Lesson 7: Creating Double Number Line Diagrams

Goals

- Comprehend and use the word “per” (orally and in writing) to mean “for each.”
- Draw and label a double number line diagram from scratch, with parallel lines and equally-spaced tick marks.
- Use double number line diagrams to find a wider range of equivalent ratios.

Learning Targets

- I can create a double number line diagram and correctly place and label tick marks to represent equivalent ratios.
- I can explain what the word per means.

Lesson Narrative

In this lesson, students create double number line diagrams from scratch. They see that it is important to use parallel lines, equally-spaced tick marks, and descriptive labels. They are also introduced to using the word **per** to refer to how much of one quantity there is for every one unit of the other quantity.

Double number lines are included in the first few activity statements to help students find an equivalent ratio involving one item or one unit. In later activities and lessons, students make their own strategic choice of an appropriate representation to support their reasoning (MP5). Regardless of method, students indicate the units that go with the numbers in a ratio, in both verbal statements and diagrams.

Note that students are not expected to use or understand the term “unit rate” in this lesson.

Alignments

Building On

- 4.NF: Grade 4 - Number and Operations---Fractions

Addressing

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Rulers

Required Preparation

It may be helpful—but not required—to bring back the blue and yellow water mixtures.

> **Student Learning Goals**
>
> Let's draw double number line diagrams like a pro.

7.1 Ordering on a Number Line

Warm Up: 10 minutes

In this warm-up, students partition a number line and locate fraction and decimal equivalents in preparation for working with double number lines in this unit. Students are purposely not asked to locate 1 on the number line to see how they reason about locating the $\frac{1}{2}$ and $\frac{1}{4}$. It is important for students to be able to identify the fractions or decimals and label tick marks correctly, interpreting the distance between tick marks, rather than the number of tick marks, as the fractional size. As students discuss with their partner, select students to share their answers to the first question during the whole-class discussion.

Building On

- 4.NF

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Display the number line for all to see. Give students 2 minutes of quiet think time and ask them to give a signal when they have an answer and a strategy. Ask students to compare their number line with a partner and share the fractions or decimals they chose to place on the number line for the second question.

Access for English Language Learners

Reading, Writing: MLR8 Discussion Supports. Briefly review the meaning of the terms "label" and "tick marks" as you or a student points to these features in the student task statement. Review the meaning of the term "locate" by acting out and thinking aloud.
Design Principle(s): Support sense-making

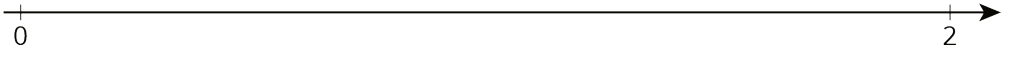
Anticipated Misconceptions

Students may place $\frac{1}{2}$ in the center of the number line, reasoning that it is half of the number line. Explain to the students they are placing the number $\frac{1}{2}$, which has a specific value and location on the number line.

Student Task Statement

1. Locate and label the following numbers on the number line:

$\frac{1}{2}$ $\frac{1}{4}$ $1\frac{3}{4}$ 1.5 1.75



2. Based on where you placed the numbers, locate and label four more fractions or decimals on the number line.

Student Response

1. Here is the number line:

2. Answers vary.

Activity Synthesis

Select students to explain how they reasoned about the location of each number on the number line. After each number, ask the class whether they agree or disagree, and if anyone else had a different way of thinking about that number. If time permits, ask students to share the fractions or decimals they located for the second question. Discuss why they chose those numbers and how they decided on their location.

7.2 Just a Little Green

10 minutes
Students continue to use double number lines to reason about equivalent ratios. Here students' attention is directed to the $1:3$ blue-to-yellow ratio in the green water recipe, which can then be used to determine any equivalent ratio. The task is also the beginning of students' exploration of finding and using ratios containing a 1.

One key idea to convey here is that finding a ratio associated with 1 unit of a quantity can be very helpful. Another is that the intervals on double number lines can be subdivided to help us find such ratios.

As students work, identify those who use division to determine the $1:3$ ratio, and then use multiplication to determine the ratios for 8 ml and 13 ml of blue water. This is a key insight for a type of reasoning that is broadly useful and will be developed further.

