Lesson 17: Two Related Quantities, Part 2

Goals

- Create a table, graph, and equation to represent the relationship between distance and time for an object moving at a constant speed.
- Identify (in writing) the independent and dependent variable in an equation.
- Interpret (orally and in writing) an equation that represents the relationship between distance and time for an object moving at a constant speed.

Learning Targets

- I can create tables and graphs to represent the relationship between distance and time for something moving at a constant speed.
- I can write an equation with variables to represent the relationship between distance and time for something moving at a constant speed.

Lesson Narrative

In this second lesson on representing relationships between two quantities, walking at a constant rate provides the context for writing an equation that represents the relationship. Students use and make connections between tables, graphs, and equations that represent the relationship between time and distance. They use their representations to compare rates and consider how each of the representations would change if the independent and dependent variables were switched.

Alignments

Addressing

- 6.EE.C.9: Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. For example, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation $d = 65t$ to represent the relationship between distance and time.
- 6.RP.A.3.a: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

- MLR7: Compare and Connect
- Notice and Wonder

Required Materials

Colored pencils

> ### Student Learning Goals
> Let’s use equations and graphs to describe stories with constant speed.

17.1 Walking to the Library

Warm Up: 10 minutes
Students reason about the relationship between distance, rate, and time to solve a problem. The purpose is to reactivate what students know about constant speed contexts, where constant speed is represented by a set of equivalent ratios associating distance traveled and elapsed time. In the longer activity that follows, students represent a constant speed context using a table, equations, and graphs.

As students work, watch for different representations used (particularly tables) as well as for students who calculate each person's speed in miles per hour or each person's pace in hours per mile.

Addressing

- 6.EE.C.9
- 6.RP.A.3.a
- 6.RP.A.3.b

Instructional Routines

- Notice and Wonder

Launch

Consider starting off by having students close their books or devices, and display the following situation. Ask students, "What do you notice?" "What do you wonder?"

Lin and Jada each leave school at 3 p.m. to walk to the library. They each walk at a steady rate. When do they arrive?

Give them 1 minute to think of at least one thing they notice and one thing they wonder.

Students might notice that they leave at 3 p.m., that they walk at a steady rate (also called a constant speed), and that there is not enough information given to answer the question.

Students might wonder many things, but in order to answer the question, they would need to know:

- How fast do they each walk?
- How far away is the library?

Ask students to open their books or devices and use the additional information to solve the problem by any method they choose. If desired, remind students of tools that may be appropriate including double number lines or tables of equivalent ratios.

Student Task Statement

Lin and Jada each walk at a steady rate from school to the library. Lin can walk 13 miles in 5 hours, and Jada can walk 25 miles in 10 hours. They each leave school at 3:00 and walk $3\frac{1}{4}$ miles to the library. What time do they each arrive?

Student Response

Lin arrives at 4:15 and Jada arrives at 4:18. Explanations vary. Sample response:

Here is a table of equivalent ratios for Lin. Instead of $3\frac{1}{4}$ use $\frac{13}{4}$.

distance (miles)	time (hours)
13	5
1	$\frac{5}{13}$
$\frac{13}{4}$	$\frac{5}{4}$

Here is a table of equivalent ratios for Jada.

distance (miles)	time (hours)
25	10
1	$\frac{10}{25}$
$\frac{13}{4}$	$\frac{130}{100}$

To travel $3\frac{1}{4}$ miles it takes Lin $\frac{5}{4}$ or 1.25 hours, which is an hour and fifteen minutes. She arrives at 4:15.

To travel $3\frac{1}{4}$ miles it takes Jada $\frac{130}{100}$ or 1.3 hours, which is an hour and eighteen minutes. She arrives at 4:18.

Activity Synthesis

Invite students who used different representations and lines of reasoning to share. If no student mentions it, demonstrate how to represent on person's trip using a table of equivalent ratios with columns representing distance and time. Ask a student to explain how to use the table to reason about the distance traveled in 1 hour and the time it takes to travel 1 mile.

