Lesson 14: Solving Percentage Problems

Goals

- Choose and create a tape diagram, double number line diagram, or table to solve problems involving percentages and explain (orally) the solution method.
- Determine what information is needed to solve a problem involving percentages. Ask questions to elicit that information.

Learning Targets

- I can choose and create diagrams to help me solve problems about percentages.

Lesson Narrative

In previous lessons, students saw that a percentage is a rate per 100. They were provided with double number line diagrams to develop this understanding and to solve problems involving percentages. In this lesson, students solve similar problems but with less support. Because double number lines are not provided, students have opportunities to choose approaches that seem appropriate. Drawing a double number line is still a good strategy, but students may opt for tables or even more abbreviated reasoning methods.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.
- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR4: Information Gap Cards
- MLR7: Compare and Connect
- MLR8: Discussion Supports

- Number Talk

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

You will need the Info Gap: Music Devices blackline master for this lesson. Make 1 copy for every 4 students, and cut them up ahead of time.

Student Learning Goals

Let's solve more percentage problems.

14.1 Number Talk: Multiplication with Decimals

Warm Up: 10 minutes
This number talk encourages students to rely on what they know about structure, patterns, decimal multiplication, and properties of operations to solve a problem mentally. Only two problems are given here so there is time to share many strategies and make connections between them.

Building On

- 5.NBT.B.7

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem, and ask students to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find the products mentally.

$6 \cdot (0.8) \cdot 2$

$(4.5) \cdot (0.6) \cdot 4$

Student Response

- 9.6. Possible strategies: $6 \cdot 2 = 12$ and $\frac{8}{10} \cdot 12 = 9.6$ (or $6 \cdot (0.8) = 4.8$ and $(4.8) \cdot 2 = 9.6$)
- 10.8. Possible strategies: $(4.5) \cdot 4 = 18$ and $\frac{6}{10} \cdot 18 = 10.8$ (or $(0.6) \cdot 4 = 2.4$ and $(2.4) \cdot (4.5) = 10.8$)

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. If not mentioned by students, ask if or how the given factors impacted their strategy choice. To involve more students in the conversation, consider asking:

- Who can restate ___'s reasoning in a different way?
- Did anyone solve the problem the same way but would explain it differently?
- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

14.2 Coupons

10 minutes

In this activity, students solve percentage problems in the context of shopping. Students consider how discounts described as percentages translate into reduced prices and the other way around. (In other words, they find A and C where $A\%$ of B is C). In each problem, students need to first determine what value is associated with 100% and reason accordingly.

Students may choose to use double number lines or tables or simply reason without using a particular representation. For instance, to find 10% of \$15 they may first find 50% by dividing \$15 by 2, and then divide the resulting \$7.50 by 5 to obtain 10% of \$15. Those who have internalized the structure of percentages may be able to multiply or divide even more efficiently (e.g., $15 \cdot \frac{1}{10} = 1.5$,

or $15 \div 10 = 1.5$). These strategies will be investigated more in the next lesson. For now, encourage students to also explain their reasoning with a double number line or table.

As students work, identify students who use different strategies so that they can share later.

Building On

- 6.RP.A.3

Addressing

- 6.RP.A.3.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Display some coupons for all to see. Point out that some coupons specify amounts to be taken off in dollars (e.g., $5 off) and some specify percentages (e.g.,10% off). Tell students that they will solve a couple of shopping problems that involve discounts.

Give students quiet think time to complete the activity and then time to share their explanation with a partner.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because...," "I noticed _____ so I...," and "How did you get...?"
Supports accessibility for: Language; Social-emotional skills

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Anticipated Misconceptions

Since the first question asks students to find the dollar amount, some students may think that $6 is the answer to the second question and not realize that it is asking them to find the percentage. Also, some students may try to find the sale price on the first question and the percentage of the sale price on the second question, instead of the discount and the percentage of the discount. Encourage them to revisit the questions or clarify what the questions ask.