Addressing

- 6.RP.A.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Ask students to recall what double number lines are and how they can be used to represent problems involving equivalent ratios. Explain that they are going to investigate the structure of double number lines in more detail. Give students 5 minutes of quiet think time, and then time to discuss their responses with a partner.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using physical objects to represent abstract concepts. Some students may benefit by starting with a kinesthetic representation of the number line on a clothesline. Students can place and adjust numbers on folded paper or cardstock on the clothesline.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Access for English Language Learners

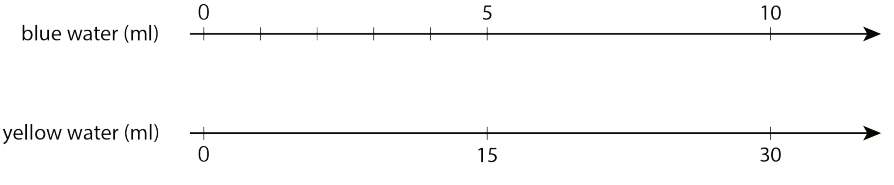
Writing, Conversing: MLR1 Stronger and Clearer Each Time. Use this routine to help students optimize for generalization related to the importance of "for every 1 ml", by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners to share their response to the question, "Why is it useful to know how much yellow water should be used with 1 ml of blue water?" Provide listeners with prompts for feedback that will help their partner add detail to strengthen and clarify their ideas. For example, students can ask their partner, "How did you use double number lines to solve this problem?" or "Can you say more about what the ratio means in this context?" Next, provide students with 3–4 minutes to revise their initial draft based on feedback from their peers. This will help students produce a written generalization that explains the importance of finding and using ratios containing a 1.
Design Principle(s): Optimize output (for generalization)

Anticipated Misconceptions

Students may have trouble figuring out that the length of a segment between consecutive tick marks is $\frac{1}{5}$ of the interval from 0 to 5, especially since there are four tick marks (not five). When focusing on blue, students' first guess about the tick marks is generally correct. For yellow, remind them that the numbers on the tick marks are made by skip counting; they are then likely to try 3's and 5's since both can make 15. Students who label the spaces between tick marks rather than the tick marks themselves may need additional work with important measurement conventions.

Student Task Statement

The other day, we made green water by mixing 5 ml of blue water with 15 ml of yellow water. We want to make a very small batch of the same shade of green water. We need to know how much yellow water to mix with only 1 ml of blue water.



1. On the number line for blue water, label the four tick marks shown.
2. On the number line for yellow water, draw and label tick marks to show the amount of yellow water needed for each amount of blue water.
3. How much yellow water should be used for 1 ml of blue water? Circle where you can see this on the double number line.
4. How much yellow water should be used for 11 ml of blue water?

5. How much yellow water should be used for 8 ml of blue water?

6. Why is it useful to know how much yellow water should be used with 1 ml of blue water?

Student Response

1. Write 1, 2, 3, 4 at the tick marks, because the fifth tick mark is 5.
2. Write 3, 6, 9, 12 at the tick marks, because the fifth tick mark is 15.
3. 3 ml of yellow water is needed.
4. 33 ml of yellow water is needed, because $33 = 3 \cdot 11$.
5. 24 ml of yellow water is needed, because $24 = 3 \cdot 8$.
6. Using this, you can multiply to figure out any amount of yellow water needed for a given amount of blue water.

Activity Synthesis

Debrief as a class after students have a chance to share their work with a partner. Focus discussions on how students determine the amount of yellow water for 1 ml of blue, and how they determine the amounts of yellow for 8 ml and 11 ml of blue. Select students who used division (to find the former) and multiplication (to find the latter) to share.

If students do not do so, frame the relationship of blue to yellow using phrases such as "for every 1 ml of . . . " or "per milliliter of . . ."

- "There are 3 milliliters of yellow water for every 1 milliliter of blue water."
- "There are 3 milliliters of yellow water **per** milliliter of blue water."

The word **per** means "for every." Ask students to think of any other situation in which they may use the word "per" as it is used here (e.g., price per bottle of water, cost per ticket, etc.) and discuss why knowing the value of one item would be helpful.

7.3 Art Paste on a Double Number Line

20 minutes

In the previous lesson, students were given blank double number line diagrams and were only responsible for labeling them to match the situation. In this activity, students draw their own double number line diagram from scratch and identify which elements are important to create a useful double number line diagram.

Addressing

- 6.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

"You just used a double number line to solve some problems. Now, you'll create a double number line from scratch. Once you know how to make double number lines, you can use them for any situation with equivalent ratios."

Arrange students in groups of 2. Ensure each student has access to a ruler. Have students check with a partner and come to an agreement about how to draw the diagrams before moving on to question 3.