One way to reason is to notice that Lin can walk 26 miles in 10 hours, so she walks slightly faster than Jada (who can complete 25 miles in 10 hours) and should arrive a bit sooner. Both of these ways of reasoning are in preparation for the following activity.

17.2 The Walk-a-thon

25 minutes (there is a digital version of this activity)
This activity revisits the familiar context of traveling at a constant rate. Students calculate and compare the unit rates in miles per hour for three people and consider the graphs and equations that describe the distance–time relationship.

Addressing

- 6.EE.C.9
- 6.RP.A.3.a

Instructional Routines

- MLR7: Compare and Connect

Launch

Give students access to colored pencils and 5–8 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts. For example, after students have completed the table about the walk-a-thon, check-in with either select groups of students or the whole class. Invite students to share the strategies they have used so far as well as any questions they have before continuing.
Supports accessibility for: Organization; Attention

Student Task Statement

Diego, Elena, and Andre participated in a walk-a-thon to raise money for cancer research. They each walked at a constant rate, but their rates were different.

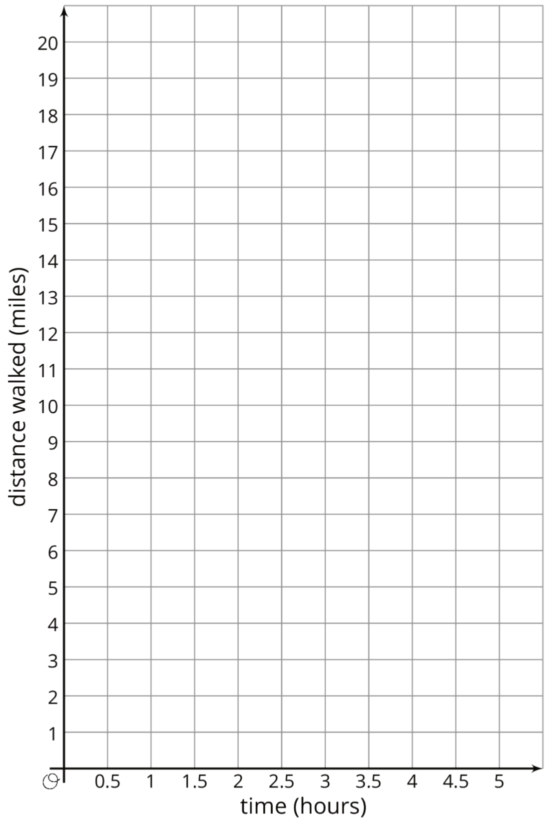
1. Complete the table to show how far each participant walked during the walk-a-thon.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1			
2	6		
	12	11	
5			17.5

2. How fast was each participant walking in miles per hour?

3. How long did it take each participant to walk one mile?

4. Graph the progress of each person in the **coordinate plane**. Use a different color for each participant.



5. Diego says that $d = 3t$ represents his walk, where d is the distance walked in miles and t is the time in hours.

   a. Explain why $d = 3t$ relates the distance Diego walked to the time it took.

b. Write two equations that relate distance and time: one for Elena and one for Andre.

6. Use the equations you wrote to predict how far each participant would walk, at their same rate, in 8 hours.

7. For Diego's equation and the equations you wrote, which is the dependent variable and which is the independent variable?

