## Lesson 8: Rewriting Quadratic Expressions in Factored Form (Part 3)

* Let’s look closely at some special kinds of factors.

### 8.1: Math Talk: Products of Large-ish Numbers

Find each product mentally.

$9⋅11$

$19⋅21$

$99⋅101$

$109⋅101$

### 8.2: Can Products Be Written as Differences?

1. Clare claims that $\left(10+3\right)\left(10−3\right)$ is equivalent to $10^{2}−3^{2}$ and $\left(20+1\right)\left(20−1\right)$ is equivalent to $20^{2}−1^{2}$. Do you agree? Show your reasoning.
	1. Use your observations from the first question and evaluate $\left(100+5\right)\left(100−5\right)$. Show your reasoning.
	2. Check your answer by computing $105⋅95$.
2. Is $\left(x+4\right)\left(x−4\right)$ equivalent to $x^{2}−4^{2}$? Support your answer:
* With a diagram:

| *
 | * $x$
 | * $4$
 |
| --- | --- | --- |
| * $x$
 | *
 | *
 |
| * $-4$
 | *
 | *
 |

* Without a diagram:
1. Is $\left(x+4\right)^{2}$ equivalent to $x^{2}+4^{2}$? Support your answer, either with or without a diagram.

#### Are you ready for more?

1. Explain how your work in the previous questions can help you mentally evaluate $22⋅18$ and $45⋅35$.
2. Here is a shortcut that can be used to mentally square any two-digit number. Let’s take $83^{2}$, for example.
	* 83 is $80+3$.
	* Compute $80^{2}$ and $3^{2}$, which give 6,400 and 9. Add these values to get 6,409.
	* Compute $80⋅3$, which is 240. Double it to get 480.
	* Add 6,409 and 480 to get 6,889.
* Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

### 8.3: What If There is No Linear Term?

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram. (Heads up: one of them is impossible.)

| factored form | standard form |
| --- | --- |
| $\left(x−10\right)\left(x+10\right)$ |   |
| $\left(2x+1\right)\left(2x−1\right)$ |   |
| $\left(4−x\right)\left(4+x\right)$ |   |
|   | $x^{2}−81$ |
|   | $49−y^{2}$ |
|   | $9z^{2}−16$ |
|   | $25t^{2}−81$ |
| $\left(c+\frac{2}{5}\right)\left(c−\frac{2}{5}\right)$ |   |
|   | $\frac{49}{16}−d^{2}$ |
| $\left(x+5\right)\left(x+5\right)$ |   |
|   | $x^{2}−6$ |
|   | $x^{2}+100$ |

### Lesson 8 Summary

Sometimes expressions in standard form don’t have a linear term. Can they still be written in factored form?

Let’s take $x^{2}−9$ as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0: $x^{2}+0x−9$. (The expression $x^{2}−0x−9$ is equivalent to $x^{2}−9$ because 0 times any number is 0, so $0x$ is 0.)

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is $\left(x+3\right)\left(x−3\right)$.

To check that this expression is indeed equivalent to $x^{2}−9$, we can expand the factored expression by applying the distributive property: $\left(x+3\right)\left(x−3\right)=x^{2}−3x+3x+\left(-9\right)$. Adding $-3x$ and $3x$ gives 0, so the expanded expression is $x^{2}−9$.

In general, a quadratic expression that is a difference of two squares and has the form:

$a^{2}−b^{2}$

can be rewritten as:

$\left(a+b\right)\left(a−b\right)$

Here is a more complicated example: $49−16y^{2}$. This expression can be written $7^{2}−\left(4y\right)^{2}$, so an equivalent expression in factored form is $\left(7+4y\right)\left(7−4y\right)$.

What about $x^{2}+9$? Can it be written in factored form?

Let’s think about this expression as $x^{2}+0x+9$. Can we find two numbers that multiply to make 9 but add up to 0? Here are factors of 9 and their sums:

* 9 and 1, sum: 10
* -9 and -1, sum: -10
* 3 and 3, sum: 6
* -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write $x^{2}+9$ in factored form using the kinds of numbers that we know about.



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