Access for English Language Learners

Representing: MLR8 Discussion Supports. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: double number line, parallel lines, tick marks, equal increments, equivalent ratios and "line up." Use visuals to support understanding of these terms in the context of this problem.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may not label tick marks with equal increments or may not align the tick marks.

Student Task Statement

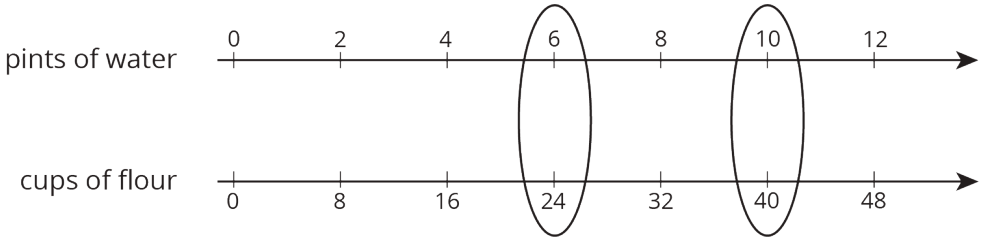
A recipe for art paste says "For every 2 pints of water, mix in 8 cups of flour."

1. Follow the instructions to draw a double number line diagram representing the recipe for art paste.
    a. Use a ruler to draw two parallel lines.
    b. Label the first line "pints of water." Label the second line "cups of flour."
    c. Draw at least 6 equally spaced tick marks that line up on both lines.
    d. Along the water line, label the tick marks with the amount of water in 0, 1, 2, 3, 4, and 5 batches of art paste.
    e. Along the flour line, label the tick marks with the amount of flour in 0, 1, 2, 3, 4, and 5 batches of art paste.
2. Compare your double number line diagram with your partner's. Discuss your thinking. If needed, revise your diagram.
3. Next, use your double number line to answer these questions:
    a. How much flour should be used with 10 pints of water?
    b. How much water should be used with 24 cups of flour?

c. How much flour **per** pint of water does this recipe use?

Student Response

1. The correctly drawn and labeled number line should look like the one below. The places to find answers to the following questions are circled.



2. No answer necessary.

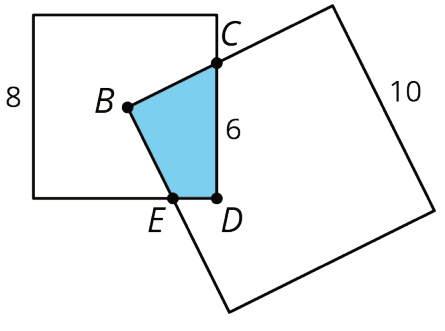
3. a. Use 40 cups of flour for 10 pints of water.

   b. Use 6 pints of water for 24 cups of flour.

   c. 4 cups of flour per pint of water

Are You Ready for More?

A square with side of 10 units overlaps a square with side of 8 units in such a way that its corner B is placed exactly at the center of the smaller square. As a result of the overlapping, the two sides of the large square intersect the two sides of the small square exactly at points C and E, as shown. The length of CD is 6 units.



What is the area of the overlapping region $CDEB$?

Student Response

16 square units. Sample reasoning: If you extend BC and BE, the smaller square is partitioned into four regions of equal area. (A rigorous argument can be made using symmetry, or side lengths and angles, for why these four regions are congruent, but at this stage of learning, students could simply reason that these appear to be four identical copies.) Since the area of the smaller square is 64 square units, the area of the shaded region is 16 square units, because $64 \div 4 = 16$.

Activity Synthesis

Select students to explain how they used their double number line diagram to answer the last question. Ask students how they can indicate the number of cups of flour per pint of water on the double number line.

If desired, capitalize on ways students might have incorrectly constructed their double number line. For example:

- Do all increments on each line need to be equal? Why or why not?
- Do the tick marks on the top line need to match those on the bottom line? Why/why not?
- Does it matter what number we use to start each line? Why or why not?

7.4 Revisiting Tuna Casserole

Optional: 10 minutes
In this activity, students revisit familiar contexts they represented with discrete diagrams in previous lessons. Here, they see how double number line diagrams are helpful for answering more questions about these situations.

Monitor for students who use a discrete diagram and for students who use a double number line diagram.

Addressing

- 6.RP.A.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2.

