Lesson 15: Equivalent Exponential Expressions

Goals

- Describe (orally) the values that result from evaluating expressions in which a fraction is raised to a power.
- Determine whether a given value is a solution to an equation that includes an exponent.
- Evaluate expressions that have a variable, an exponent, and one other operation for a given value of the variable, carrying out the operations in the conventional order.

Learning Targets

- I can find solutions to equations with exponents in a list of numbers.
- I can replace a variable with a number in an expression with exponents and operations and use the correct order to evaluate the expression.

Lesson Narrative

In this lesson, students encounter expressions and equations with variables that also involve exponents. Students first evaluate expressions for given values of their variables. They learn that multiplication can be expressed without a dot or other symbol by placing a number, known as a coefficient, next to a variable or variable expression. In the next activity, students are presented with equations that contain a variable. They engage in MP7 by considering the structure of the equations and apply their understanding of exponents and operations to select a number from a list that, when replaced for the variable, makes the equation true. That number is a solution of the equation.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.
- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

Student Learning Goals

Let's investigate expressions with variables and exponents.

15.1 Up or Down?

Warm Up: 10 minutes

The purpose of this warm-up is for students to take two numbers to different powers and look for patterns. One number is a whole number and the other is a fraction that is the reciprocal of the whole number. Some students may notice they do not need to multiply $\frac{1}{3}$ after they complete the column for 3 because it is the reciprocal. This is a helpful pattern for students to notice, but also ask these students if their answer makes sense to ensure they understand the product is getting smaller as they multiply by further factors of $\frac{1}{3}$. Aside from the presence of exponents, these observations are largely a review of work from grade 5.

As students complete the table, monitor and select students who can describe some of the following patterns:

- The products in the 3 column increase in value as the exponent increases.
- The products in the $\frac{1}{3}$ column decrease in value as the exponent increases.
- The products in the $\frac{1}{3}$ column are reciprocals of the products in the corresponding 3 column.

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Launch

Give students 2 minutes of quiet work time, followed by a whole-group discussion.

Anticipated Misconceptions

Some students may need to sketch a diagram to help them find a fraction of another fraction.

Student Task Statement

Find the values of 3^x and $\left(\frac{1}{3}\right)^x$ for different values of x. What patterns do you notice?

x	3^x	$\left(\frac{1}{3}\right)^x$
1		
2		
3		
4		

Student Response

x	3^x	$\left(\frac{1}{3}\right)^x$
1	3	$\frac{1}{3}$
2	9	$\frac{1}{9}$
3	27	$\frac{1}{27}$
4	81	$\frac{1}{81}$

Many observations are possible. Possible responses:

- The products in the 3 column increase in value as the exponent increases.
- The products in the 3 column are multiplied by 3 each time you go down a row.
- The products in the $\frac{1}{3}$ column decrease in value as the exponent increases.
- The products in the $\frac{1}{3}$ column are multiplied by $\frac{1}{3}$ each time you go down a row.
- The products in the $\frac{1}{3}$ column are reciprocals of the products in the corresponding 3 column.

Activity Synthesis

Ask students to share responses to complete the table. Record and display the responses for all to see. Ask selected students to share the patterns they noticed in the table and ask others to explain why they think these patterns happen. If the following ideas do not arise from the students in the conversation, bring them to students' attention:

- The products in the 3 column increase in value as the exponent increases.
- The products in the 3 column are multiplied by 3 each time you go down a row.
- The products in the $\frac{1}{3}$ column decrease in value as the exponent increases.

- The products in the $\frac{1}{3}$ column are multiplied by $\frac{1}{3}$ each time you go down a row.
- The products in the $\frac{1}{3}$ column are reciprocals of the products in the corresponding 3 column.

15.2 What's the Value?

10 minutes

In this activity, students first encounter exponential expressions with variables.

Addressing

- 6.EE.A.1
- 6.EE.A.2.c

Instructional Routines

- MLR8: Discussion Supports

Launch

Recall with students that when we write $6x^2$, we mean to multiply 6 by the result of x^2. Remind them that the number part of such a product is called the *coefficient* of the expression, so in this example, 6 is the coefficient of x^2.