Student Task Statement

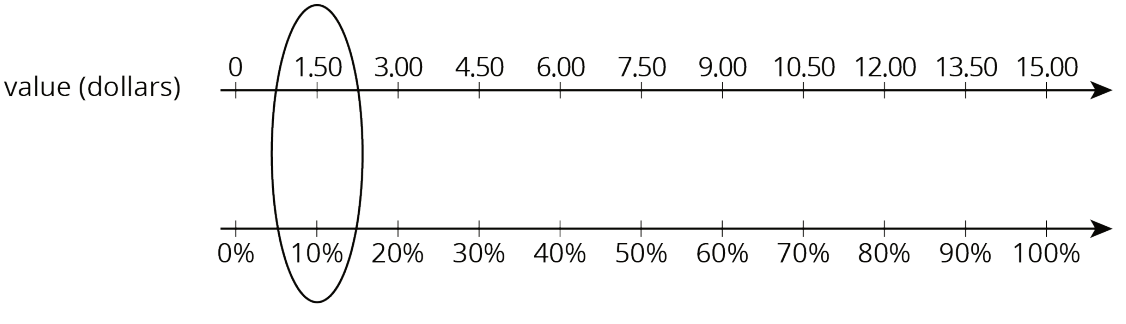
Han and Clare go shopping, and they each have a coupon. Answer each question and show your reasoning.

1. Han buys an item with a normal price of $15, and uses a 10% off coupon. How much does he save by using the coupon?

2. Clare buys an item with a normal price of $24, but saves $6 by using a coupon. For what percentage off is this coupon?

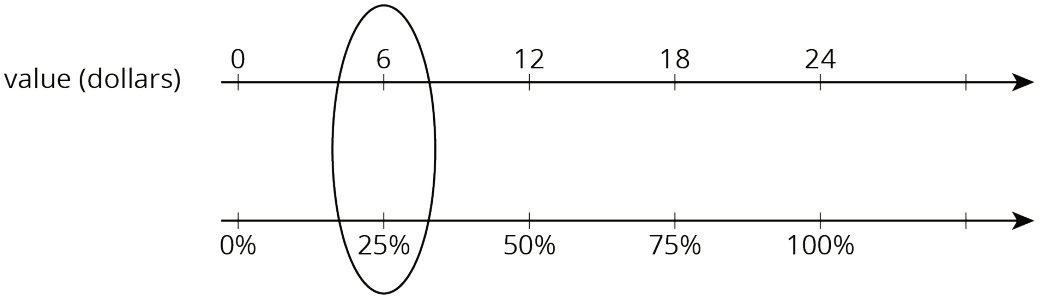
Student Response

1. $1.50. Possible strategies: $\frac{10}{100} \cdot 15 = 1.50$ or $(0.10) \cdot 15 = 1.50$



value (dollars)	percentage
15	100
1.5	10

2. 25%. Possible strategies: $6 \div 24 = \frac{25}{100}$ or $6 \div 24 = 0.25$



value (dollars)	percentage
24	100
6	25

Are You Ready for More?

Clare paid full price for an item. Han bought the same item for 80% of the full price. Clare said, "I can't believe I paid 125% of what you paid, Han!" Is what she said true? Explain.

Student Response

Yes. Han paid 80% or $\frac{4}{5}$ what Clare paid. So Clare paid $\frac{5}{4}$ or 125% of what Han paid.

