Lesson 12: Edge Lengths and Volumes

Goals

- Comprehend the term “cube root of a” (in spoken language) and the notation $\sqrt[3]{a}$ (in written language) to mean the side length of a cube whose volume is a cubic units.
- Coordinate representations of a cube root, including cube root notation, decimal representation, the side length of a cube of given volume, and a point on the number line.

Learning Targets

- I can approximate cube roots.
- I know what a cube root is.
- I understand the meaning of expressions like $\sqrt[3]{5}$.

Lesson Narrative

This is the first of two lessons in which students learn about cube roots. In the first lesson, students learn the notation and meaning of cube roots, e.g., $\sqrt[3]{8}$. In the warm-up, they order solutions to equations of the form $a^2 = 9$ and $b^3 = 8$. They already know about square roots, so in the discussion of the warm-up, they learn about the parallel definition of cube roots. In the following classroom activity, students use cube roots to find the edge length of a cube with given volume. A card sort activity helps them make connections between cube roots as values, as solutions to equations, and as points on the number line.

In the next lesson, students will find out that it is possible to find cube roots of negative numbers.

Alignments

Addressing

- 8.EE.A.2: Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
- 8.NS.A.2: Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2). For example, by truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Copies of the blackline master for this lesson. Prepare 1 copy for every 3 students, and cut them up ahead of time.

Student Learning Goals

Let’s explore the relationship between volume and edge lengths of cubes.

12.1 Ordering Squares and Cubes

Warm Up: 10 minutes
The purpose of this warm-up is to introduce students to **cube roots** during the discussion. This activity provides an opportunity to use cube root language and notation during the discussion. Students will explore the possibility of negative cube roots in the next lesson.

At first, students should be able to order the values of a, b, c, and f without a calculator. However, d and e will be easier with a calculator. Encourage students to use estimated values for d and e to order the values before using a calculator. As students work, identify students who use different strategies for ordering.

Addressing

- 8.EE.A.2

Instructional Routines

- Think Pair Share

Launch

Students in groups of 2. Give students 2–3 minutes to order the options to the best of their ability without a calculator, and to share their reasoning with a partner. Pause to discuss which are easy to order (likely f, b, c, and a) and which ones students are not sure about (likely d and e, which are between b and a). Then give students 1 minute with a calculator to finish ordering the options. Follow with a whole-class discussion.

Student Task Statement

Let a, b, c, d, e, and f be positive numbers.

Given these equations, arrange a, b, c, d, e, and f from least to greatest. Explain your reasoning.

- $a^2 = 9$

- $b^3 = 8$
- $c^2 = 10$
- $d^3 = 9$
- $e^2 = 8$
- $f^3 = 7$

Student Response

The order is f, b, d, e, a, c. Sample response: We know that $a = \sqrt{9}$, which is equal to 3. Since $e = \sqrt{8}$, it is slightly less than 3. Since $c = \sqrt{10}$, it is slightly greater than 3. We know that $b = \sqrt[3]{8}$, which is equal to 2 because $2^3 = 8$. Since $d = \sqrt[3]{9}$, it is slightly greater than 2. Since $f = \sqrt[3]{7}$, it is slightly less than 2.

Activity Synthesis

Ask a student to share their order of a, b, c, d, e, and f from least to greatest. Record and display their responses for all to see. Ask the class if they agree or disagree. If the class agrees, select previously identified students to share their strategies for ordering the values. If the class is in disagreement, ask students to share their reasoning until an agreement is reached.

As students share, ask students which were the easiest to find and which were the hardest to find. Introduce students to cube root language and notation. Remind students that they previously learned that the equation $c^2 = 10$ has solution $c = \sqrt{10}$. Similarly, we can say that the equation $d^3 = 9$ has solution $d = \sqrt[3]{9}$. Ask students to write the solution to $f^3 = 7$ ($f = \sqrt[3]{7}$).