Give students 5 minutes of quiet work time to evaluate the expressions. Follow with whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. After students have evaluated the first 2-3 expressions, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students may use the wrong order when evaluating expressions, such as in multiplying by 3 first in $3x^2$ and then squaring. Have them recall the last lesson where they practiced applying the agreed on order of operations in numeric expressions.

Students may interpret an expression like $3x$ as meaning 3 next to the digit x instead of as multiplication. For example, students may think that $3x$ means 35 instead of 15 when $x = 5$. Discuss how the shorthand notation of coefficients next to variables with no symbols between them tells us to multiply, and that it will simplify future work with expressions and equations.

Student Task Statement

Evaluate each expression for the given value of x.

1. $3x^2$ when x is 10
2. $3x^2$ when x is $\frac{1}{9}$
3. $\frac{x^3}{4}$ when x is 4
4. $\frac{x^3}{4}$ when x is $\frac{1}{2}$
5. $9 + x^7$ when x is 1
6. $9 + x^7$ when x is $\frac{1}{2}$

Student Response

1. 300
2. $\frac{1}{27}$
3. 16
4. $\frac{1}{32}$
5. 10
6. $9\frac{1}{128}$

Activity Synthesis

The purpose of the discussion is to ensure that students understand how to evaluate expressions with variables for a given value of the variable. It is also an opportunity for students to practice interpreting and using vocabulary like *coefficient, variable, power,* and *exponent*.

Some guiding questions:

- "In each expression, what is the coefficient?" (3, 3, $\frac{1}{4}$, $\frac{1}{4}$, 1, 1.)
- "Choose one of the expressions. Describe the steps that you carried out to evaluate the expression." (Sample response: I rewrote $9 + x^7$ as $9 + \left(\frac{1}{2}\right)^7$. I needed to first evaluate the exponent. I knew that $\left(\frac{1}{2}\right)^7$ means $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$, which is $\frac{1}{128}$. So then I wrote $9 + \frac{1}{128}$, which can also be written $9\frac{1}{128}$.
- "How is evaluating the expressions when x is a fraction similar to when x is a whole number? How is it different?" (It's similar because you're still just multiplying x by itself a certain number of times. It's different because multiplying a fraction by a fraction is a bit more complicated than multiplying a whole number by itself.)

Access for English Language Learners

Speaking, Writing: MLR8 Discussion supports. Use this to amplify mathematical uses of language to communicate about how to evaluate exponential expressions. Revoice the term "coefficient" during student discussion and press for detail when identifying the coefficients and the order in which they are multiplied in each expression.
Design Principle(s): Support sense-making

15.3 Exponent Experimentation

15 minutes
In this activity, students continue their work with exponential expressions and recall what is meant by a *solution* to an equation as they look to replace a variable with a number that makes two expressions equivalent. Note that some of the equations also have solutions that are negative; however, since operations on negative numbers are not part of grade 6 standards, students are only expected to consider positive values in this task. This activity addresses student understanding of the meaning of the equal sign as one that supports work in algebra, namely, that the expressions on either side have the same value.

Addressing

- 6.EE.A.2.c
- 6.EE.B.5

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Ask students to close their books or devices. Display the equation $x^2 = 100$ and discuss what it would mean to find a *solution* to the equation. Remind students that a *solution* is a value for x that makes the equation true. Discuss why 10 is a solution, and why 50 is not a solution.

Give students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about exponential expressions and finding a solution to an equation. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Find a solution to each equation in the list. (Numbers in the list may be a solution to more than one equation, and not all numbers in the list will be used.)

1. $64 = x^2$
2. $64 = x^3$
3. $2^x = 32$
4. $x = \left(\frac{2}{5}\right)^3$
5. $\frac{16}{9} = x^2$
6. $2 \cdot 2^5 = 2^x$
7. $2x = 2^4$
8. $4^3 = 8^x$

List:

$\frac{8}{125}$	$\frac{6}{15}$	$\frac{5}{8}$	$\frac{8}{9}$	1	$\frac{4}{3}$	2	3	4	5	6	8

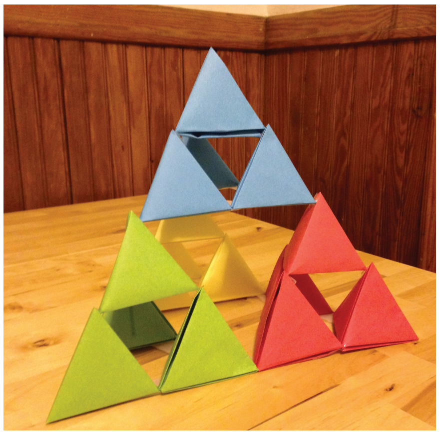
Student Response

1. 8
2. 4
3. 5
4. $\frac{8}{125}$
5. $\frac{4}{3}$
6. 6
7. 8
8. 2

Are You Ready for More?

