Lesson 2: Corresponding Parts and Scale Factors

Goals

- Comprehend the phrase "scale factor" and explain (orally) how it relates corresponding lengths of a figure and its scaled copy.
- Explain (orally) what it means to say one part in a figure "corresponds" to a part in another figure.
- Identify and describe (orally and in writing) corresponding points, corresponding segments, or corresponding angles in a pair of figures.

Learning Targets

- I can describe what the scale factor has to do with a figure and its scaled copy.
- In a pair of figures, I can identify corresponding points, corresponding segments, and corresponding angles.

Lesson Narrative

This lesson develops the vocabulary for talking about scaling and scaled copies more precisely (MP6), and identifying the structures in common between two figures (MP7).

Specifically, students learn to use the term **corresponding** to refer to a pair of points, segments, or angles in two figures that are scaled copies. Students also begin to describe the numerical relationship between the corresponding lengths in two figures using a **scale factor**. They see that when two figures are scaled copies of one another, the same scale factor relates their corresponding lengths. They practice identifying scale factors.

A look at the angles of scaled copies also begins here. Students use tracing paper to trace and compare angles in an original figure and its copies. They observe that in scaled copies the measures of corresponding angles are equal.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- 5.NF.B.4: Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A.2: Recognize and represent proportional relationships between quantities.

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder
- Number Talk
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Prepare to display the images of the railroad crossing sign for the Corresponding Parts activity. Make sure students have access to their geometry toolkits, especially tracing paper and graph paper.

> **Student Learning Goals**
>
> Let's describe features of scaled copies.

2.1 Number Talk: Multiplying by a Unit Fraction

Warm Up: 5 minutes
This number talk allows students to review multiplication strategies, refreshing the idea that multiplying by a unit fraction is the same as dividing by its whole number reciprocal. It encourages students to use the structure of base ten numbers and the properties of operations to find the product of two whole numbers (MP7). For example, a student might find $72 \cdot \frac{1}{9}$ (or $72 \div 9$) and then shift the decimal one place to the right in order to evaluate $(7.2) \cdot \frac{1}{9}$. Each problem was chosen to

elicit different approaches, so as students share theirs, ask how the factors in each problem impacted their strategies.

Before students begin, consider establishing a small, discreet hand signal (such as a thumbs-up) students can display to indicate they have an answer that they can support by reasoning. Discreet signaling is a quick way for teachers to gather feedback about timing. It also keeps students from being distracted or rushed by raised hands around the class.

Building On

- 5.NBT.B.7
- 5.NF.B.4

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students up to 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a brief whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find each product mentally.

$\frac{1}{4} \cdot 32$

$(7.2) \cdot \frac{1}{9}$

$\frac{1}{4} \cdot (5.6)$

Student Response

- $\frac{1}{4} \cdot 32 = 8$. Possible strategy: $32 \div 4 = 8$
- $(7.2) \cdot \frac{1}{9} = 0.8$. Possible strategy: $72 \div 9 = 8$ so $(7.2) \div 9 = 0.8$

- $\frac{1}{4} \cdot (5.6) = 1.4$. Possible strategy: $(5.6) \div 4 = 1.4$

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If students express strategies in terms of division, ask if that strategy would work for any multiplication problem involving fractions. Highlight that these problems only involve unit fractions and division by the denominator is a strategy that works when multiplying by a unit fraction.

To involve more students in the conversation, consider asking:

- Who can restate ___'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

2.2 Corresponding Parts

15 minutes (there is a digital version of this activity)
This activity introduces important language students will apply to describe scaled copies. In particular, it introduces the important idea of corresponding parts. Students have previously analyzed corresponding sides in figures. Here they will begin to examine angles explicitly as well, understanding that corresponding angles in a figure and its scaled copy have the same measure.

Addressing

- 7.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Launch

Tell students that in this lesson, they will look more closely at copies of figures and describe specific parts in them.

