Lesson 12: Units in Scale Drawings

Goals

- Comprehend that the phrase “equivalent scales” refers to different scales that relate scaled and actual measurements by the same scale factor.
- Generate a scale without units that is equivalent to a given scale with units, or vice versa.
- Justify (orally and in writing) that scales are equivalent, including scales with and without units.

Learning Targets

- I can tell whether two scales are equivalent.
- I can write scales with units as scales without units.

Lesson Narrative

In previous lessons, students learned to express scales with or without units that can be the same or different. In this lesson, they analyze various scales and find that sometimes it is helpful to rewrite scales with units as scales without units in order to compare them. They see that equivalent scales relate scaled and actual measurements by the same scale factor, even though the scales may be expressed differently. For example, the scale 1 inch to 2.5 feet is equivalent to the scale 5 m to 150 m, because they are both at a scale of 1 to 30.

This lesson is also the culmination of students' work on scaling and area. Students have seen many examples of the relationship between scaled area and actual area, and now they must use this realization to find the area of an irregularly-shaped pool (MP7, MP8).

Here is some information about equal lengths that students may want to refer to during these activities.

Customary Units

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

Metric Units

1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters

Equal Lengths in Different Systems

1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers

1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

Alignments

Building On

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Addressing

- 7.G.A.1: Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Notice and Wonder
- Take Turns
- Think Pair Share

Required Materials

Copies of blackline master
Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Metric and customary unit conversion charts
Pre-printed slips, cut from copies of the blackline master

Required Preparation

Note: This lesson contains optional activities. Decide which activities you will do before preparing the materials!

- For the Card Sort: Scales activity, print and cut the slips from the blackline master, so that each group of 3–4 students gets one complete set. If possible, copy each complete set on a different color of paper, so that a stray slip can quickly be put back.
- For the Pondering Pools activity, prepare one copy of the blackline master for every two students.

Ensure students have access to geometry toolkits. It is also recommended that a conversion chart for metric and customary units of length be provided while students are working on the activities in this lesson.

Student Learning Goals

Let's use different scales to describe the same drawing.

12.1 Centimeters in a Mile

Warm Up: 5 minutes
The goal of this warm-up is to review expressions in the context of conversions. This lesson will examine in depth equivalent scales, that is, scales that lead to the same size scale drawing. Checking whether or not two scales are equivalent often involves converting quantities to common units.

Building On

- 6.RP.A.3.d

Student Task Statement

There are 2.54 cm in an inch, 12 inches in a foot, and 5,280 feet in a mile. Which expression gives the number of centimeters in a mile? Explain your reasoning.

1. $\frac{2.54}{12 \cdot 5{,}280}$
2. $5{,}280 \cdot 12 \cdot (2.54)$
3. $\frac{1}{5{,}280 \cdot 12 \cdot (2.54)}$
4. $5{,}280 + 12 + 2.54$
5. $\frac{5{,}280 \cdot 12}{2.54}$

Student Response

B

Activity Synthesis

Ask one or more students to explain their reasoning for the correct choice $5{,}280 \cdot 12 \cdot (2.54)$. There are 2.54 centimeters in an inch and 12 inches in a foot, so that means there are $12 \cdot (2.54)$ centimeters in a foot. Then there are 5,280 feet in a mile, so that makes $5{,}280 \cdot 12 \cdot (2.54)$ centimeters in a mile. Students can also use common sense about measurements. A centimeter is a small unit of measure while a mile is quite large, so there have to be many centimeters in a mile.

Make sure to ask students what option C, $\frac{1}{2.54 \cdot 12 \cdot 5{,}280}$, represents in this setting. (The scale factor to convert from miles to centimeters.)

12.2 Card Sort: Scales

Optional: 15 minutes

The purpose of this activity is to give students more practice identifying equivalent scales, including some expressed without units and some with units. Students work with their group to sort slips into groups of equivalent scales and explain their reasoning. A key insight to uncover here is that when comparing scales, it can be helpful to convert them into equivalent scales in a particular format (e.g., without units, or using the same units).

You will need the Scales Card Sort blackline master for this activity.

Addressing

- 7.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Demonstrate how to set up and do the matching activity. Choose a student to be your partner. Mix up the rest of the cards and place them face up. Select two cards and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree (e.g., by explaining your mathematical thinking, asking clarifying questions, etc.).

