Lesson 8: Scale Drawings and Maps

Goals

- Justify (orally and in writing) which of two objects was moving faster.
- Use a scale drawing to estimate the distance an object traveled, as well as its speed or elapsed time, and explain (orally and in writing) the solution method.

Learning Targets

- I can use a map and its scale to solve problems about traveling.

Lesson Narrative

This lesson is optional. In this lesson, students apply what they have learned about scale drawings to solve problems involving constant speed (MP1, MP2). Students are given a map with scale as well as a starting and ending point. In addition, they are either given the time the trip takes and are asked to estimate the speed or they are given the speed and asked to estimate how long the trip takes. In both cases, they need to make strategic use of the map and scale and they will need to estimate distances because the roads are not straight.

In the sixth grade, students have examined many contexts involving travel at constant speed. If a car travels at 30 mph, there is a ratio between the time of travel and the distance traveled. This can be represented in a ratio table, or on a graph, or with an equation. If d is the distance traveled in miles, and t is the amount of time in hours, then traveling at 30 mph can be represented by the equation $d = 30t$. Students may or may not use this representation as they work on the activities in this lesson. But they will gain further familiarity with this important context which they will examine in greater depth when they study ratios and proportional reasoning in grade 7, starting in the next unit.

Alignments

Building On

- 6.NS.B.2: Fluently divide multi-digit numbers using the standard algorithm.
- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.
- 7.RP.A.2.b: Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Ensure students have access to geometry toolkits.

Student Learning Goals

Let's use scale drawings to solve problems.

8.1 A Train and a Car

Warm Up: 5 minutes
This warm-up serves two purposes. It refreshes the concept of distance, rate, and time of travel from grade 6, preparing students to use scale drawings to solve speed-related problems. It also allows students to estimate decimal calculations.

Students are likely to approach the question in a few different ways. As students work, notice students using each strategy.

- By finding or estimating the speed of the train in miles per hour and comparing this to the speed of the car
- By finding the distance the car travels in 4 hours and comparing it to the distance the train travels in 4 hours

Building On

- 6.NS.B.2
- 6.RP.A.3.b

Launch

Give students 3 minutes of quiet think time. Ask students to calculate the answer mentally and to give a signal when they have an answer and explanation. Follow with a whole-class discussion.

> **Student Task Statement**
>
> Two cities are 243 miles apart.
>
> - It takes a train 4 hours to travel between the two cities at a constant speed.
> - A car travels between the two cities at a constant speed of 65 miles per hour.
>
> Which is traveling faster, the car or the train? Be prepared to explain your reasoning.

Student Response

The car is traveling faster. Sample strategy: the speed of the train in miles per hour is $243 \div 4$. This is $(240 \div 4) + (3 \div 4) = 60\frac{3}{4}$, and that's slower than the car. Alternatively, in 4 hours, the car would travel $4 \cdot 65$ or 260 miles, and that's farther than the distance between the cities. So again, the conclusion is that the car is traveling faster.

Activity Synthesis

Invite students to share their strategies. Make sure to highlight different strategies, such as calculating the train's speed from the information and calculating how far the car would travel in 4 hours.

Record and display student explanations for all to see. To involve more students in the conversation, consider asking:

- Did anyone solve the problem in a different way?
- Does anyone want to add on to _____'s strategy?
- Do you agree or disagree? Why?

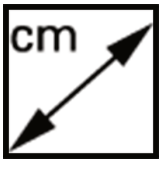
8.2 Driving on I-90

Optional: 15 minutes (there is a digital version of this activity)
Here, students use a scale and a scale drawing to answer a speed-related question. The task involves at least a couple of steps beyond finding the distance of travel and can be approached in several ways. Minimal scaffolding is given here, allowing students to model with mathematics more independently (MP4).

As students work, notice the different approaches they use to find the actual distance and to determine if the driver was speeding. Some likely variations:

- Comparing the *speed in miles per minute* (calculating the car's speed in miles per minute and converting the speed limit to miles per minute).
- Comparing the *speed in miles per hour* (finding the car's speed in miles per minute and converting it to miles per hour so it can be compared to the speed limit in miles per hour).
- Comparing the *time* it would take to travel the same distance at two different speeds (the car's and the limit).
- Comparing the *distance* traveled in the same amount of time at two different speeds (the car's and the limit).

Identify students using each method so they can share later.



For classrooms using the digital version of the activity, students will be measuring with the Distance or Length tool The tool will measure the shortest distance between two points or the length of a segment.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A.2.b

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

Tell students that they will now use a scale drawing (a map) to solve a problem about speed of travel. Survey the class on their familiarity with highway travel and speed limits. If some students are not familiar with speed limits, ask those who are to explain.