Display the designs (the three images in the activity statement) and the following descriptions for all to see. Ask students what they notice and what they wonder. After discussion, explain that the original design and its two copies have parts that correspond to one another. Point out some of their corresponding parts:

- The X-pattern going across each figure
- The curved outline of each figure
- The points K in the original sign, A in Copy 1, and U in Copy 2

Arrange students in groups of 2 and provide access to their geometry toolkits (especially tracing paper). Give students 2–3 minutes to complete the first two questions and another 2 minutes to discuss their responses with their partner. Ask students to pause their work for a quick class discussion afterwards.

Have a few students name a set of corresponding points, segments, and angles.

Then, ask students to indicate whether they think either copy is a scaled copy. Invite a couple of students to share their reasoning. When the class reaches an agreement that Copy 1 is a scaled copy and Copy 2 is not, ask students to complete the remaining questions individually and to use tracing paper as a tool.

Consider demonstrating to the class how to use tracing paper to compare angles. Tell or show students that the line segments forming an angle could be extended for easier tracing and comparison.

For classrooms using the digital version of the activity, the applet has a moveable angle tool to compare the angles in the copies with the angles in the original.

Access for Students with Disabilities

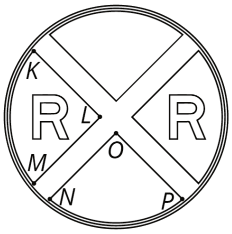
Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: corresponding points, corresponding line segments, and corresponding angles.
Supports accessibility for: Memory; Language

Access for English Language Learners

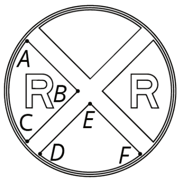
Speaking: MLR8 Discussion Supports. Use this routine to amplify mathematical uses of language to communicate about corresponding points, segments, and angles. As students share what they noticed between the three images, revoice their statements using the term "corresponding." Then, invite students to use the term "corresponding" when describing what they noticed. Some students may benefit from chorally repeating the phrases that include the word "corresponding" in context.
Design Principle(s): Optimize output (for explanation)

Student Task Statement

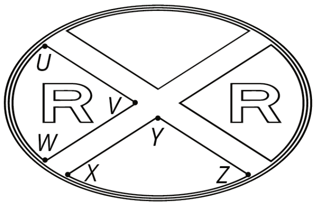
Here is a figure and two copies, each with some points labeled.



ORIGINAL



COPY 1



COPY 2

1. Complete this table to show **corresponding parts** in the three figures.

original	copy 1	copy 2
point P		
segment LM		
	segment EF	
		point W
angle KLM		
		angle XYZ

2. Is either copy a scaled copy of the original figure? Explain your reasoning.

3. Use tracing paper to compare angle KLM with its corresponding angles in Copy 1 and Copy 2. What do you notice?

4. Use tracing paper to compare angle NOP with its corresponding angles in Copy 1 and Copy 2. What do you notice?

Student Response

1.

original	copy 1	copy 2
point P	point F	point Z
segment LM	segment BC	segment VW
segment OP	segment EF	segment YZ
point M	point C	point W
angle KLM	angle ABC	angle UVW
angle NOP	angle DEF	angle XYZ

2. Copy 1 is a scaled copy, but Copy 2 is not. Sample explanation: The original sign is a circle. Copy 1 is also a circle, only smaller. Copy 2 has been stretched sideways and shrunken vertically; its shape has changed into an oval, so it is not a scaled copy.
3. Angle ABC in Copy 1 corresponds to and has the same size as angle KLM. Angle UVW in Copy 2 also corresponds to angle KLM but is smaller in size than the original angle.
4. Angle DEF in Copy 1 corresponds to and has the same size as angle NOP. Angle XYZ in Copy 2 also corresponds to angle NOP but is larger in size than the original angle.