Finally, tell students that while square roots are a way to write the exact value of the side length of a square with a known area, cube roots are a way to write the exact value of the edge length of a cube with a known volume, which students will do in a following activity.

12.2 Name That Edge Length!

10 minutes

Now that students have been introduced to the notation for cube roots, the goal of this task is for students to connect their understanding of the relationship between the volume and edge lengths of cubes with that of cube roots.

Addressing

- 8.EE.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

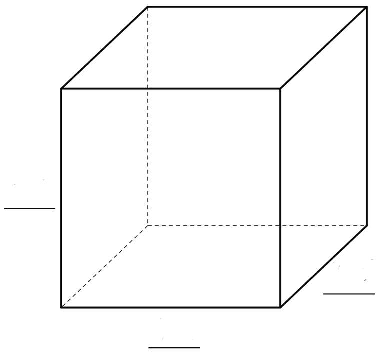
Students should not use a calculator, and all values should be exact.
If students are unsure what to put for the "volume equation," remind them that the volume of a cube is the cube of the edge length.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, review activities from lesson 2 in which students explored square root by completing a table based on the side length and the area of a square. *Supports accessibility for: Social-emotional skills; Conceptual processing*

Student Task Statement

Fill in the missing values using the information provided:



sides	volume	volume equation
	27 in^3	
$\sqrt[3]{5}$		
		$(\sqrt[3]{16})^3 = 16$

Student Response

1. Each length is 3 in. Equation: $3^3 = 27$.
2. Volume is 5. Equation: $\sqrt[3]{5}^3 = 5$.
3. Each length is $\sqrt[3]{16}$. The volume is 16.

Are You Ready for More?

A cube has a volume of 8 cubic centimeters. A square has the same value for its area as the value for the surface area of the cube. How long is each side of the square?

Student Response

The cube has sides of length 2 and surface area 24, so the square has side lengths $\sqrt{24}$.

Activity Synthesis

Previously, students learned that knowing the area of a square is sufficient information for stating the exact length of the side of the square using square roots. The purpose of this discussion is for students to make that same connection with the volume of cubes and cube roots. The cube root of the volume of a cube represents the *exact* value of the edge length of a cube in ways that measuring or approximating by cubing values do not (except in special cases such as perfect cubes).

Discuss:

- "What integers is $\sqrt[3]{5}$ between?" ($\sqrt[3]{5}$ is between 1 and 2 because $1^3 = 1$ and $2^3 = 8$.)
- "What integers is $\sqrt[3]{16}$ between?" ($\sqrt[3]{16}$ is between 2 and 3 because $2^3 = 8$ and $3^3 = 27$.)
- "Name another volume for a cube with edge lengths between 2 and 3." (A cube with volume 26 has edge lengths between 2 and 3 since since $3^3 = 27$, meaning $\sqrt[3]{26}$ has a value slightly less than 3.)

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Display sentence frames for students to use as a support when they explain their strategy. For example, "I noticed _____, so I . . ." or "First, I _____ because _____." Invite students to share with a partner to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to clarify their thinking, and to consider how they will communicate their reasoning.
Design Principle(s): Optimize output (for explanation)

12.3 Card Sort: Rooted in the Number Line

15 minutes

The purpose of this activity is for students to use rational approximations of irrational numbers and to match irrational numbers to equations they are solutions to. To do this, students sort cards into sets of three consisting of:

- A square or cube root value
- An equation of the form $x^2 = p$ or $x^3 = p$ that the value is a solution to
- A number line showing the location of the value

Identify groups using clear explanations for how they chose to arrange their sets of 3, particularly when matching the cards with a value plotted on a number line.

Addressing

- 8.EE.A.2
- 8.NS.A.2

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Arrange students in groups of 2–4. Students should not use a calculator for this activity. Tell students that they will be given cards to sort into sets of three. Distribute 27 pre-cut slips from the blackline master to each group. Group work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards (using multiples of 3) to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organization

Anticipated Misconceptions

Some students may mix up cube and square roots. Encourage these students to pay close attention to the notation. They may want to fill in the "2" for the square roots similar to how the "3" is used for cube roots.