Arrange students in groups of 4. If desired, arrange students in groups of 4–6 in two dimensions. (Assign each student into a group and then to a label within it, so that new groups—consisting one student from each of the original groups—can be formed later).

Give students 5–6 minutes to sort the slips, and another 2–3 minutes to check another group's work, followed by whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students cards a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organization

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Access for English Language Learners

Conversing: MLR8 Discussion Supports. Use this routine to help students describe the reasons for their card sorts. In groups of 4, students should take turns sorting 1–2 cards and explaining their reasoning to their group. Display the following sentence frames for all to see: “ ____ and ____ are equivalent because . . .”, and “I noticed ___ , so I matched . . .” Encourage students to challenge each other when they disagree. This will help students clarify their reasoning about equivalent scales.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

If groups have trouble getting started, encourage them to think about different ways to express a scale, both with units and without units.

Students may sort the cards by the types (metric or customary; with units or without units) rather than by common scale factors. Remind students that scales that are equivalent have the same factor relating its scaled lengths to actual lengths.

Students may think that scales in metric units and those in customary units cannot be equivalent. For example, they may think that “1 inch to 1,000 inches” belongs in one group and “1 cm to 10 m” belongs in another. If this misconception arises and is not resolved in group discussions, address it during the activity synthesis.

Student Task Statement

Your teacher will give you some cards with a scale on each card.

1. Sort the cards into sets of equivalent scales. Be prepared to explain how you know that the scales in each set are equivalent. Each set should have at least two cards.
2. Trade places with another group and check each other’s work. If you disagree about how the scales should be sorted, work to reach an agreement.

   Pause here so your teacher can review your work.
3. Next, record one of the sets with three equivalent scales and explain why they are equivalent.

Student Response

1.
    - 1 centimeter to 1 meter, and 1 to 100
    - 1 centimeter to 1 kilometer, $\frac{1}{2}$ cm to 500 m, and 1 to 100,000
    - 1 inch to 8 feet, $\frac{1}{8}$ inch to 1 foot, and 1 to 96

    - 1 centimeter to 10 meters, 1 inch to 1,000 inches, and 1 millimeter to 1 meter
    - 1 foot to 1 mile, and 1 to 5,280
    - 1 inch to 1 mile, and 1 to 63,360
2. No answer necessary.
3. Answers vary. Sample response: There are 100,000 centimeters in one kilometer. Also, 100,000 groups of $\frac{1}{2}$ centimeter is 50,000 centimeters. This is the same length as 500 meters, because $50{,}000 \div 100 = 500$. That means the scales 1 cm to 1 km and $\frac{1}{2}$ cm to 500 m are both equivalent to the scale 1 to 100,000.

Activity Synthesis

Much of the discussion will happen in and between small groups, so a whole-class debrief may only be necessary to tie any loose ends. Invite a few students to share how their group reasoned about a couple of the scales (e.g., $\frac{1}{2}$ cm to 500 m, 1 mm to 1 m).

Address any questions that arose during sorting, common misconceptions, or unsettled disagreements between groups. For example, students may still be unclear about whether scales in customary and metric units can be equivalent. (i.e., Can “1 inch to 1,000 inches” and “1 centimeter to 10 meters” both go in the same group? Why or why not?) Help students see that as long as the two scales represent the same scale factor, they are equivalent and will produce the same scale drawing.

If time permits, consider asking students to order their groups of equivalent scales, starting with the ones that would produce the smallest drawing of the same actual thing to the ones that would produce the largest drawing. Invite students to explain their reasoning.

12.3 The World’s Largest Flag

15 minutes

In this activity, students use a scale without units to find actual and scaled distances that involve a wider range of numbers, from 0.02 to 2,000. They also return to thinking about how the area of a scale drawing relates to the area of the actual thing.

Students are likely to find scaled lengths in one of two ways: 1) by first converting the measurement in meters to centimeters and then dividing by 2,000; or 2) by dividing the measurement by 2,000 and then converting the result to centimeters. To find actual lengths, the same paths are likely, except that students will multiply by 2,000 and reverse the unit conversion. Identify students who use different approaches so they can share later.