Activity Synthesis

Select a few students to share their observations about angles. Discuss the size of corresponding angles in figures that are scaled copies and those that are not. Ask questions such as:

- In the scaled copy, Copy 1, did the size of any angle change compared to its corresponding angle in the original sign? (No)
- In Copy 2, did the size of any angle change relative to its corresponding angle in the original sign? (Yes) Which ones? (Angle UVW has a different measure than angle KLM, for example.)
- What can you say about corresponding angles in two figures that are scaled copies of one another? (They have the same measure.)
- What can you say about corresponding angles in two figures that are *not* scaled copies? (They *might* not have the same measure.)

2.3 Scaled Triangles

15 minutes

In this activity, students continue to practice identifying corresponding parts of scaled copies. By organizing corresponding lengths in a table, students see that there is a single factor that relates each length in the original triangle to its corresponding length in a copy (MP8). They learn that this number is called a **scale factor**.

As students work on the first question, listen to how they reason about which triangles are scaled copies. Identify groups who use side lengths and angles as the basis for deciding. (Students are not expected to reason formally yet, but should begin to look to lengths and angles for clues.)

As students identify corresponding sides and their measures in the second and third questions, look out for confusion about corresponding parts. Notice how students decide which sides of the right triangles correspond.

If students still have access to tracing paper, monitor for students who use this tool strategically (MP5).

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2

Instructional Routines

- Think Pair Share

Launch

Arrange students into groups of 4. Assign each student one of the following pairs of triangles in the first question.

- A and E
- B and F
- C and G
- D and H

Give students 2 minutes of quiet think time to determine if their assigned triangles are scaled copies of the original triangle. Give another 2–3 minutes to discuss their responses and complete the first question in groups.

Discuss briefly as a class which triangles are scaled copies and select a couple of groups who reasoned in terms of lengths and angles to explain their reasoning. Some guiding questions:

- What information did you use to tell scaled copies from those that are not?
- How were you able to tell right away that some figures are not scaled copies?

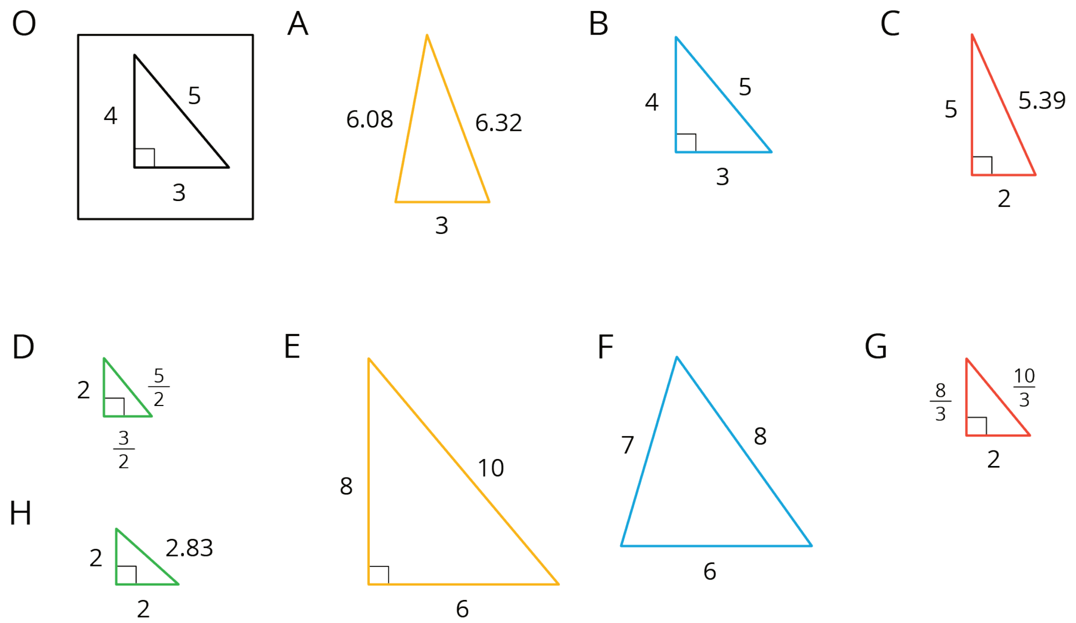
Give students quiet work time to complete the rest of the task after the class recognizes that A, C, F, and H are not scaled copies.

