by a student) to discuss how that is not the case.

Some students may not be familiar with words such as “twice,” “double,” or “triple.” Clarify the meanings by saying “two times as long” or “three times as long.”

1.3 Pairs of Scaled Polygons

15 minutes (there is a digital version of this activity)
In this activity, students hone their understanding of scaled copies by working with more complex figures. Students work with a partner to match pairs of polygons that are scaled copies. The polygons appear comparable to one another, so students need to look very closely at all side lengths of the polygons to tell if they are scaled copies.

As students confer with one another, notice how they go about looking for a match. Monitor for students who use precise language (MP6) to articulate their reasoning (e.g., “The top side of A is half the length of the top side of G, but the vertical sides of A are a third of the lengths of those in G.”).

You will need the Pairs of Scaled Polygons blackline master for this activity.

Addressing

- 7.G.A.1

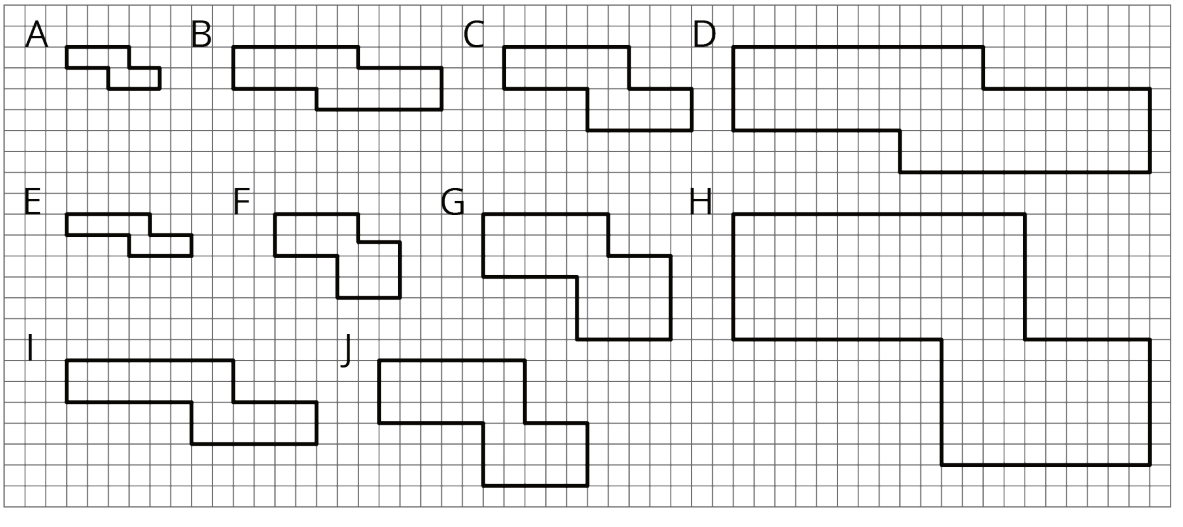
Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the cards and place them face-up. Tell them that each polygon has one and only one match (i.e., for each polygon, there is one and only one scaled copy of the polygon). Select two cards and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 2. Give each group a set of 10 slips cut from the blackline master. Encourage students to refer to a running list of statements and diagrams to refine their language and explanations of how they know one figure is a scaled copy of the other.



Access for Students with Disabilities

Representation: Internalize Comprehension. Provide a range of examples and counterexamples. During the demonstration of how to set up and do the matching activity, select two cards that do not match, and invite students to come up with a shared justification.
Supports accessibility for: Conceptual processing

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support small-group discussion. As students take turns finding a match of two polygons that are scaled copies of one another and explaining their reasoning to their partner, display the following sentence frames for all to see: "____ matches ____ because . . ." and "I noticed ___ , so I matched . . . ." Encourage students to challenge each other when they disagree with the sentence frames "I agree because . . .", and "I disagree because . . . ." This will help students clarify their reasoning about scaled copies of polygons.
Design Principle(s): Support sense-making; Optimize output (for explanation)

Anticipated Misconceptions

Some students may think a figure has more than one match. Remind them that there is only one scaled copy for each polygon and ask them to recheck all the side lengths.