Activity Synthesis

Select students who used different representations: first a tape diagram, then a double number line or a table (or both, time permitting). As students explain, illustrate and display those representations for all to see. If no students mention using a double number line or a table, demonstrate at least one of these methods. When discussing double number lines (or tables), ask students if the same double number line (or table) could be used to solve both problems and discuss why not. Emphasize the fact that two separate double number lines (or tables) are necessary because the value for 100% is different in each case.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students share how they made sense of the first question, make sure you hear from students with different strategies for finding the dollar amount Han saves by using the coupon. Some students may find 10% of $15 by multiplying 15 by $\frac{1}{10}$ or dividing 15 by 10. Others may draw a double number line where $15 corresponds with 100% and figure out the dollar amount that corresponds with 10%. Others may use a table and reason that 10% is $\frac{1}{10}$ of 100%, so the amount Han saves must be $\frac{1}{10}$ of $15. Encourage students to make comparisons and connections between the various representations of the situation. Ask questions such as, "What is especially clear in this representation?" and "Where do you see the product of 15 and $\frac{1}{10}$ represented in the diagram?" This will foster students' meta-awareness and support constructive conversations as they compare and connect the various ways to find a percentage of a quantity.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

14.3 Info Gap: Music Devices

20 minutes
In this info gap activity, students find both A and C (where $A\%$ of B is C) in the context of buying a music device. The value of B is different in each of the two questions about the music device, so students who choose to draw diagrams or tables need to draw two. When answering the second question—expressing $24 as a percentage of $25—students may notice that drawing a complete double number line diagram with all 25 tick marks is rather time consuming. Encourage students to look for and make use of any noticeable structure (MP7) or pattern (MP8) to help them solve more efficiently.

Some students may, for example, see that if $25 corresponds to 100%, then each dollar—and thus each tick mark—is $100 \div 25$ or 4. They can then bypass drawing the rest of the tick marks and simply multiply $4 \cdot 24$ to obtain 96. Some may notice that $24 is $1 away from $25 and simply subtract the corresponding percentage from 100% ($100 - 4 = 96$). A table provides another efficient structure for reasoning about this. Monitor for students who use different strategies.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Here is the text of the cards for reference and planning:

Info Gap: Music Devices
Problem Card 1

A store sells 3 different music devices: Device A, Device B, and Device C.

1. Which of the devices can Jada afford?
2. What percentage of the money needed for Device B does she have?

Info Gap: Music Devices
Data Card 1

- Device A costs $15.
- Device B costs $25.
- Device C costs $40.
- Jada has 60% of the money needed to buy Device C.

Info Gap: Music Devices
Problem Card 2

The store starts selling another music device. Jada is interested in Device D, though she does not have enough money to buy it. How much does Device D cost?

Info Gap: Music Devices
Data Card 2

- Jada has $24.
- Jada has 40% of the money needed to buy Device D.

Note: If time is short, the second set of cards can be considered optional. It would be better for students to thoroughly understand one of these problems than to rush through both of them with less understanding.

Addressing

- 6.RP.A.3.c

Instructional Routines

- MLR4: Information Gap Cards

Launch

Arrange students in groups of 2. In each group, distribute the first problem card to one student and a data card to the other student. After debriefing on the first problem, distribute the cards for the second problem, in which students switch roles.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organization

Access for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to solve problems involving percentages. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

If your teacher gives you the *data card*:

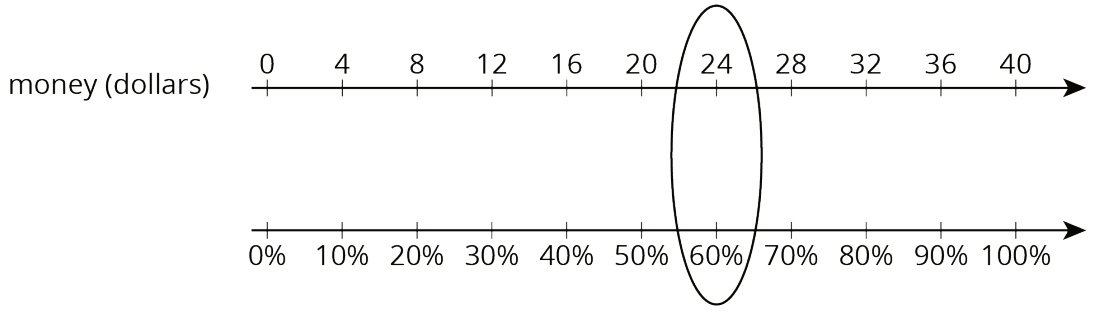
1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Student Response