Anticipated Misconceptions

Students may think that Triangle F is a scaled copy because just like the 3-4-5 triangle, the sides are also three consecutive whole numbers. Point out that corresponding angles are not equal.

Student Task Statement

Here is Triangle O, followed by a number of other triangles.



Your teacher will assign you two of the triangles to look at.

1. For each of your assigned triangles, is it a scaled copy of Triangle O? Be prepared to explain your reasoning.
2. As a group, identify *all* the scaled copies of Triangle O in the collection. Discuss your thinking. If you disagree, work to reach an agreement.
3. List all the triangles that are scaled copies in the table. Record the side lengths that correspond to the side lengths of Triangle O listed in each column.

Triangle O	3	4	5

4. Explain or show how each copy has been scaled from the original (Triangle O).

Student Response

1. Answers vary depending on the pair of triangles students have. Triangles B, D, E, and G are scaled copies.

2. Triangles B, D, E, and G are scaled copies. Sample reasoning: B, D, E, and G have not changed in shape (they are still right triangles). Each of their sides are the same number of times as long as the corresponding sides in the original triangle. Triangles A and F do not have the same shape as Triangle O (their angles are all different), so they are not scaled copies. Triangles C, G, and H are right triangles but their sides are not the same number of times as long as the corresponding sides in the original triangle.

3.

Triangle O	3	4	5
Triangle B	3	4	5
Triangle D	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E	6	8	10
Triangle G	2	$\frac{8}{3}$	$\frac{10}{3}$

4. Explanations vary. Sample explanations:
    - Triangle B is a same-size copy of the original. All the lengths stay the same.
    - In Triangle D, all the lengths are half of the original ones.
    - In Triangle E, all sides double in length.
    - In Triangle G, the lengths are $\frac{2}{3}$ times the corresponding lengths in the original triangle.

Are You Ready for More?

Choose one of the triangles that is not a scaled copy of Triangle O. Describe how you could change at least one side to make a scaled copy, while leaving at least one side unchanged.

Student Response

Answers vary. Sample response: on Triangle F, the side of length 7 could be extended to have length 10.

Activity Synthesis

Display the image of all triangles and invite a couple of students to share how they knew which sides of the triangles correspond. Then, display a completed table in the third question for all to see. Ask each group to present its observations about one triangle and how the triangle has been scaled from the original. Encourage the use of “corresponding” in their explanations. As students present, record or illustrate their reasoning on the table, e.g., by drawing arrows between rows and annotating with the operation students are describing, as shown here.

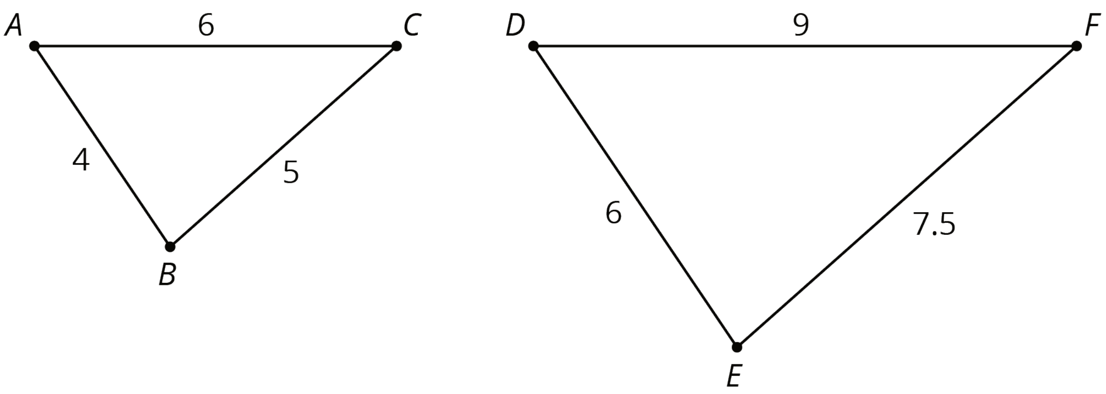
Triangle O	3	4	5
Triangle D $\cdot\frac{1}{2}$	$\frac{3}{2}$	2	$\frac{5}{2}$
Triangle E $\cdot 2$	6	8	10
Triangle B $\cdot 1$	3	4	5
Triangle G $\cdot\frac{2}{3}$	2	$\frac{8}{3}$	$\frac{10}{3}$