This fractal is called a Sierpinski Tetrahedron. A tetrahedron is a polyhedron that has four faces. (The plural of tetrahedron is tetrahedra.)

The small tetrahedra form four medium-sized tetrahedra: blue, red, yellow, and green. The medium-sized tetrahedra form one large tetrahedron.



1. How many small faces does this fractal have? Be sure to include faces you can't see. Try to find a way to figure this out so that you don't have to count every face.
2. How many small tetrahedra are in the bottom layer, touching the table?
3. To make an even bigger version of this fractal, you could take four fractals like the one pictured and put them together. Explain where you would attach the fractals to make a bigger tetrahedron.
4. How many small faces would this bigger fractal have? How many small tetrahedra would be in the bottom layer?
5. What other patterns can you find?

Student Response

1. 64 faces. Each small tetrahedron has 4 faces, and there are 16 small tetrahedra in the entire fractal. $4 \cdot 16 = 64$.
2. 9 tetrahedra
3. In the picture, we see four mid-sized tetrahedra. There is a mid-sized tetrahedron at the top. At each of the three bottom vertices the apex of a mid-sized tetrahedron is attached. We can follow this pattern with the large tetrahedra—attach the apex of a large tetrahedron to each of three bottom vertices of the kite in the picture.
4. 256 small faces and 27 tetrahedra in the bottom layer. Since we've quadrupled the number of small tetrahedra, the number of faces also quadruples. The new bottom layer will contain three copies of the bottom layer of the fractal pictured. Since the fractal we see has 9 tetrahedra in the bottom layer, the new kite should have 3 times more tetrahedra.
5. Answers vary.

Activity Synthesis

The discussion should focus on the meaning of a solution to an equation, the meaning of the equal sign, and how the meaning of exponents can help find solutions. Discussion:

- Explain what the equal sign in these equations tell us about the expressions on either side.
- Describe your strategy for finding a number that makes each equation true.
- Were there any equations that needed a different approach?
- What was your strategy when x was the exponent?
- Compare your strategy for the first question to your strategy for the last question.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Present an incorrect solution based on a common misconception about solving equations with exponents. For example, "The solution to the equation $x = (\frac{2}{5})^3$ is $\frac{6}{15}$ because $(\frac{2}{5})^3$ is equivalent to $\frac{2}{5} \cdot 3$, which is $\frac{6}{15}$." Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who clarify the meaning of exponents and the equal sign. For example, $(\frac{2}{5})^3$ means $\frac{2}{5} \cdot \frac{2}{5} \cdot \frac{2}{5}$, which is not equivalent to $\frac{2}{5} \cdot 3$. This routine will engage students in meta-awareness as they critique and correct the language used to find solutions to equations with exponents.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students to reflect on their thinking in the activities they completed. Some questions to consider:

- "How did you make use of the meaning of operations and exponents in each activity?"
- "How did thinking about the meaning of operations and exponents help to find the value of x in the activity with the equations?"
- "Was there anything you learned in the first activity that helped you with the second activity?"
- "What have you noticed about a number less than 1 raised to a power? How does this compare to a number greater than 1 raised to a power?"

15.4 True Statements

Cool Down: 5 minutes

Addressing

- 6.EE.A.1
- 6.EE.B.5

Student Task Statement

Match each equation to a solution.

1. $2^x = 64$
2. $x = \left(\frac{2}{5}\right)^3$
3. $3 \cdot \left(3^4\right) = 3^x$
4. $\frac{16}{25} = x^2$

- $\frac{8}{125}$
- $\frac{4}{5}$
- 5
- 6

Student Response

1. 6
2. $\frac{8}{125}$
3. 5
4. $\frac{4}{5}$

Student Lesson Summary

In this lesson, we saw expressions that used the letter x as a variable. We evaluated these expressions for different values of x.