Addressing

- 7.G.A.1

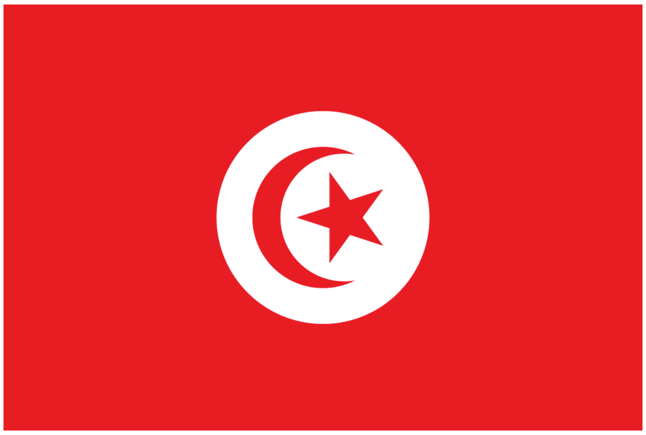
Instructional Routines

- Think Pair Share

Launch

Have students close their books or devices. Display an image of Tunisia’s flag. Explain that Tunisia holds the world record for the largest version of a country flag. The record-breaking flag is nearly four soccer fields in length. Solicit from students a few guesses for a scale that would be appropriate to create a scale drawing of the flag on a sheet of paper. If asked, provide the length of the flag (396 m) and the size of the paper (letter size: $8\frac{1}{2}$ inches by 11 inches, or about 21.5 cm by 28 cm).

After hearing some guesses, explain to students that they will now solve problems about the scale and scale drawing of the giant Tunisian flag.



Arrange students in groups of 3–4. Provide access to a metric unit conversion chart. Give students 4–5 minutes of quiet work time, and then another 5 minutes to collaborate and discuss their work in groups.

During work time, assign one sub-problem from the second question for each group to present.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Students may be confused about whether to multiply or divide by 2,000 (or to multiply by 2,000 or by $\frac{1}{2,000}$) when finding the missing lengths. Encourage students to articulate what a scale of 1 to 2,000 means, or remind them that it is a shorthand for saying “1 unit on a scale drawing represents

2,000 of the same units in the object it represents." Ask them to now think about which of the two—actual or scaled lengths—is 2,000 times the other and which is $\frac{1}{2,000}$ of the other.

For the third question relating the area of the real flag to the scale model, if students are stuck, encourage them to work out the dimensions of each explicitly and to use this to calculate the scale factor between the areas.

Student Task Statement

As of 2016, Tunisia holds the world record for the largest version of a national flag. It was almost as long as four soccer fields. The flag has a circle in the center, a crescent moon inside the circle, and a star inside the crescent moon.

1. Complete the table. Explain or show your reasoning.

	flag length	flag height	height of crescent moon
actual	396 m		99 m
at 1 to 2,000 scale		13.2 cm	

2. Complete each scale with the value that makes it equivalent to the scale of 1 to 2,000. Explain or show your reasoning.

   a. 1 cm to ___________ cm

   b. 1 cm to ___________ m

   c. 1 cm to ___________ km

   d. 2 m to ___________ m

   e. 5 cm to __________ m

   f. ___________ cm to 1,000 m

   g. ___________ mm to 20 m

3. a. What is the area of the large flag?

   b. What is the area of the smaller flag?

   c. The area of the large flag is how many times the area of the smaller flag?

Student Response

1.

	flag length	flag height	height of crescent moon
actual	396 m	264 m	99 m
at 1 to 2,000 scale	19.8 cm	13.2 cm	4.95 cm

Sample reasoning:

- Length of flag: $396 \div 2,000 = 0.198$. 0.198 m is 19.8 cm.
- Height of flag: $(13.2) \cdot 2,000 = 26,400$. 26,400 cm is 264 m.
- Height of crescent moon on flag: $99 \div 2,000 = 0.0495$. 0.0495 m is 4.95 cm.

2. a. 1 cm to **2,000** cm. 2,000 times 1 cm is 2,000 cm.

   b. 1 cm to **20** m. I converted 2,000 cm to m. $2,000 \div 100 = 20$.

   c. 1 cm to **0.02** km. I converted 2000 cm to km. $2,000 \div 100,000 = 0.02$.

   d. 2 m to **4,000** m. 2,000 times 2 m is 4,000 m.

   e. 5 cm to **100** m. I know that 1 cm represents 20 m, so 5 cm represents $5 \cdot 20$.

   f. **50** cm to 1,000 m. I divided 1,000 m by 2,000, which is 0.5 m or 50 cm.

   g. **10** mm to 20 m. I know that 1 cm represents 20 m and 1 cm is 10 mm, so 10 mm represents 20 m.