Use the language that students use to describe the side lengths and the numerical relationships in the table to guide students toward **scale factor**. For example: “You explained that the lengths in Triangle F are all twice those in the original triangle, so we can write those as “2 times” the original numbers. Lengths in Triangle A are half of those in the original; we can write “$\frac{1}{2}$ times” the original numbers. We call those multipliers—the 2 and the $\frac{1}{2}$-scale factors. We say that scaling Triangle O by a scale factor of 2 produces Triangle F, and that scaling Triangle O by $\frac{1}{2}$ produces Triangle A.”

Lesson Synthesis

- What do we mean by **corresponding parts?**
- What is a **scale factor**? How does it work?

Students can use informal language to describe corresponding parts, and recognize a scale factor as a common ratio between the lengths of corresponding side lengths. In the figure, triangle DEF is a scaled copy of triangle ABC. We call parts that have the same position within each figure *corresponding parts*. For example, we refer to vertex E in triangle DEF and vertex B in triangle ABC as *corresponding points*; segment BC and segment EF as *corresponding segments*; and angle C (or angle BCA) and angle F (or angle EFD) as *corresponding angles*.



The segments in a scaled copy are always a certain number of times as long as the corresponding segments in the original figure. We call that number the *scale factor*. For example, the scale factor between ABC and its copy triangle DEF is $\frac{3}{2}$ or 1.5 because all lengths in triangle DEF are 1.5 times as long as the corresponding lengths in triangle ABC.

2.4 Comparing Polygons $ABCD$ and $PQRS$

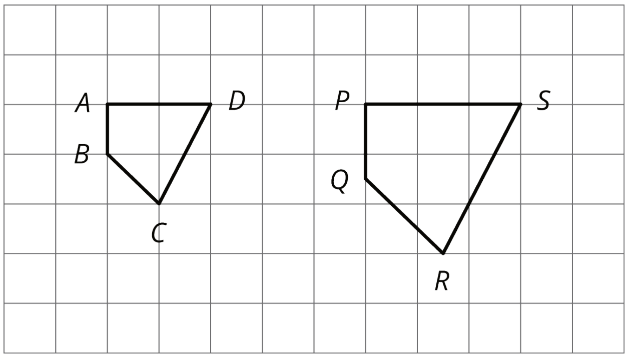
Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Polygon $PQRS$ is a scaled copy of polygon $ABCD$.



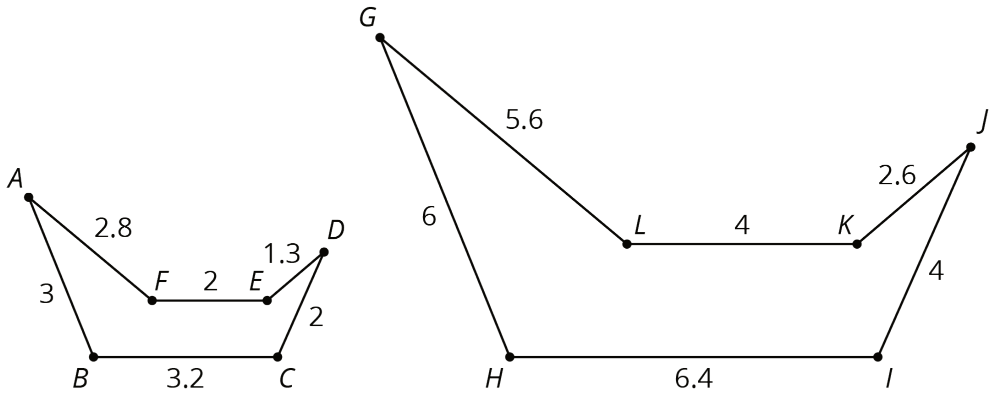
1. Name the angle in the scaled copy that corresponds to angle ABC.
2. Name the segment in the scaled copy that corresponds to segment AD.
3. What is the scale factor from polygon $ABCD$ to polygon $PQRS$?