Student Response

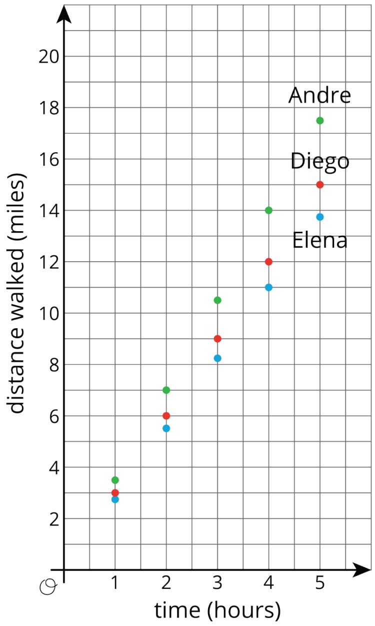
1.

time in hours	miles walked by Diego	miles walked by Elena	miles walked by Andre
1	3	2.75 or $2\frac{3}{4}$	3.5
2	6	5.5 or $5\frac{1}{2}$	7
4	12	11	14
5	15	13.75 or $13\frac{3}{4}$	17.5

2. Diego: 3 miles per hour, Elena: 2.75 miles per hour, Andre: 3.5 miles per hour

3. Diego: $\frac{1}{3}$ hour, Elena: $\frac{4}{11}$ hour, Andre: $\frac{2}{7}$ hour

4.



5. a. Answers vary. Sample response: Diego walked 3 miles in 1 hour. So you can multiply the number of hours by 3 to find the distance.

b. Elena: $d = 2.75t$, Andre: $d = 3.5t$ or equivalent

6. Diego: 24 miles, Elena: 22 miles, Andre: 28 miles
7. Answers vary. Sample response: In Diego's equation, time is the independent variable and distance is the dependent variable. If the equations look instead like $t = \frac{1}{3}d$, the distance is the independent variable and time is the dependent variable.

Are You Ready for More?

1. Two trains are traveling toward each other, on parallel tracks. Train A is moving at a constant speed of 70 miles per hour. Train B is moving at a constant speed of 50 miles per hour. The trains are initially 320 miles apart. How long will it take them to meet? One way to start thinking about this problem is to make a table. Add as many rows as you like.

	train A	train B
starting position	0 miles	320 miles
after 1 hour	70 miles	270 miles
after 2 hours		

2. How long will it take a train traveling at 120 miles per hour to go 320 miles?
3. Explain the connection between these two problems.

Student Response

1. $2\frac{2}{3}$ hours, or 2 hours and 40 minutes.
2. 2 hours and 40 minutes.
3. Since trains A and B are moving toward each other, we can add their two speeds to find the rate at which their distance decreases. 70 miles per hour + 50 miles per hour = 120 miles per hour.

Activity Synthesis

The goal of the discussion is to ensure that students understand how each of the table, graph, and equations represent the situation and how they are connected to each other. Consider asking some of the following questions:

- “How can you determine from the table who walked the fastest and slowest?”
- “How can you determine from the graph who walked the fastest and slowest?”
- “How can you determine from the equations who walked the fastest and slowest?”

- “If distance was the independent variable, how would the equations and graphs be different?”

Access for English Language Learners

Representing, Speaking, Listening: MLR7 Compare and Connect. Invite students to prepare a visual display of their table, graph, and equations that relate distance and time for each participant. As students analyze each others’ work, ask them to share what is especially clear in a particular representation. Listen for and amplify the language students use to describe how the distance traveled increases by a constant amount per hour and how this pattern can be seen on the table and graph. This will foster students’ meta-awareness and support constructive conversations as they compare and connect the tables, graphs, and equations that represent the same situation.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students to think about the different representations they used for a situation involving time, distance, and a constant rate. Invite their thoughts on which representations would be most helpful in finding unknown quantities in different situations. Ask what factors they would consider in deciding which quantity to set as the independent variable when writing an equation to describe a situation.

17.3 Interpret the Point

Cool Down: 5 minutes

Addressing

- 6.EE.C.9

Student Task Statement

During a walk-a-thon, Noah's time in hours, t, and distance in miles, d, are related by the equation $\frac{1}{3}d = t$. A graph of the equation includes the point $(12, 4)$.

1. Identify the independent variable.
2. What does the point $(12, 4)$ represent in this situation?
3. What point would represent the time it took to walk $7\frac{1}{2}$ miles?