3. a. The large flag is 396 m by 264 m, so its area is 104,544 m^2.

   b. The small flag is 19.8 cm by 13.2 cm, so its area is 261.36 cm^2.

   c. The scale factor for the height is 2,000 and the scale factor for the length is 2,000, so the area of the actual flag is $2{,}000 \cdot 2{,}000$, or 4,000,000 times the area of the scale drawing.

Activity Synthesis

Select a few students with differing solution paths to share their responses to the first question. Record and display their reasoning for all to see. Highlight two different ways for dealing with unit conversions. For example, in finding scaled lengths, one can either first convert the actual length in meters to centimeters and then multiply by $\frac{1}{2{,}000}$, or multiply by $\frac{1}{2{,}000}$ first, and then convert the quotient into centimeters.

Invite previously identified students to display and share their responses for the sub-problems in the second question. After each person shares, solicit questions or comments from the class.

Emphasize that all of the scales are equivalent because in each scale, a factor of 2,000 relates scaled distances to actual distances.

Reiterate the fact that a scale does not have to be expressed in terms of 1 scaled unit, as is shown in the last three sub-questions, but that 1 is often chosen because it makes the scale factor easier to see and can make calculations more efficient.

Make sure students understand why the scale factor for the area of the two flags is 4,000,000. (Both the length and the height of the large flag are 2,000 times the length and height of the small flag. So the area of the large flag is $2{,}000 \cdot 2{,}000$ times the area of the small flag. Alternatively, there are 10,000 square centimeters in a square meter, so in square centimeters, the area of the large flag is 1,053,360,000. Dividing this by the area of the small flag in square centimeters, 261.36, also gives 4,000,000.)

12.4 Pondering Pools

Optional: 10 minutes

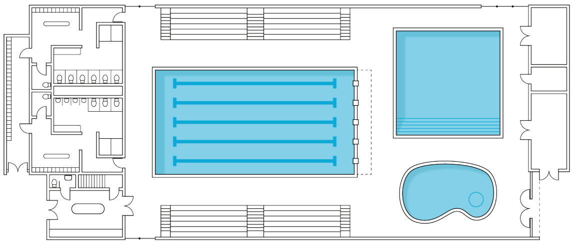
Previously, whenever students were asked to use a scale drawing to calculate the area of an actual region, they were able to find the dimensions of the actual region as an intermediate step. Each time, students were prompted to notice that the actual area was related to the scaled area by the $(\text{scale factor})^2$. Some students may have already become comfortable using this relationship to calculate the actual area directly from the scaled area, without needing to calculate the actual dimensions as an intermediate step.

The purpose of this activity is to help all students internalize this more efficient method. The question about the rectangular pool can be solved either way, but for the question about the kidney-shaped pool, students must rely on the relationship between scaled area and actual area.

As students work, monitor for those who express the scale of the drawing in different but equivalent ways (e.g., 3 cm to 15 m, 1 cm to 5 m, 1 to 500). Also monitor the different ways students find the area of the large rectangular pool:

- By first finding the actual side lengths of the pool in meters and then multiplying them
- By calculating the scaled area in square centimeters and multiplying it by 25 (or 5^2)

You will need the Pondering Pools blackline master for this activity.



Addressing

- 7.G.A.1

Instructional Routines

- MLR5: Co-Craft Questions
- Notice and Wonder

Launch

Give each student a copy of the blackline master. Invite students to share what they notice and what they wonder about the floor plan of the aquatic center.

Some things they might notice include:

- There are three different swimming pools on the floor plan.
- This floor plan has more details than others they have worked with previously, such as stairs and doors.

Some things they might wonder include:

- What is the scale of this drawing?
- How deep are these pools?
- Where is this aquatic center located?
- How much does it cost to get to use these pools?

Provide access to centimeter rulers. Give students 4–5 minutes of quiet work time, followed by whole-class discussion.