Student Task Statement

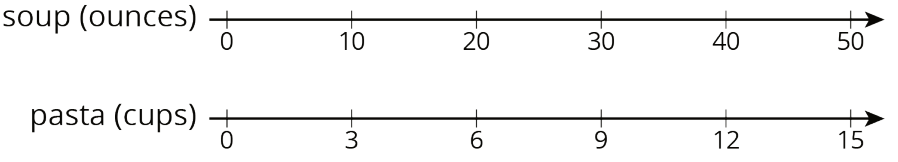
The other day, we looked at a recipe for tuna casserole that called for 10 ounces of cream of chicken soup for every 3 cups of elbow-shaped pasta.

1. Draw a double number line diagram that represents the amounts of soup and pasta in different-sized batches of this recipe.
2. If you made a large amount of tuna casserole by mixing 40 ounces of soup with 15 cups of pasta, would it taste the same as the original recipe? Explain or show your reasoning.

3. The original recipe called for 6 ounces of tuna for every 3 cups of pasta. Add a line to your diagram to represent the amount of tuna in different batches of casserole.
4. How many ounces of soup should you mix with 30 ounces of tuna to make a casserole that tastes the same as the original recipe?

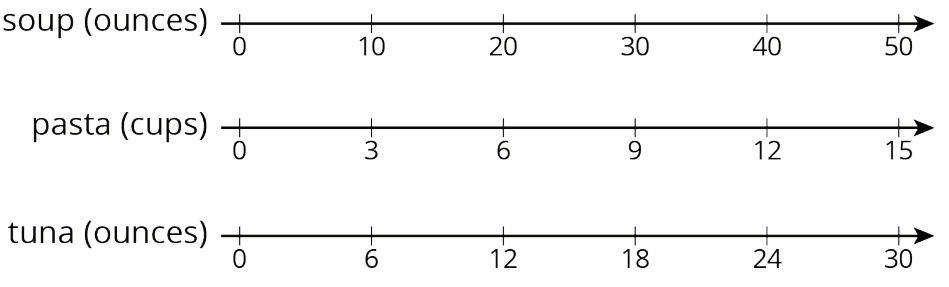
Student Response

1.



2. No, it would not taste the same. Explanations vary. Sample responses:
   - If you want to use 40 ounces of soup, then you should use 12 cups of pasta, because 12 is directly below the 40 on the double number line.
   - If you want to use 15 cups of pasta, then you should use 50 ounces of soup, because 50 is directly above the 15 on the double number line.

3.



4. 50 ounces of soup, because the 50 on the top line of the diagram lines up with the 30 on the bottom line of the diagram.

Activity Synthesis

Select students to present their solutions. Sequence discrete diagrams first and double number line diagrams second. Help students see connections between the two representations.

Access for English Language Learners

Conversing, Representing: MLR7 Compare and Connect. Use this routine to support student understanding of the connections between discrete diagrams and double number line diagrams. After students have presented their solutions, give students quiet think time to consider what is the same and what is different about the two types of representations. Next, ask students to discuss what they noticed with a partner. Listen for and amplify mathematical language students use to describe the connections they notice about how each representation shows equivalent ratios.
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

Create a double number line with the help of the class. Start by asking, "What are some important things to pay attention to when you create a double number line?" Then, "What situation should we represent?" It is fine to choose a situation that students have already encountered in this lesson or an earlier lesson.

As you are creating the double number line together, write down anything mentioned that it is important to pay attention to. For example:

- The two lines you draw should be parallel to each other. One practice is to use both edges of a ruler to create two parallel lines. But double number lines are tools for reasoning, so they don't have to be perfect.
- Each line should be labeled with what it represents. Include units of measure.
- Tick marks should be evenly spaced, and the two sets of tick marks should be lined up vertically in pairs.

One strategy might be to intentionally do something wrong, and ask students how you should fix it. For example, draw tick marks that are very obviously *not* evenly spaced, or neglect to include units of measure in your labels.