Student Response

1. Distance, d, is the independent variable.
2. Answers vary. Sample responses: Noah can walk 12 miles in 4 hours. It takes Noah 4 hours to walk 12 miles.

3. $(7\frac{1}{2}, 2\frac{1}{2})$. $\frac{1}{3}(7\frac{1}{2}) = t$, $t = 2\frac{1}{2}$

Student Lesson Summary

Equations are very useful for solving problems with constant speeds. Here is an example.

A boat is traveling at a constant speed of 25 miles per hour.

1. How far can the boat travel in 3.25 hours?
2. How long does it take for the boat to travel 60 miles?

We can write equations to help us answer questions like these.

Let's use t to represent the time in hours and d to represent the distance in miles that the boat travels.

When we know the time and want to find the distance, we can write:

$$d = 25t$$

In this equation, if t changes, d is affected by the change, so we t is the independent variable and d is the dependent variable.

This equation can help us find d when we have any value of t. In 3.25 hours, the boat can travel $25(3.25)$ or 81.25 miles.

When we know the distance and want to find the time, we can write:

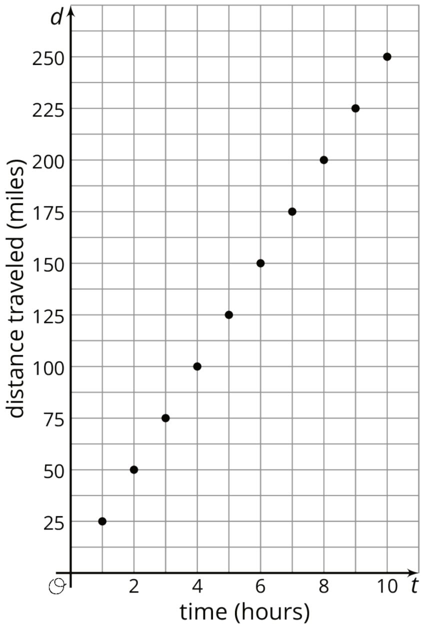
$$t = \frac{d}{25}$$

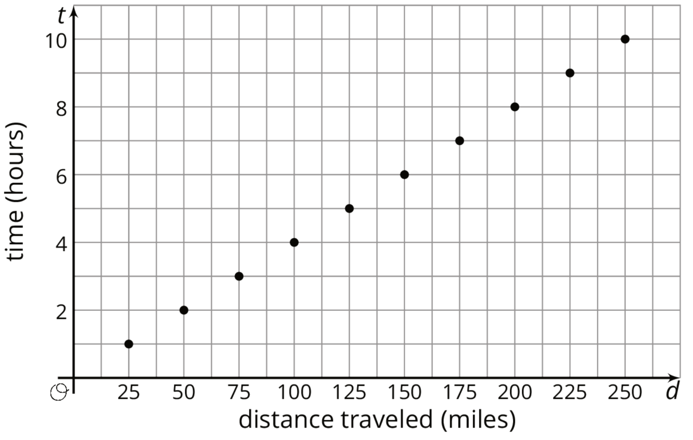
In this equation, if d changes, t is affected by the change, so we d is the independent variable and t is the dependent variable.

This equation can help us find t when for any value of d. To travel 60 miles, it will take $\frac{60}{25}$ or $2\frac{2}{5}$ hours.

These problems can also be solved using important ratio techniques such as a table of equivalent ratios. The equations are particularly valuable in this case because the answers are not round numbers or easy to quickly evaluate.