Access for English Language Learners

Representing, Conversing, Writing: Math Language Routine 5: Co-Craft Questions. This is the first time Math Language Routine 5 is suggested as a support in this course. In this routine, students are given a context or situation, often in the form of a problem stem with or without numerical values. Students develop mathematical questions that can be asked about the situation. A typical prompt is: “What mathematical questions could you ask about this situation?" The purpose of this routine is to allow students to make sense of a context before feeling pressure to produce answers, and to develop students’ awareness of the language used in mathematics problems.
Design Principle(s): Cultivate conversation; Support sense-making

How It Happens:

1. Give each student a copy of the floor plan of the aquatic center. Do not allow students to see the follow-up questions for this situation.

   Ask students, “What mathematical questions could you ask about this situation?”

2. Give students 1 minute of individual time to jot some notes, and then 3 minutes to share ideas with a partner.

   As pairs discuss, support students in using conversation skills to generate and refine their questions collaboratively by seeking clarity, referring to students’ written notes, and revoicing oral responses as necessary. Listen for how students refer to the scale of the drawing and talk about area in their discussion.

3. Ask each pair of students to contribute one written question to a poster, the whiteboard, or digital projection. Call on 2–3 pairs of students to present their question to the whole class, and invite the class to make comparisons among the questions shared and their own questions.

   Listen for questions intended to ask about the scale of the floor plan and the areas of the pools, and take note of those that use units and those that do not use units. Revoice student ideas with an emphasis on different but equivalent scales, as well as various methods for finding the areas of the pools, wherever it serves to clarify a question.

4. Reveal the three follow-up questions for this situation and give students a couple of minutes to compare them to their own and to those of their classmates. Identify similarities and differences.

   Consider providing these prompts: “Which of your questions is most similar to/different than the ones provided? Why?”, “Is there a main mathematical concept that is present in both your questions and those provided? If so, describe it.”, and “How do your questions relate to one of the lesson goals of comprehending equivalent scales?”

5. Invite students to choose one question to answer (from the class or from the curriculum), and then have students move on to the following problems.

Anticipated Misconceptions

Students may multiply the scaled area by 5 instead of by 5^2. Remind them to consider what 1 *square* centimeter represents, rather than what 1 centimeter represents.

Students may think that the last question cannot be answered because not enough information is given. Encourage them to revisit their previous work regarding how scaled area relates to actual area.

Student Task Statement

Your teacher will give you a floor plan of a recreation center.

1. What is the scale of the floor plan if the actual side length of the square pool is 15 m? Express your answer both as a scale with units and without units.

2. Find the actual area of the large rectangular pool. Show your reasoning.

3. The kidney-shaped pool has an area of 3.2 cm^2 on the drawing. What is its actual area? Explain or show your reasoning.

Student Response

1. The scale is 1 to 500, which could also be expressed as 1 cm to 5 m or 3 cm to 15 m. Sample reasoning: The side length of the square pool is 15 m. On the drawing, the side length measures 3 cm. The scale is 3 cm to 15 m, or 1 cm to 5 m. There are 500 cm in 5 m.

2. About 412.5 m^2. Sample explanations:

   - On the drawing, the pool is about 5.5 cm by 3 cm. The pool's actual measurements are 27.5 m by 15 m. Its area is 412.5 m^2, because $(27.5) \boldsymbol{\cdot} 15 = 412.5$.

   - On the drawing, the pool is about 5.5 cm by 3 cm, so its area is about 16.5 cm^2. If 1 cm represents 5 m, then 1 cm^2 is 25 m^2 in actual area, so the area is 412.5 m^2, because $(16.5) \boldsymbol{\cdot} 25 = 412.5$.

3. About 80 m^2. Sample explanation: I know that 1 cm^2 represents 25 m^2, so I multiplied 3.2 cm^2 by 25. $(3.2) \boldsymbol{\cdot} 25 = 80$.

Are You Ready for More?

1. Square A is a scaled copy of Square B with scale factor 2. If the area of Square A is 10 units^2, what is the area of Square B?
2. Cube A is a scaled copy of Cube B with scale factor 2. If the volume of Cube A is 10 units^3, what is the volume of Cube B?
3. The four-dimensional Hypercube A is a scaled copy of Hypercube B with scale factor 2. If the “volume” of Hypercube A is 10 units^4, what do you think the “volume” of Hypercube B is?