7.5 Revisiting Paws, Ears, and Tails

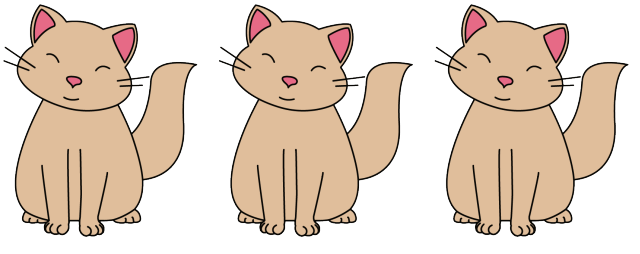
Cool Down: 5 minutes

Addressing

- 6.RP.A.3

Student Task Statement

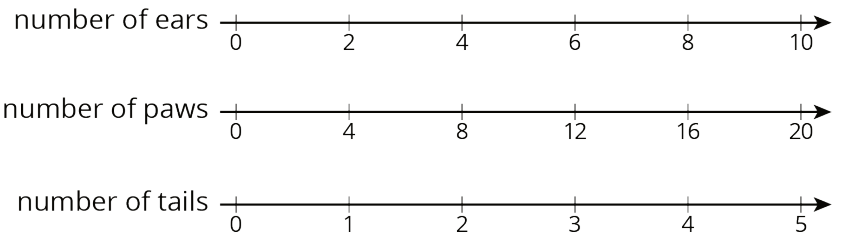
Each of these cats has 2 ears, 4 paws, and 1 tail.



1. Draw a double number line diagram that represents a ratio in the situation.
2. Write a sentence that describes this situation and that uses the word *per*.

Student Response

1. Students may draw any 2 of the 3 number lines shown.



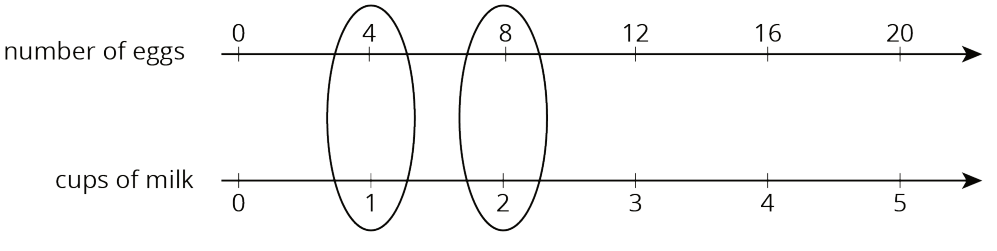
2. Answers vary. Samples responses:
    - There are 2 ears per tail.
    - There are 4 paws per tail.
    - There are 2 paws per ear.
    - There is $\frac{1}{2}$ tail per ear.

Student Lesson Summary

Here are some guidelines to keep in mind when drawing a double number line diagram:

- The two parallel lines should have labels that describe what the numbers represent.
- The tick marks and numbers should be spaced at equal intervals.
- Numbers that line up vertically make equivalent ratios.

For example, the ratio of the number of eggs to cups of milk in a recipe is $4 : 1$. Here is a double number line that represents the situation:



We can also say that this recipe uses "4 eggs per cup of milk" because the word **per** means "for each."

Glossary

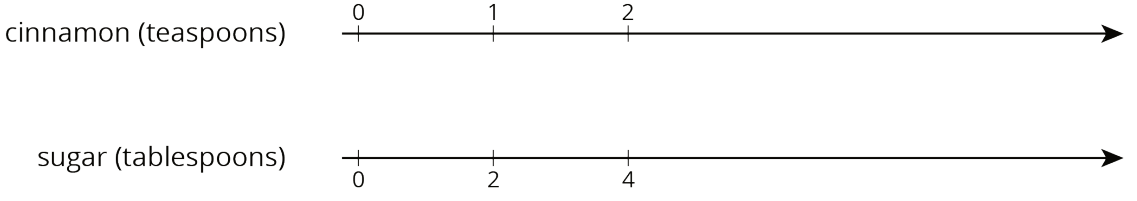
- per

Lesson 7 Practice Problems

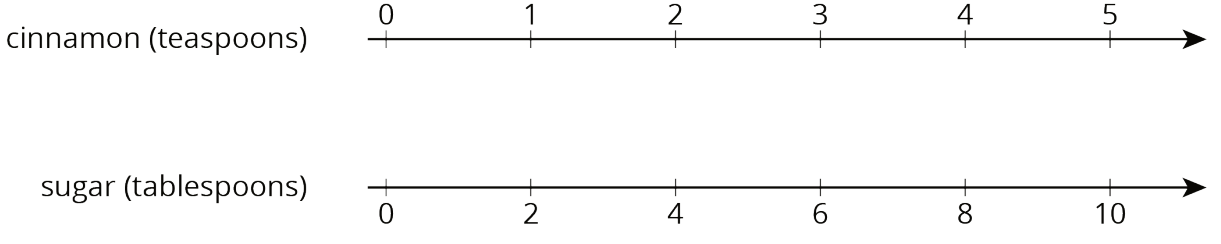
Problem 1

Statement

A recipe for cinnamon rolls uses 2 tablespoons of sugar per teaspoon of cinnamon for the filling. Complete the double number line diagram to show the amount of cinnamon and sugar in 3, 4, and 5 batches.