Student Response

1. Angle PQR corresponds to angle ABC.
2. Segment PS corresponds to segment AD.
3. The scale factor is $\frac{3}{2}$ since $PS = 3$ and $AD = 2$.

Student Lesson Summary

A figure and its scaled copy have **corresponding parts**, or parts that are in the same position in relation to the rest of each figure. These parts could be points, segments, or angles. For example, Polygon 2 is a scaled copy of Polygon 1.



Polygon 1 Polygon 2

- Each point in Polygon 1 has a *corresponding point* in Polygon 2.
  For example, point B corresponds to point H and point C corresponds to point I.
- Each segment in Polygon 1 has a *corresponding segment* in Polygon 2.
  For example, segment AF corresponds to segment GL.
- Each angle in Polygon 1 also has a *corresponding angle* in Polygon 2.
  For example, angle DEF corresponds to angle JKL.

The **scale factor** between Polygon 1 and Polygon 2 is 2, because all of the lengths in Polygon 2 are 2 times the corresponding lengths in Polygon 1. The angle measures in Polygon 2 are the same as the corresponding angle measures in Polygon 1. For example, the measure of angle JKL is the same as the measure of angle DEF.

Glossary

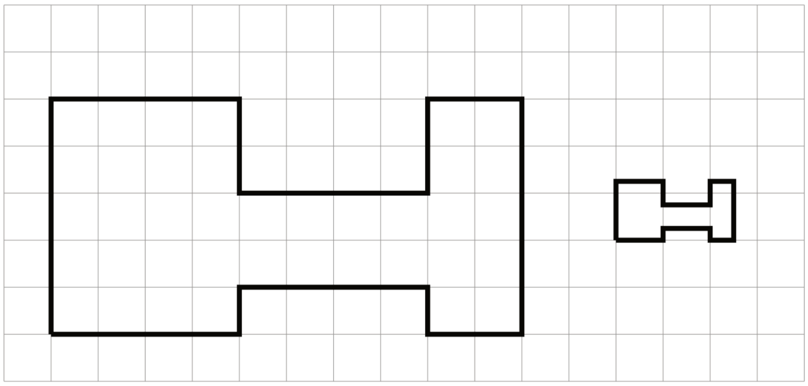
- corresponding
- scale factor

Lesson 2 Practice Problems

Problem 1

Statement

The second H-shaped polygon is a scaled copy of the first.

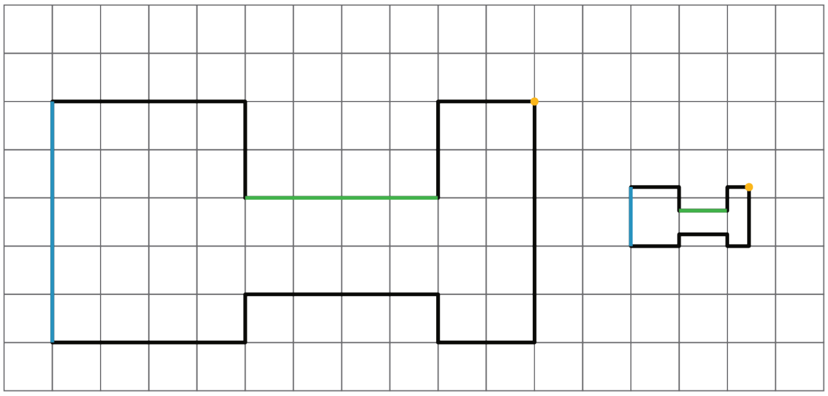


a. Show one pair of corresponding points and two pairs of corresponding sides in the original polygon and its copy. Consider using colored pencils to highlight corresponding parts or labeling some of the vertices.

b. What scale factor takes the original polygon to its smaller copy? Explain or show your reasoning.