Student Response

1. $\frac{10}{2^2}$
2. $\frac{10}{2^3}$
3. The answer to this depends on what it means to scale a hypercube in 4 dimensions! Assuming the pattern we see in 2 and 3 dimensions holds, we might suspect that the answer is $\frac{10}{2^4}$. That might even help us think about how to define scaling in four dimensions. If we use coordinates and think of scaling by a factor of 2 as multiplying all of the coordinates by a factor of 2, then it does, in fact, work the way we think it should based on the pattern in 2 and 3 dimensions.

Activity Synthesis

The goals of this discussion are to reinforce that there is more than one way to express the scale of a scale drawing and to see that, for a given problem, one way of expressing the scale may be more helpful than another.

First, invite students to share the scales they wrote for the first question. Record the answers for all to see. For each answer, poll the class on whether they agree that the scale is equivalent.

Next, ask selected students to share how they solved the questions about the area of the pools. Discuss:

- Were any of these scales easier to use when finding the actual area? Were any more difficult? Which ones?
- What might be some benefits of using one method over another for finding the actual area?

Lesson Synthesis

Scales can be expressed in many different ways, including using different units or not using any units.

- How can we express the scale 1 inch to 5 miles without units? (Since there are 12 inches in a foot and 5,280 feet in a mile, this is the same as 1 inch to 63,360 inches, or 1 to 63,360.)

A scale tells us how a distance on a scale drawing corresponds to an actual distance, and it can also tell us how an area on a drawing corresponds to an actual area.

If a map uses the scale 1 inch to 5 miles:

- How can we find the actual area of a region represented on the map? (Find the area on the map in square inches and multiply by 25, because 1 square inch represents 25 square miles.)
- How can we find a region's scaled area if we know its actual area? (Multiply the area of the actual region by $\frac{1}{25}$.)

12.5 Drawing the Backyard

Cool Down: 5 minutes

Addressing

- 7.G.A.1

Student Task Statement

Lin and her brother each created a scale drawing of their backyard, but at different scales. Lin used a scale of 1 inch to 1 foot. Her brother used a scale of 1 inch to 1 yard.

1. Express the scales for the drawings without units.
2. Whose drawing is larger? How many times as large is it? Explain or show your reasoning.

Student Response

1. Lin's scale of 1 inch to 1 foot can be written as 1 to 12. Her brother's scale of 1 inch to 1 yard can be written as 1 to 36.
2. Lin's drawing is larger. Sample explanations:
   - The lengths on Lin's plan are 3 times the corresponding lengths on her brother's drawing. The area of Lin's drawing is 9 times the area of her brother's drawing.
   - Since 1 yard equals 3 feet, the scale of Lin's brother's drawing is equivalent to 1 inch to 3 feet. Each inch on his drawing represents a longer distance than on Lin's drawing, so his drawing will require less space on paper.
   - At 1 inch to 1 foot, Lin's drawing will show $\frac{1}{12}$ of actual the distances. At 1 inch to 1 yard, or 1 inch to 3 feet, her brother's drawing will show $\frac{1}{36}$ of the actual distances. Since $\frac{1}{12}$ is larger than $\frac{1}{36}$, her drawing will be larger.

Student Lesson Summary

Sometimes scales come with units, and sometimes they don't. For example, a map of Nebraska may have a scale of 1 mm to 1 km. This means that each millimeter of distance on the map represents 1 kilometer of distance in Nebraska. Notice that there are 1,000 millimeters in 1 meter and 1,000 meters in 1 kilometer. This means there are $1{,}000 \cdot 1{,}000$ or 1,000,000 millimeters in 1 kilometer. So, the same scale without units is 1 to 1,000,000, which means that each unit of distance on the map represents 1,000,000 units of distance in Nebraska. This is true for *any* choice of unit to express the scale of this map.

Sometimes when a scale comes with units, it is useful to rewrite it without units. For example, let's say we have a different map of Rhode Island, and we want to use the two maps to compare the size of Nebraska and Rhode Island. It is important to know if the maps are at the same scale. The scale of the map of Rhode Island is 1 inch to 10 miles. There are 5,280 feet in 1 mile, and 12 inches in 1 foot, so there are 63,360 inches in 1 mile (because $5{,}280 \cdot 12 = 63{,}360$). Therefore, there are 633,600 inches in 10 miles. The scale of the map of Rhode Island without units is 1 to 633,600. The two maps are not at the same scale, so we should not use these maps to compare the size of Nebraska to the size of Rhode Island.