Arrange students in groups of 2 and provide access to geometry toolkits. Give students 5 minutes to work on the problem either individually or with their partner.

In the Digital Activity, students have choices about the number of points to plot along the route and whether or not they want to draw segments. Students need to pay attention to the map legend; turning on the grid helps them see that one unit on the grid is equivalent to 0.5 miles.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, draw students attention to the warm-up and remind them how they calculated the speed of the train in miles per hour. Ask students how they can use this method to calculate the speed of driver from Point A to Point B.
Supports accessibility for: Social-emotional skills; Conceptual processing

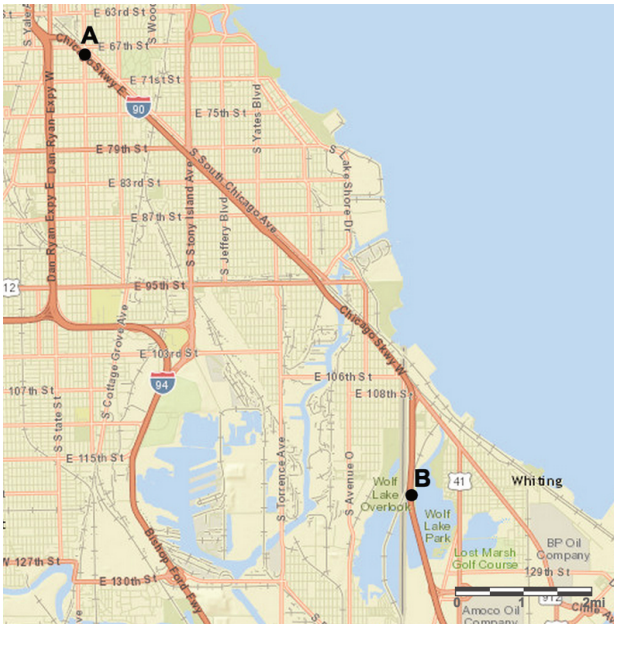
Anticipated Misconceptions

Students might not realize that they need to compare two quantities (either two speeds, two distances traveled in the same amount of time, or two durations of travel) in order to answer the question about speeding. Remind them that there are two potential scenarios here: the driver is obeying the speed limit or the driver is not obeying it.

Once students have found the distance between A and B to be about 8.5 miles, they might be inclined to divide 55 by 8.5 simply because 55 is a larger number. Using double number lines or a table to show the relationship between miles traveled and number of hours might be helpful, as might using friendlier examples of distances (e.g., "How long would it take to travel 110 miles? 11 miles?").

Student Task Statement

1. A driver is traveling at a constant speed on Interstate 90 outside Chicago. If she traveled from Point A to Point B in 8 minutes, did she obey the speed limit of 55 miles per hour? Explain your reasoning.



2. A traffic helicopter flew directly from Point A to Point B in 8 minutes. Did the helicopter travel faster or slower than the driver? Explain or show your reasoning.

Student Response

1. No, she did not. Sample explanations:
    - Using the scale and paper, I found the distance from Point A to Point B to be about 8.5 times the scale representing 1 mile, or 8.5 miles. If she traveled at 55 miles per hour, it would take about 0.15 hour or 9 minutes to travel 8.5 miles, since $8.5 \div 55 \approx 0.15$. Since she got from A to B in 8 minutes, she must have been going faster than the speed limit.
    - The distance between A and B is about 8.5 times the length of the segment representing 1 mile, so the distance is about 8.5 miles. She traveled 8.5 miles in 8 minutes, so her speed was about 1.06 miles per minute. The speed of 55 miles per hour is about 0.917 mile per minute, so the driver did not obey the speed limit.
2. The helicopter traveled slower, since the direct (straight-line) distance is shorter than the distance along the highway. Since it took the same amount of time to travel a shorter distance, the helicopter traveled more slowly.

Activity Synthesis

Ask students to indicate whether they believe the driver was speeding or not. Invite students who approached the task in different ways to share, highlighting methods that focus on:

- Calculating or estimating the speed the driver is traveling (in miles per minute or miles per hour)
- Finding how long it would take to make the trip at the speed limit
- Finding how far the driver would travel in 8 minutes going at the speed limit

Display their work or record or summarize it for all to see.

Ask students if they thought any method seems more efficient than others and why. Highlight that all the methods involved finding the distance traveled, and that the scale drawing and scale enabled us to find that distance.

One method for solving this problem which avoids the decimals that approximate fractional quantities is to observe that 55 miles per hour is the same as 55 miles in 60 minutes, so that is less than one mile per minute. So it will take more than 8.5 minutes to travel 8.5 miles at 55 miles per hour, and the driver must have been speeding.