For Problem Card 1:

1. Jada can only afford Device A because 60% of $40 is $24. Possible strategy:



2. Jada has 96% of the money needed for Device B. Possible strategy: Set up a table, and reason that since 100 is $25 \cdot 4$, we also multiply 24 by 4.

money (dollars)	percentage
25	100
1	4
24	96

For Problem Card 2:

$60. Sample reasoning: if $24 is 40%, then $6 is 10%. Therefore, $60 is 100%.

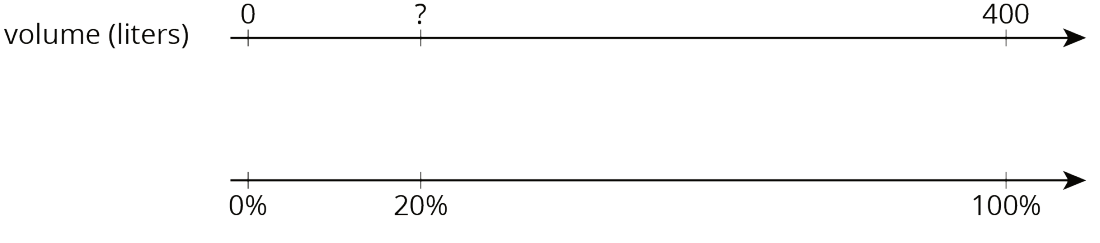
Activity Synthesis

Select students with different strategies to share their approaches to the first question, starting with less efficient methods and ending with more efficient methods. Then, ask the class to predict how the same strategies might be used to solve the second question, and how the second problem could be solved more quickly.

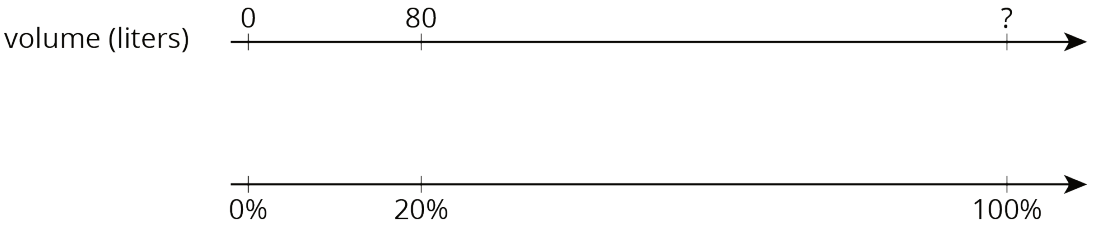
Lesson Synthesis

We know that 20% of 400 liters is 80 liters. There are three different questions we can ask:

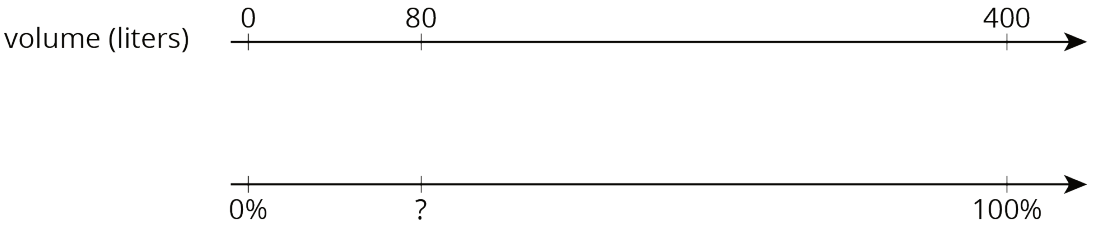
1. What is 20% of a 400 liter tank?