Here is some information about equal lengths that you may find useful.

Customary Units

1 foot (ft) = 12 inches (in)
1 yard (yd) = 36 inches
1 yard = 3 feet
1 mile = 5,280 feet

Metric Units

1 meter (m) = 1,000 millimeters (mm)
1 meter = 100 centimeters
1 kilometer (km) = 1,000 meters

Equal Lengths in Different Systems

1 inch = 2.54 centimeters
1 foot ≈ 0.30 meter
1 mile ≈ 1.61 kilometers

1 centimeter ≈ 0.39 inch
1 meter ≈ 39.37 inches
1 kilometer ≈ 0.62 mile

Lesson 12 Practice Problems

Problem 1

Statement

The Empire State Building in New York City is about 1,450 feet high (including the antenna at the top) and 400 feet wide. Andre wants to make a scale drawing of the front view of the Empire State Building on an $8\frac{1}{2}$-inch-by-11-inch piece of paper. Select a scale that you think is the most appropriate for the scale drawing. Explain your reasoning.

a. 1 inch to 1 foot

b. 1 inch to 100 feet

c. 1 inch to 1 mile

d. 1 centimeter to 1 meter

e. 1 centimeter to 50 meters

f. 1 centimeter to 1 kilometer

Solution

E, or 1 cm to 50 m, would be most appropriate. Explanations vary. Sample explanation: With A, B, and D, the scaled image will not fit on the page. For C and F, the image will be too small. Option E is just right because at 1 cm to 50 m, the height of the building is about 10 cm, and the width is about 3 cm.

Problem 2

Statement

Elena finds that the area of a house on a scale drawing is 25 square inches. The actual area of the house is 2,025 square feet. What is the scale of the drawing?

Solution

1 inch to 9 feet

Problem 3

Statement

Which of these scales are equivalent to 3 cm to 4 km? Select **all** that apply. Recall that 1 inch is 2.54 centimeters.

A. 0.75 cm to 1 km

B. 1 cm to 12 km

C. 6 mm to 2 km

D. 0.3 mm to 40 m

E. 1 inch to 7.62 km

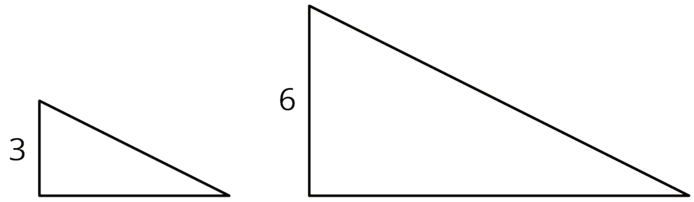
Solution

["A", "D"]

Problem 4

Statement

These two triangles are scaled copies of one another. The area of the smaller triangle is 9 square units. What is the area of the larger triangle? Explain or show how you know.



Solution

36 square units. When the lengths of a scaled copy are 2 times those of the original figure, the area of the copy is 4 times that of the original area: $4 \cdot 9 = 36$.

Problem 5

Statement

Water costs $1.25 per bottle. At this rate, what is the cost of:

a. 10 bottles?

b. 20 bottles?

c. 50 bottles?

Solution

a. $12.50 (because $10 \cdot 1.25 = 12.5$)

b. $25 (because $20 \cdot 1.25 = 25$)

c. $62.50 (because $50 \cdot 1.25 = 62.5$)

Problem 6

Statement

The first row of the table shows the amount of dish detergent and water needed to make a soap solution.

a. Complete the table for 2, 3, and 4 batches.

b. How much water and detergent is needed for 8 batches? Explain your reasoning.

number of batches	cups of water	cups of detergent
1	6	1
2		
3		
4		

Solution

a.

number of batches	cups of water	cups of detergent
1	6	1
2	12	2
3	18	3
4	24	4

b. 48 cups of water and 8 cups of dish detergent. Explanations vary. Sample response: 8 batches is 2 times 4 batches. Doubling 24 gives 48 and doubling 4 gives 8.