- To evaluate the expression $2x^3$ when x is 5, we replace the letter x with 5 to get $2 \cdot 5^3$. This is equal to $2 \cdot 125$ or just 250. So the value of $2x^3$ is 250 when x is 5.
- To evaluate $\frac{x^2}{8}$ when x is 4, we replace the letter x with 4 to get $\frac{4^2}{8} = \frac{16}{8}$, which equals 2. So $\frac{x^2}{8}$ has a value of 2 when x is 4.

We also saw equations with the variable x and had to decide what value of x would make the equation true.

- Suppose we have an equation $10 \cdot 3^x = 90$ and a list of possible solutions: $1, 2, 3, 9, 11$. The only value of x that makes the equation true is 2 because $10 \cdot 3^2 = 10 \cdot 3 \cdot 3$, which equals 90. So 2 is the solution to the equation.

Lesson 15 Practice Problems

Problem 1

Statement

Evaluate each expression if $x = 3$.

a. 2^x

b. x^2

c. 1^x

d. x^1

e. $\left(\frac{1}{2}\right)^x$

Solution

a. 8

b. 9

c. 1

d. 3

e. $\frac{1}{8}$

Problem 2

Statement

Evaluate each expression for the given value of each variable.

a. $2 + x^3$, x is 3

b. x^2, x is $\frac{1}{2}$

c. $3x^2 + y$, x is 5 y is 3

d. $10y + x^2$, x is 6 y is 4

Solution

a. 29

b. $\frac{1}{4}$

c. 78

d. 76

Problem 3

Statement

Decide if the expressions have the same value. If not, determine which expression has the larger value.

a. 2^3 and 3^2

b. 1^{31} and 31^1

c. 4^2 and 2^4

d. $\left(\frac{1}{2}\right)^3$ and $\left(\frac{1}{3}\right)^2$

Solution

a. Not equal. 3^2 has the larger value, because $2^3 = 8$ and $3^2 = 9$.

b. Not equal. 31^1 has the larger value, because $1^{31} = 1$ and $31^1 = 31$.

c. Equal. They both have 16 as their value.

d. Not equal. $\left(\frac{1}{2}\right)^3$, because $\left(\frac{1}{2}\right)^3 = \frac{1}{8}$ and $\left(\frac{1}{3}\right)^2 = \frac{1}{9}$ and $\frac{1}{8} > \frac{1}{9}$.

Problem 4

Statement

Match each equation to its solution.

A. $7 + x^2 = 16$

B. $5 - x^2 = 1$

C. $2 \cdot 2^3 = 2^x$

D. $\frac{3^4}{3^x} = 27$

1. $x = 1$

2. $x = 2$

3. $x = 3$

4. $x = 4$

Solution

- A: 3
- B: 2
- C: 4
- D: 1

Problem 5

Statement

An adult pass at the amusement park costs 1.6 times as much as a child’s pass.

a. How many dollars does an adult pass cost if a child’s pass costs:

$5? $10? w dollars?

b. A child’s pass costs $15. How many dollars does an adult pass cost?

Solution

a. 8 dollars ($1.6 \cdot 5 = 8$), 16 dollars, ($1.6 \cdot 10 = 16$), $1.6w$ dollars

b. 24 dollars ($1.6 \cdot 15 = 24$)

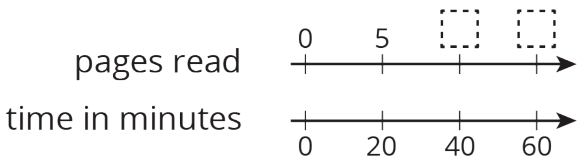
(From Unit 6, Lesson 6.)

Problem 6

Statement

Jada reads 5 pages every 20 minutes. At this rate, how many pages can she read in 1 hour?

- Use a double number line to find the answer.



- Use a table to find the answer.

pages read	time in minutes
5	20

Which strategy do you think is better, and why?

Solution

- 15 pages. The missing labels should be 10 and 15.
- Answers vary. Sample responses:

pages read	time in minutes
5	20
0.25	1
15	60

pages read	time in minutes
5	20
10	40
15	60

Answers vary. Sample response: The table is more efficient, because I can skip values.

(From Unit 2, Lesson 14.)