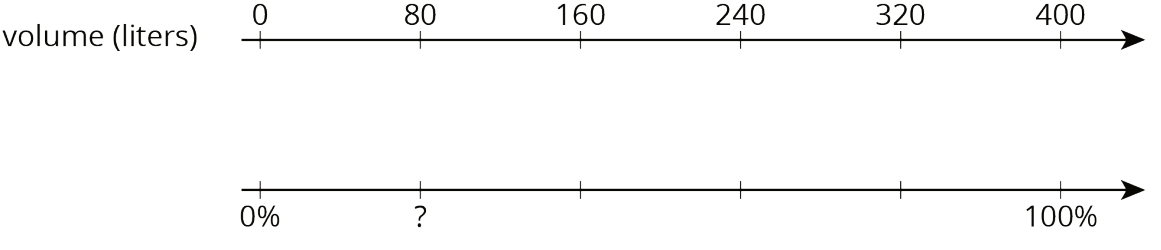
2. 20% of a full tank is 80 liters. How many liters are in a full tank?



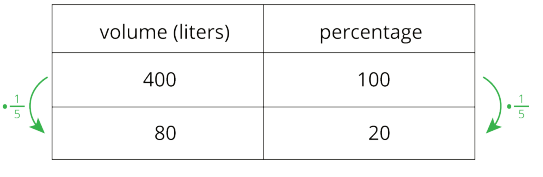
3. 80 liters is what percentage of a 400 liter tank?



We can use a double number line to answer all three of these questions. In a previous lesson, we learned how to solve the first two kinds of problems. In this lesson, we also addressed the third kind of problem. We can begin solving by asking ourselves: what fraction of 400 is 80? The answer can tell us how many tick marks to place on the number line or how to divide the segment between 0 and 100%. $80 \div 400 = \frac{1}{5}$, so we can divide segment between 0 and 100% into five equal parts, as shown here:



Since $\frac{1}{5}$ of 100 is 20, we know the percentage is 20. We can also use a table:



14.4 Walking to School

Cool Down: 5 minutes

Addressing

- 6.RP.A.3.c

Student Task Statement

It takes Jada 20 minutes to walk to school. It takes Andre 80% as long to walk to school.

How long does it take Andre to walk to school?

Student Response

16 minutes. Possible strategies:

- 10% of 20 minutes is 2 minutes. $8 \cdot 2 = 16$, so it takes 16 minutes for Andre to walk to school.
- Using a table:

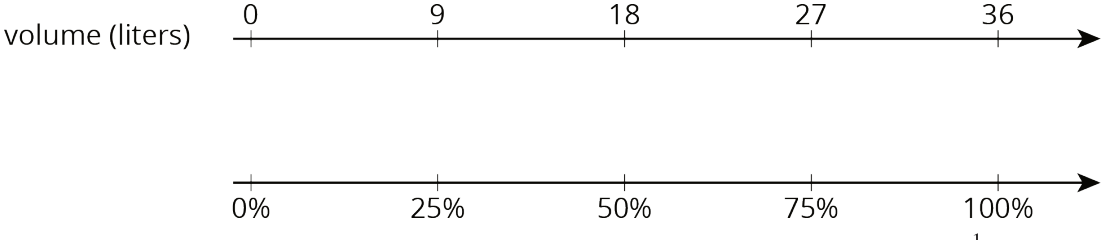
time (minutes)	percentage
20	100
2	10
16	80

Student Lesson Summary

A pot can hold 36 liters of water. What percentage of the pot is filled when it contains 9 liters of water?

Here are two different ways to solve this problem:

- Using a double number line:



We can divide the distance between 0 and 36 into four equal intervals, so 9 is $\frac{1}{4}$ of 36, or 9 is 25% of 36.

- Using a table:

volume (liters)	percentage
36	100
9	25

$\cdot \frac{1}{4}$ $\cdot \frac{1}{4}$

Lesson 14 Practice Problems

Problem 1

Statement

For each problem, explain or show your reasoning.

a. 160 is what percentage of 40?

b. 40 is 160% of what number?

c. What number is 40% of 160?