Student Task Statement

Your teacher will give your group a set of cards. For each card with a letter and value, find the two other cards that match. One shows the location on a number line where the value exists, and the other shows an equation that the value satisfies. Be prepared to explain your reasoning.

Student Response

The blackline master shows the solution to the matching.

Activity Synthesis

Select 3–4 previously identified groups to share one of their sets of three cards.

Also discuss:

- “What was a match that was hard to make?” (Two of the number lines have values between 4 and 5, and it took some extra reasoning to figure out that the one with the value closer to 4 was $\sqrt{18}$ while the one with the point closer to 5 was $\sqrt[3]{100}$.
- “If another class was going to sort these cards, what is something you would recommend they have or do that you found helpful?” (I would recommend they have a list of perfect squares and perfect cubes to help think about where the different roots are plotted on the number line.)

Access for English Language Learners

Speaking, Writing: MLR8 Discussion Supports. Display sentence frames to support students as they justify their reasoning for each match. For example, “The solution to ________ ($x^3 = 25$) is ________ ($\sqrt[3]{25}$) and it is between ______ (2 and 3) on the number line, because . . .” The helps students place extra attention on language used to engage in mathematical communication, without reducing the cognitive demand of the task. It also emphasizes the importance of justification.
Design Principle: Maximize meta-awareness

Lesson Synthesis

In this lesson, students learned about cube roots. Similar to square roots, cube roots can be thought of in the context of shapes; a cube with volume 64 cubic units has an edge length of 4 units, which is $\sqrt[3]{64}$, because $4^3 = 64$.

- “If a cube has a volume of 27 cubic inches, what is its side length?” (The cube with volume 27 cubic inches has an edge length of 3 inches, since $\sqrt[3]{27} = 3$.)
- “What is the solution to $x^3 = 150$, and what two integers would it fall between on a number line?” (The solution to $x^3 = 150$ is $\sqrt[3]{150}$ and it is between 5 and 6 on a number line, because $5^3 = 125$ and $6^3 = 216$.)

12.4 Roots of 36

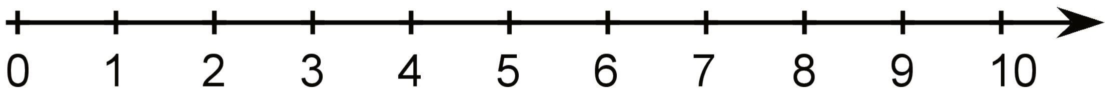
Cool Down: 5 minutes

Addressing

- 8.NS.A.2

Student Task Statement

Plot $\sqrt{36}$ and $\sqrt[3]{36}$ on the number line.

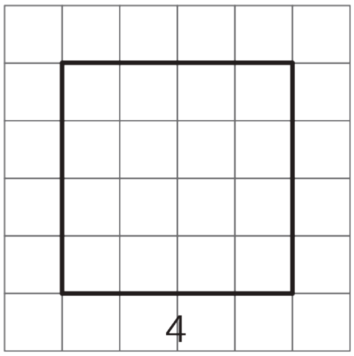


Student Response

$\sqrt{36}$ should be at 6, and $\sqrt[3]{36}$ should be between 3 and 4.

Student Lesson Summary

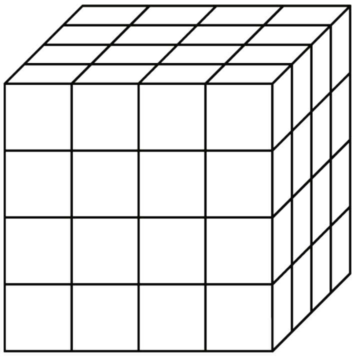
To review, the side length of the square is the square root of its area. In this diagram, the square has an area of 16 units and a side length of 4 units.