8.3 Biking through Kansas

Optional: 10 minutes
In the previous activity, students were given a map with a scale and the amount of time it took to get from one place to another. They used this to estimate the speed of the trip. In this activity, students work with the speed and a map with scale to find the amount of time a trip will take.

The main strategy to expect is to measure the distance between the two locations on the map and use the scale to convert this to the distance between the actual cities. Then students can calculate how long it will take at 15 mph.

Addressing

- 7.G.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Tell students that they will now use a scale drawing (a map) to solve a different problem about travel, this time focusing on how long it will take. Ask students what is the farthest they have ever biked. How long did it take? Do they know someone who has biked farther or for longer? If so, how far and how long?

Keep students in the same groups. Give students 4–5 minutes of quiet work time followed by partner and whole-class discussion.

Anticipated Misconceptions

The road from Garden City to Dodge City has many twists and bends. Students may not be sure how to treat these. Tell them to make their best estimate. Measuring many small segments of the road will have the advantage that those short segments are straight but it is time consuming. A good estimate will be sufficient here.

Student Task Statement

A cyclist rides at a constant speed of 15 miles per hour. At this speed, about how long would it take the cyclist to ride from Garden City to Dodge City, Kansas?

Student Response

Answers vary. Sample response 1: Using the scale, it appears to be about 50 miles from Garden City. In 3 hours, the cyclist would ride 45 miles, and the remaining 5 miles would take $\frac{1}{3}$ of an hour or 20 minutes. It would take the cyclist about 3 hours and 20 minutes.

Sample response 2: 15 mph is 15 miles in 60 minutes or 1 mile every 4 minutes. So 4 miles take 16 minutes. The (4 mile) scale fit a little more than 12 times, so that means the trip will take a little more than $12 \cdot 16$ minutes. That's 192 minutes or 3 hours and 12 minutes.

Are You Ready for More?

Jada finds a map that says, “Note: This map is not to scale.” What do you think this means? Why is this information important?

Student Response

Answers vary. Sample response: it means that there is no one scale factor that relates distances on the map to distances in the place represented by the map. Some distances are distorted. If Jada were using her map to calculate how long it would take her to travel from one point to another on the map, her prediction may be inaccurate.

Activity Synthesis

First, have students compare answers with a partner and discuss their reasoning until they reach an agreement.

Next, invite students to share how they estimated the distance between the two cities (and how long it takes the cyclist to travel this distance). Ask students to consider the different distances students estimated the trip to be. What are some reasons for the differences? Possible explanations include:

- Measurement error

- The road is not straight and so needs to be approximated
- For students who lay out the scale over and over again to cover the distance, it is difficult to estimate the fraction of the scale at the last step

Because of these different sources of inaccuracy, reporting the distance as 50 miles is reasonable; reporting it as 52 miles would require a lot of time and measurements; and reporting it as 51.6 miles is not reasonable with the given scale and map.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students work to determine the duration of the trip from Garden City to Dodge City, look for students with different strategies for estimating the distance between the two cities. As students investigate each other's work, ask students to share what worked well in a particular approach. During this discussion, listen for any comments that make the estimation of the distance more precise. Then encourage students to make connections between the various uses of constant speed to calculate the duration of the trip. Amplify language students use to make sense of the cyclist's constant speed and how it could be represented in the map. This will support constructive conversations as students compare strategies for calculating the duration of a trip and make connections between the quantity and visual representations of constant speed on the map. *Design Principle(s): Cultivate conversation; Maximize meta-awareness*

Lesson Synthesis

A map with a scale helps estimate the distance between two places by measuring the distance on the map and using the scale to find the actual distance. Once the distance between two places is known:

- If we know how long the trip takes, we can calculate the speed by finding the quotient of the distance and the time.
- If we know the speed, we can calculate how long the trip takes by finding the quotient of the distance and the speed.

In both cases, care has to be taken regarding units. For example, if a 130-mile trip at a constant speed takes two hours, then the speed is 65 miles per hour, because $130 \div 2 = 65$. A 35-mile trip at 70 miles per hour takes $\frac{1}{2}$ hour, because $35 \div 70 = \frac{1}{2}$.