Solution

a. Answers vary. Sample markings:



b. $\frac{1}{4}$ or 0.25. Sample explanation: The sides that are 4 units long in the original polygon are 1 unit long in the copy, which is one fourth of the original length.

Problem 2

Statement

Figure B is a scaled copy of Figure A. Select **all** of the statements that must be true:

A. Figure B is larger than Figure A.

B. Figure B has the same number of edges as Figure A.

C. Figure B has the same perimeter as Figure A.

D. Figure B has the same number of angles as Figure A.

E. Figure B has angles with the same measures as Figure A.

Solution

["B", "D", "E"]

Problem 3

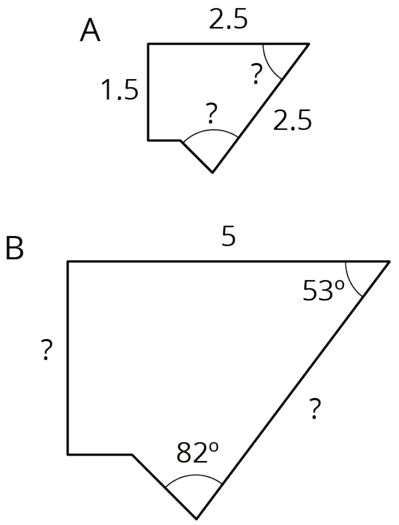
Statement

Polygon B is a scaled copy of Polygon A.

a. What is the scale factor from Polygon A to Polygon B? Explain your reasoning.

b. Find the missing length of each side marked with ? in Polygon B.

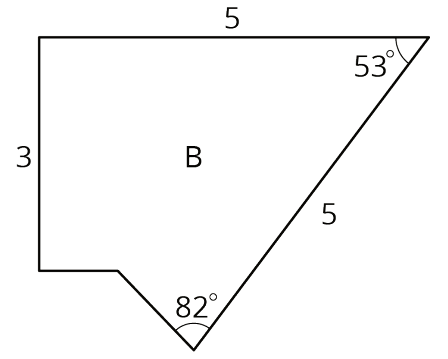
c. Determine the measure of each angle marked with ? in Polygon A.



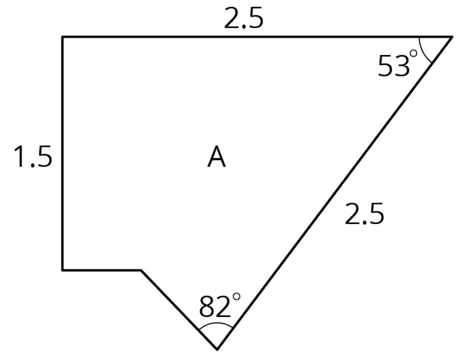
Solution

a. 2 because the top horizontal side has length 2.5 units in Polygon A and 5 units in Polygon B

b. All sides scale by the same factor of 2, so the side that is 2.5 units in Polygon A is 5 units in the copy, and the 1.5-unit-long one is 3 units in the copy



c. 53° and 82° because scaled copies have the same corresponding angles



Problem 4

Statement

Complete each equation with a number that makes it true.

a. $8 \cdot$ ______ $= 40$

b. $8 +$ ______ $= 40$

c. $21 \div$ ______ $= 7$

d. $21 -$ ______ $= 7$

e. $21 \cdot$ ______ $= 7$

Solution

a. 5

b. 32

c. 3

d. 14

e. $\frac{1}{3}$