These equations are both true:

$$4^2 = 16$$
$$\sqrt{16} = 4$$

Now think about a solid cube. The cube has a volume, and the edge length of the cube is called the **cube root** of its volume. In this diagram, the cube has a volume of 64 units and an edge length of 4 units:



These equations are both true:

$$4^3 = 64$$

$$\sqrt[3]{64} = 4$$

$\sqrt[3]{64}$ is pronounced "The cube root of 64." Here are some other values of cube roots:

$$\sqrt[3]{8} = 2, \text{ because } 2^3 = 8$$

$$\sqrt[3]{27} = 3, \text{ because } 3^3 = 27$$

$$\sqrt[3]{125} = 5, \text{ because } 5^3 = 125$$

Glossary

- cube root

Lesson 12 Practice Problems

Problem 1

Statement

a. What is the volume of a cube with a side length of
   i. 4 centimeters?
   ii. $\sqrt[3]{11}$ feet?
   iii. s units?

b. What is the side length of a cube with a volume of
   i. 1,000 cubic centimeters?
   ii. 23 cubic inches?
   iii. v cubic units?

Solution

a.
   i. 64 cubic centimeters
   ii. 11 cubic feet
   iii. s^3 cubic units

b.
   i. 10 cm
   ii. $\sqrt[3]{23}$ inches
   iii. $\sqrt[3]{v}$ units

Problem 2

Statement

Write an equivalent expression that doesn't use a cube root symbol.

a. $\sqrt[3]{1}$

b. $\sqrt[3]{216}$

c. $\sqrt[3]{8000}$

d. $\sqrt[3]{\frac{1}{64}}$

e. $\sqrt[3]{\frac{27}{125}}$

f. $\sqrt[3]{0.027}$

g. $\sqrt[3]{0.000125}$

Solution

a. 1

b. 6

c. 20

d. $\frac{1}{4}$

e. $\frac{3}{5}$

f. 0.3

g. 0.05

Problem 3

Statement

Find the distance between each pair of points. If you get stuck, try plotting the points on graph paper.

a. $X = (5, 0)$ and $Y = (-4, 0)$

b. $K = (-21, -29)$ and $L = (0, 0)$

Solution

a. 9

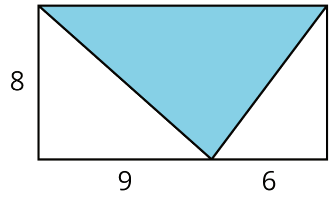
b. $\sqrt{1282}$

(From Unit 8, Lesson 11.)

Problem 4

Statement

Here is a 15-by-8 rectangle divided into triangles. Is the shaded triangle a right triangle? Explain or show your reasoning.



Solution

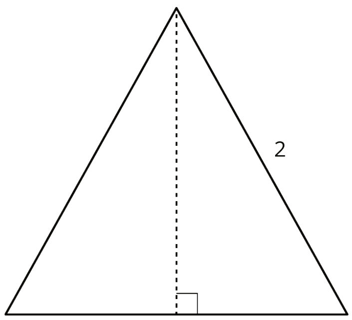
No, it is not. Use the Pythagorean Theorem to find the length of the interior sides of the triangle: the lengths are $\sqrt{145}$ and 10. The longest side is 15, the length of the rectangle. Now check whether this triangle's side lengths make $a^2 + b^2 = c^2$. Because $145 + 100 = 245$, not 225, the converse of the Pythagorean Theorem states this triangle is not a right triangle.

(From Unit 8, Lesson 9.)

Problem 5

Statement

Here is an equilateral triangle. The length of each side is 2 units. A height is drawn. In an equilateral triangle, the height divides the opposite side into two pieces of equal length.



a. Find the exact height.

b. Find the area of the equilateral triangle.

c. (Challenge) Using x for the length of each side in an equilateral triangle, express its area in terms of x.

Solution

a. $\sqrt{3}$ units

b. $\sqrt{3}$ units^2

c. The area is $\frac{x^2\sqrt{3}}{4}$ units^2

(From Unit 8, Lesson 10.)