8.4 Walking Around the Botanical Garden

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Building Towards

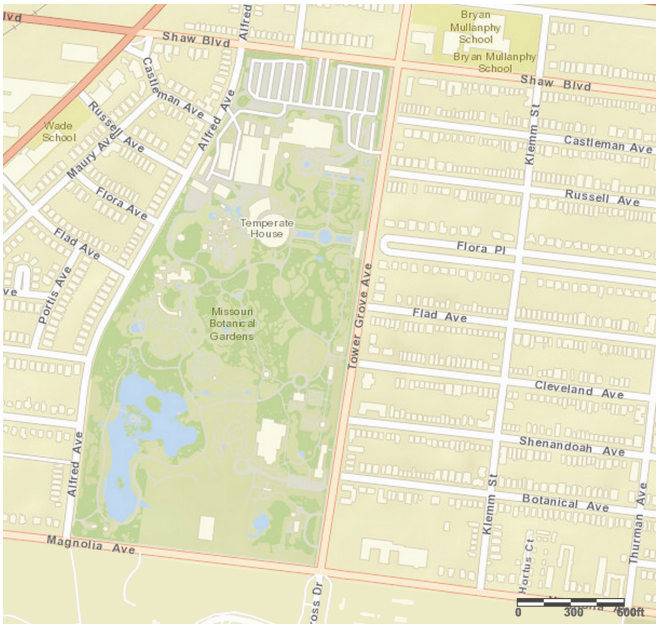
- 7.RP.A

Launch

Provide access to geometry toolkits. Make sure students know where the boundaries of the Botanical Garden are on the map.

Student Task Statement

Here is a map of the Missouri Botanical Garden. Clare walked all the way around the garden.



1. What is the actual distance around the garden? Show your reasoning.
2. It took Clare 30 minutes to walk around the garden at a constant speed. At what speed was she walking? Show your reasoning.

Student Response

1. It takes about 14 segments of the scale to measure the perimeter of the garden, and $14 \cdot 600 = 8{,}400$. So the distance around is about 8,400 feet.
2. If she walks for 30 minutes, that means she was traveling at about 280 feet per minute ($8{,}400 \div 30 = 280$), or about 16,800 feet per hour ($280 \cdot 60 \approx 16{,}800$).

Student Lesson Summary

Maps with scales are useful for making calculations involving speed, time, and distance. Here is a map of part of Alabama.

Suppose it takes a car 1 hour and 30 minutes to travel at constant speed from Birmingham to Montgomery. How fast is the car traveling?

To make an estimate, we need to know about how far it is from Birmingham to Montgomery. The scale of the map represents 20 miles, so we can estimate the distance between these cities is about 90 miles.

Since 90 miles in 1.5 hours is the same speed as 180 miles in 3 hours, the car is traveling about 60 miles per hour.

time (hours)	distance (miles)
1.5	90
3	180
1	60

$\cdot 2$, $\cdot \frac{1}{3}$ (between time rows); $\cdot 2$, $\cdot \frac{1}{3}$ (between distance rows)

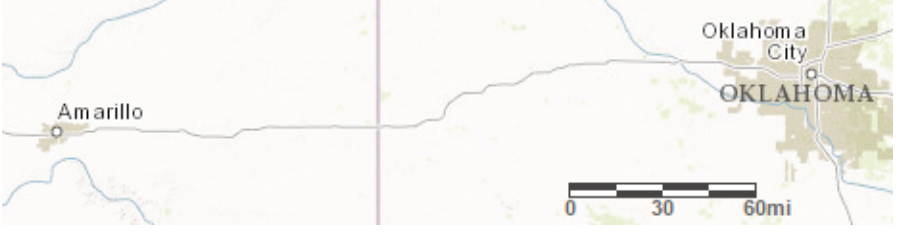
Suppose a car is traveling at a constant speed of 60 miles per hour from Montgomery to Centreville. How long will it take the car to make the trip? Using the scale, we can estimate that it is about 70 miles. Since 60 miles per hour is the same as 1 mile per minute, it will take the car about 70 minutes (or 1 hour and 10 minutes) to make this trip.

Lesson 8 Practice Problems

Problem 1

Statement

Here is a map that shows parts of Texas and Oklahoma.



a. About how far is it from Amarillo to Oklahoma City? Explain your reasoning.

b. Driving at a constant speed of 70 miles per hour, will it be possible to make this trip in 3 hours? Explain how you know.

Solution

a. About 260 miles (but the road is not straight, so it is hard to tell the exact distance from the map)

b. No, a traveler can only go 210 miles in 3 hours, and the distance between the cities is definitely farther than that.

Problem 2

Statement

A local park is in the shape of a square. A map of the local park is made with the scale 1 inch to 200 feet.

a. If the park is shown as a square on the map, each side of which is one foot long, how long is each side of the square park?

b. If a straight path in the park is 900 feet long, how long would the path be when represented on the map?

Solution

a. 2,400 feet

b. 4.5 inches