We can also graph the two equations we wrote to get a visual picture of the relationship between the two quantities:





Glossary

- coordinate plane

Lesson 17 Practice Problems

Problem 1

Statement

A car is traveling down a road at a constant speed of 50 miles per hour.

a. Complete the table with the amounts of time it takes the car to travel certain distances, or the distances traveled for certain amounts of time.

b. Write an equation that represents the distance traveled by the car, d, for an amount of time, t.

c. In your equation, which is the dependent variable and which is the independent variable?

time (hours)	distance (miles)
2	
1.5	
t	
	50
	300
	d

Solution

time (hours)	distance (miles)
2	100
1.5	75
t	$50t$
1	50
6	300
$\frac{1}{50}d$	d

a. see table

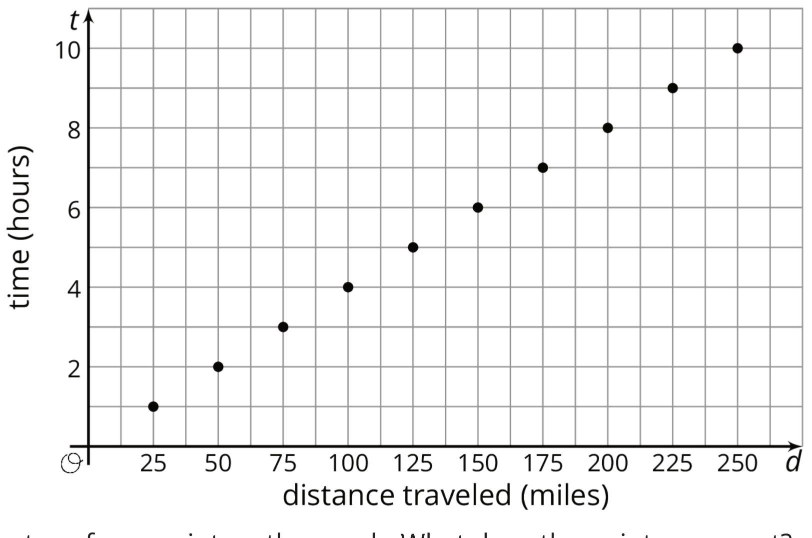
b. $d = 50t$

c. t is the independent variable and d is the dependent variable.

Problem 2

Statement

The graph represents the amount of time in hours it takes a ship to travel various distances in miles.



a. Write the coordinates of one point on the graph. What does the point represent?

b. What is the speed of the ship in miles per hour?

c. Write an equation that relates the time, t, it takes to travel a given distance, d.

Solution

a. Answers vary. Sample response: $(75, 3)$. This point represents that the ship travels 75 miles in 3 hours.

b. 25 miles per hour

c. $d = 25t$ or $t = \frac{d}{25}$

Problem 3

Statement

Find a solution to each equation in the list that follows (not all numbers will be used):

a. $2^x = 8$

b. $2^x = 2$

c. $x^2 = 100$

d. $x^2 = \frac{1}{100}$

e. $x^1 = 7$

f. $2^x \cdot 2^3 = 2^7$

g. $\frac{2^x}{2^3} = 2^5$

List: $\frac{1}{10}$ $\frac{1}{3}$ 1 2 3 4 5 7 8 10 16

Solution

a. 3

b. 1

c. 10

d. $\frac{1}{10}$

e. 7

f. 4

g. 8

(From Unit 6, Lesson 15.)

Problem 4

Statement

Select **all** the expressions that are equivalent to $5x + 30x - 15x$.

A. $5(x + 6x - 3x)$

B. $(5 + 30 - 15) \cdot x$

C. $x(5 + 30x - 15x)$

D. $5x(1 + 6 - 3)$

E. $5(x + 30x - 15x)$

Solution

["A", "B", "D"]

(From Unit 6, Lesson 11.)

Problem 5

Statement

Evaluate each expression if x is 1, y is 2, and z is 3.

a. $7x^2 - z$

b. $(x + 4)^3 - y$

c. $y(x + 3^3)$

d. $(7 - y + z)^2$

e. $0.241x + x^3$

Solution

a. 4

b. 123

c. 56

d. 64

e. 1.241

(From Unit 6, Lesson 15.)