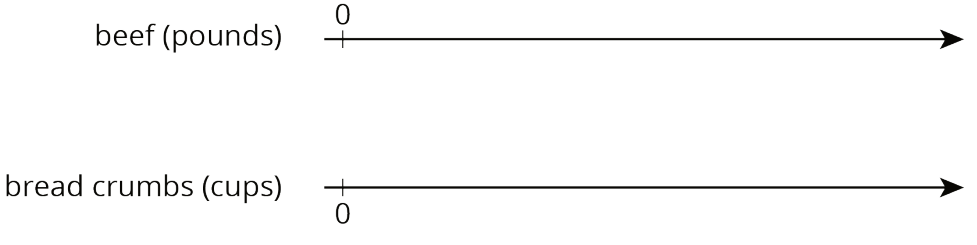
Solution



Problem 2

Statement

One batch of meatloaf contains 2 pounds of beef and $\frac{1}{2}$ cup of bread crumbs. Complete the double number line diagram to show the amounts of beef and bread crumbs needed for 1, 2, 3, and 4 batches of meatloaf.



Solution

1 batch: 2 pounds of beef, $\frac{1}{2}$ cup of bread crumbs. 2 batches: 4 pounds of beef, 1 cup of bread crumbs. 3 batches: 6 pounds of beef, $1\frac{1}{2}$ cups of bread crumbs. On a double number line, the top line is labeled 2, 4, 6, 8 and the bottom line is labeled $\frac{1}{2}$, 1, $1\frac{1}{2}$, 2.

Problem 3

Statement

A recipe for tropical fruit punch says, “Combine 4 cups of pineapple juice with 5 cups of orange juice.”

a. Create a double number showing the amount of each type of juice in 1, 2, 3, 4, and 5 batches of the recipe.

b. If 12 cups of pineapple juice are used with 20 cups of orange juice, will the recipe taste the same? Explain your reasoning.

c. The recipe also calls for $\frac{1}{3}$ cup of lime juice for every 5 cups of orange juice. Add a line to your diagram to represent the amount of lime juice in different batches of tropical fruit punch.

Solution

a. Answers vary. A correct double number line will have equally spaced tick marks. A line labeled “cups of pineapple juice” is labeled 0, 4, 8, 12, 16, 20 and a line labeled “cups of orange juice” is labeled 0, 5, 10, 15, 20, 25.

b. No, it will not taste the same. 12 cups of pineapple juice should be mixed with 15 cups of orange juice.

c. A line labeled “cups of lime juice” is labeled $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}$.

Problem 4

Statement

One batch of pink paint uses 2 cups of red paint and 7 cups of white paint. Mai made a large amount of pink paint using 14 cups of red paint.

a. How many batches of pink paint did she make?

b. How many cups of white paint did she use?

Solution

a. 7 batches (because 14 is $7 \cdot 2$)

b. 49 cups (because $7 \cdot 7 = 49$)

(From Unit 2, Lesson 4.)

Problem 5

Statement

a. Find three different ratios that are equivalent to the ratio $3 : 11$.

b. Explain why your ratios are equivalent.

Solution

a. Answers vary. Sample response: $6 : 22$, $9 : 33$, $12 : 44$.

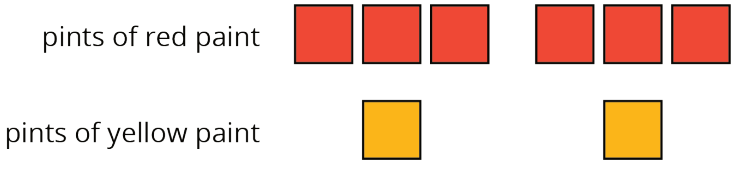
b. Answers vary. Sample response: These ratios come from $3 : 11$ by multiplying both numbers in the ratio by 2, 3, and 4 respectively.

(From Unit 2, Lesson 5.)

Problem 6

Statement

Here is a diagram that represents the pints of red and yellow paint in a mixture.



Select **all** statements that accurately describe the diagram.

A. The ratio of yellow paint to red paint is 2 to 6.

B. For every 3 pints of red paint, there is 1 pint of yellow paint.

C. For every pint of yellow paint, there are 3 pints of red paint.

D. For every pint of yellow paint there are 6 pints of red paint.

E. The ratio of red paint to yellow paint is $6 : 2$.

Solution

["A", "B", "C", "E"]

(From Unit 2, Lesson 2.)