Solution

Reasoning varies. Sample responses:

a. 400%, because $4 \cdot 40 = 160$.

b. 25, because $40 \div 8 = 5$ is 20% of that number, and $5 \cdot 5 = 25$ is 100% of that number.

c. 64, because 10% of 160 is 16, and $4 \cdot 16 = 64$.

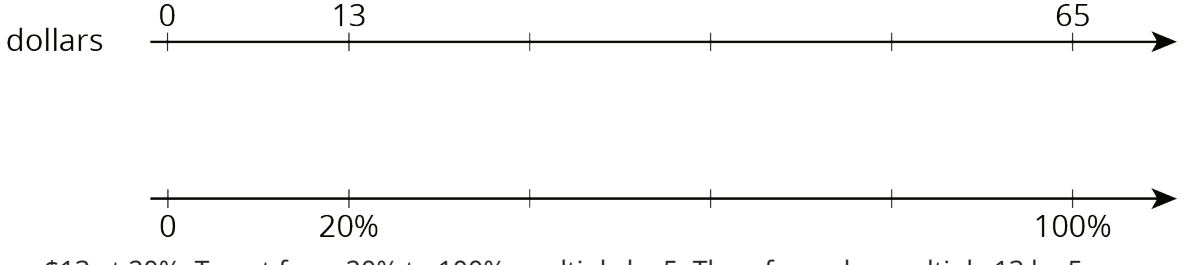
Problem 2

Statement

A store is having a 20%-off sale on all merchandise. If Mai buys one item and saves $13, what was the original price of her purchase? Explain or show your reasoning.

Solution

$65. Possible reasoning:



Place $13 at 20%. To get from 20% to 100%, multiply by 5. Therefore, also multiply 13 by 5.

Problem 3

Statement

The original price of a scarf was $16. During a store-closing sale, a shopper saved $12 on the scarf. What percentage discount did she receive? Explain or show your reasoning.

Solution

75%. Possible explanations:

- $12 \div 16 = \frac{75}{100}$ (or $12 \div 16 = 0.75$)
-

value (dollars)	percentage
16	100
12	75

Problem 4

Statement

Select **all** the expressions whose value is larger than 100.

A. 120% of 100

B. 50% of 150

C. 150% of 50

D. 20% of 800

E. 200% of 30

F. 500% of 400

G. 1% of 1,000

Solution

["A", "D", "F"]

Problem 5

Statement

An ant travels at a constant rate of 30 cm every 2 minutes.

a. At what pace does the ant travel per centimeter?

b. At what speed does the ant travel per minute?

Solution

a. The pace is $\frac{1}{15}$ of a minute per centimeter.

b. The speed is 15 centimeters per minute.

(From Unit 3, Lesson 8.)

Problem 6

Statement

Is $3\frac{1}{2}$ cups more or less than 1 liter? Explain or show your reasoning. (Note: 1 cup $\approx$ 236.6 milliliters)

Solution

Less. Explanations vary. Possible explanation:

cups	milliliters
1	236.6
0.5	118.3
3	709.8
3.5	828.1

(From Unit 3, Lesson 4.)

Problem 7

Statement

Name a unit of measurement that is about the same size as each object.

a. The distance of a doorknob from the floor is about 1 _____.

b. The thickness of a fingernail is about 1 _____.

c. The volume of a drop of honey is about 1 _____.

d. The weight or mass of a pineapple is about 1 _____.

e. The thickness of a picture book is about 1 _____.

f. The weight or mass of a buffalo is about 1 _____.

g. The volume of a flower vase is about 1 _____.

h. The weight or mass of 20 staples is about 1 _____.

i. The volume of a melon is about 1 _____.

j. The length of a piece of printer paper is about 1 _____.

Solution

a. Yard or meter

b. Millimeter

c. Milliliter

d. Kilogram or pound

e. Centimeter or inch

f. Ton

g. Cup, quart, or liter

h. Gram

i. Gallon

j. Foot

(From Unit 3, Lesson 2.)