Some students may think that vertices must land at intersections of grid lines and conclude that, e.g., G cannot be a copy of F because not all vertices on F are on such intersections. Ask them to consider how a 1-unit-long segment would change if scaled to be half its original size. Where must one or both of its vertices land?

Student Task Statement

Your teacher will give you a set of cards that have polygons drawn on a grid. Mix up the cards and place them all face up.

1. Take turns with your partner to match a pair of polygons that are scaled copies of one another.
    a. For each match you find, explain to your partner how you know it's a match.
    b. For each match your partner finds, listen carefully to their explanation, and if you disagree, explain your thinking.
2. When you agree on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.

3. Select one pair of polygons to examine further. Draw both polygons on the grid. Explain or show how you know that one polygon is a scaled copy of the other.

Student Response

1. The following polygons are scaled versions of one another:
    - A and C
    - B and D
    - E and I
    - F and G
    - H and J
2. No answer needed.
3. Answers vary. Sample explanation for A and C: All the side lengths in C are twice as long as the lengths of the matching sides in A.

Are You Ready for More?

Is it possible to draw a polygon that is a scaled copy of both Polygon A and Polygon B? Either draw such a polygon, or explain how you know this is impossible.

Student Response

It's impossible to draw a polygon that is a scaled copy of both Polygon A and Polygon B. Sample explanations:

- If I draw a polygon that is a scaled copy of A, all the side lengths would be the same number of times larger or smaller than A, but they won't be the same number of times larger or smaller than B.

- A and B are not scaled copies of each other, so if I draw a scaled copy of one, it will not be a scaled copy of the other.

Activity Synthesis

The purpose of this discussion is to draw out concrete methods for deciding whether or not two polygons are scaled copies of one another, and in particular, to understand that just eyeballing to see whether they look roughly the same is not enough to determine that they are scaled copies.

Display the image of all the polygons. Ask students to share their pairings and guide a discussion about how students went about finding the scaled copies. Ask questions such as:

- When you look at another polygon, what exactly did you check or look for? (General shape, side lengths)
- How many sides did you compare before you decided that the polygon was or was not a scaled copy? (Two sides can be enough to tell that polygons are not scaled copies; all sides are needed to make sure a polygon is a scaled copy.)
- Did anyone check the angles of the polygons? Why or why not? (No; the sides of the polygons all follow grid lines.)

If students do not agree about some pairings after the discussion, ask the groups to explain their case and discuss which of the pairings is correct. Highlight the use of quantitative descriptors such as “half as long” or “three times as long” in the discussion. Ensure that students see that when a figure is a scaled copy of another, all of its segments are the same number of times as long as the corresponding segments in the other.

Lesson Synthesis

In this lesson, we encountered copies of a figure that are both scaled and not scaled. We saw different versions of a portrait of a student and of a letter F, as well as a variety of polygons that had some things in common.
In each case, we decided that some were scaled copies of one another and some were not.
Consider asking students:

- What is a **scaled copy**?
- What are some characteristics of scaled copies? How are they different from figures that are not scaled copies?
- What specific information did you look for when determining if something was a scaled copy of an original?

While initial answers need not be particularly precise at this stage of the unit (for example, “scaled copies look the same but are a different size”), guide the discussion toward making careful statements that one could test. The lengths of segments in a scaled copy are related to the lengths in the original figure in a consistent way. For instance, if a segment in a scaled copy is half the length of its counterpart in the original, then all other segments in the copy are also half the length of their

original counterparts. We might say, “All the segments are twice as long,” or “All the segments are one-third the size of the segments in the original.”

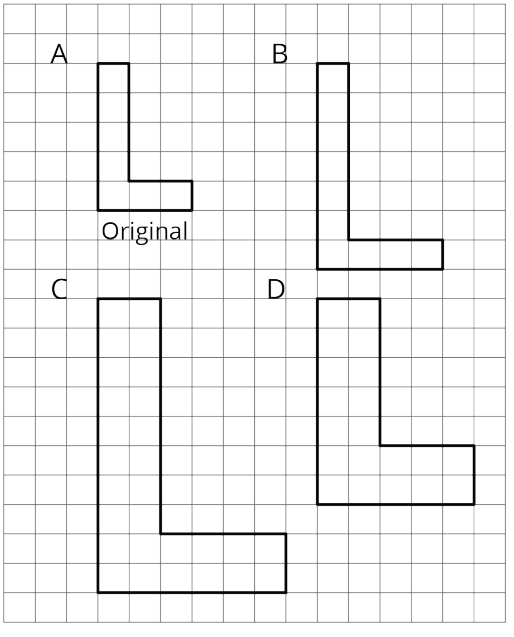
1.4 Scaling L

Cool Down: 5 minutes
Addressing

- 7.G.A.1

Student Task Statement

Are any of the figures B, C, or D scaled copies of figure A? Explain how you know.



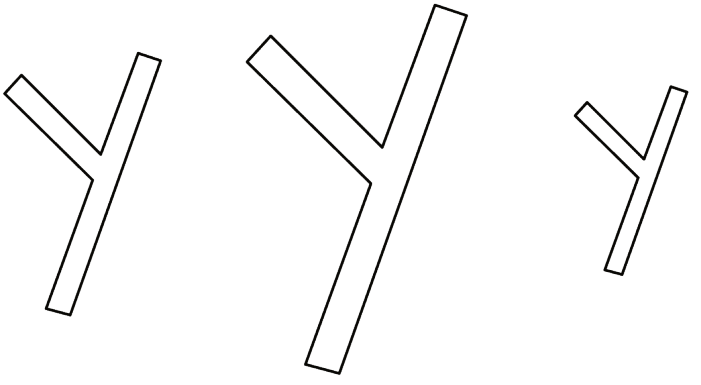
Student Response

Only figure C is a scaled copy of figure A. Sample explanation: In figure C, the length of each segment of the letter L is twice the length of the matching segment in A. In B, none of the segments are double the length. In figure D, some segments are double in length and some are not. So the block letters in B and D are not enlarged evenly.

Student Lesson Summary

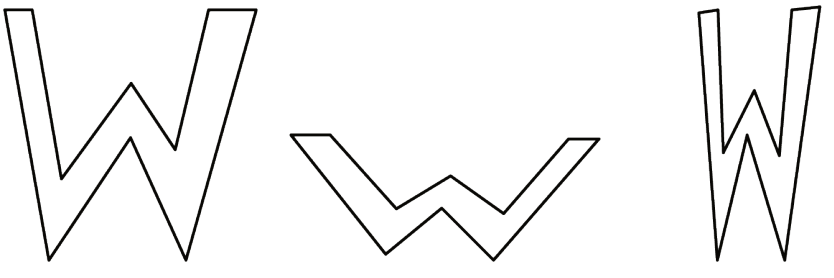
What is a **scaled copy** of a figure? Let’s look at some examples.

The second and third drawings are both scaled copies of the original Y.



Original

However, here, the second and third drawings are *not* scaled copies of the original W.



Original

The second drawing is spread out (wider and shorter). The third drawing is squished in (narrower, but the same height).

We will learn more about what it means for one figure to be a scaled copy of another in upcoming lessons.

Glossary

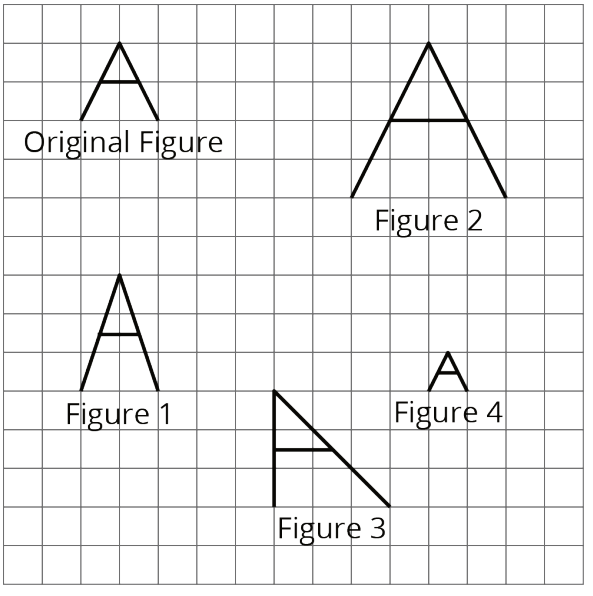
- scaled copy

Lesson 1 Practice Problems

Problem 1

Statement

Here is a figure that looks like the letter A, along with several other figures. Which figures are scaled copies of the original A? Explain how you know.



Solution

Figures 2 and 4 are scaled copies. Sample explanations:

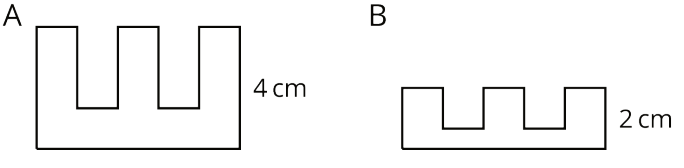
- The original A fits inside a square. The horizontal segment is halfway up the height of the square. The tip of the A is at the midpoint of the horizontal side of the square.
- Figure 1 inside a rectangle, not a square, so it is not a scaled copy. Figure 3 fits inside a square but the shape is different than the original letter A, since one of the legs of the A in Figure 3 is now vertical, so it also is not a scaled copy.
- Figure 2 is twice as high and twice as wide as the original A, and Figure 4 is half as tall and as wide, but in both figures the locations of the horizontal segment and the tip of the letter A still match the original.

Problem 2

Statement

Tyler says that Figure B is a scaled copy of Figure A because all of the peaks are half as tall.

Do you agree with Tyler? Explain your reasoning.



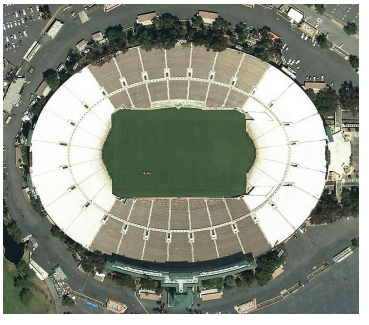
Solution

No. For the smaller figure to be a scaled copy, the figure would have to be half as wide as well.

Problem 3

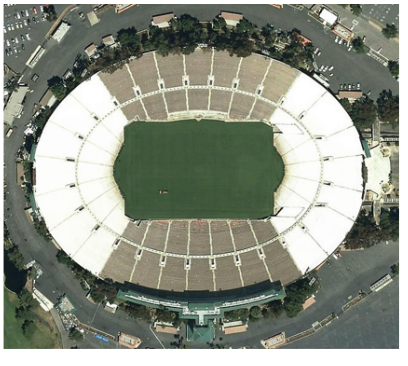
Statement

Here is a picture of the Rose Bowl Stadium in Pasadena, CA.

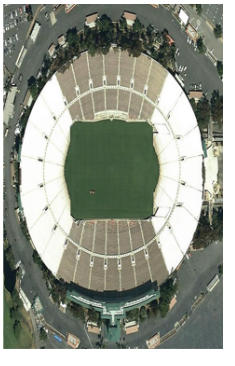


Here are some copies of the picture. Select **all** the pictures that are scaled copies of the original picture.

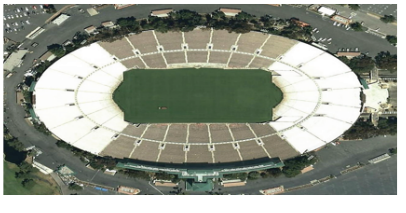
A



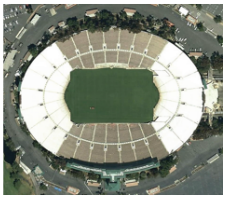
B



C



D



Solution

["A", "D"]

Problem 4

Statement

Complete each equation with a number that makes it true.

a. $5 \cdot \_\_\_\_\_ = 15$

b. $4 \cdot \_\_\_\_\_ = 32$

c. $6 \cdot \_\_\_\_\_ = 9$

d. $12 \cdot \_\_\_\_\_ = 3$

Solution

a. 3

b. 8

c. 1.5, $\frac{3}{2}$, or equivalent

d. 0.25, $\frac{1}{4}$, or equivalent