Lesson 9: Constant Speed

Goals

- Calculate the distance an object travels in 1 unit of time and express it using a phrase like "meters per second" (orally and in writing).
- For an object moving at a constant speed, use a double number line diagram to represent equivalent ratios between the distance traveled and elapsed time.
- Justify (orally and in writing) which of two objects is moving faster, by identifying that it travels more distance in the same amount of time or that it travels the same distance in less time.

Learning Targets

- I can choose and create diagrams to help me reason about constant speed.
- If I know an object is moving at a constant speed, and I know two of these things: the distance it travels, the amount of time it takes, and its speed, I can find the other thing.

Lesson Narrative

In the previous lesson, students used the context of shopping to explore how equivalent ratios and ratios involving one can be used to find unknown amounts. In this lesson, they revisit these ideas in a new context—constant speed—and through concrete experiences. Students measure the time it takes them to travel a predetermined distance—first by moving slowly, then quickly—and use it to calculate and compare the speed they traveled in **meters per second**.

Here, double number lines are used to represent the association between distance and time, and to convey the idea of constant speed as a set of equivalent ratios (e.g., 10 meters traveled in 20 seconds at a constant speed means that 0.5 meters is traveled in 1 second, and 5 meters is traveled in 10 seconds). Students come to understand that, like price, speed can be described using the terms *per* and *at this rate*.

The idea of a constant speed relating the quantities of distance and time is foundational for the later, more abstract idea of a constant rate, and is important in the development of students' ability to reason abstractly about quantities (MP2).

Alignments

Building On

- 5.NBT.A.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.

Addressing

- 6.RP.A.3.b: Solve unit rate problems including those involving unit pricing and constant speed. For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?

Instructional Routines

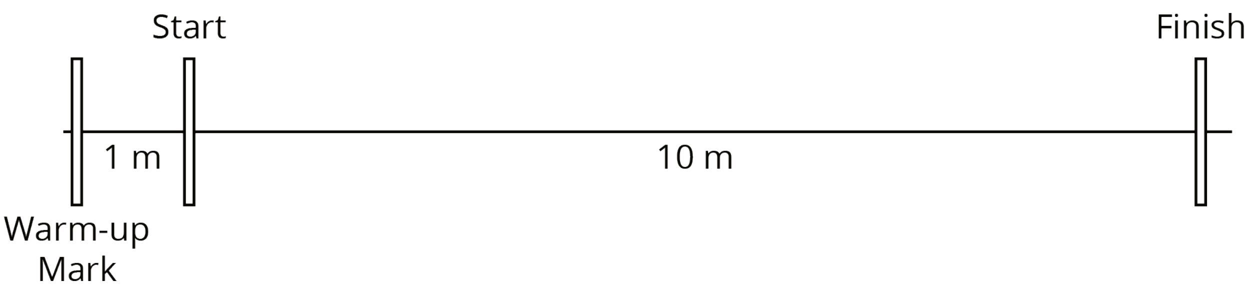
- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Masking tape
Meter sticks
Stopwatches
String

Required Preparation

Before class, set up 4 paths with a 1-meter warm-up zone and a 10-meter measuring zone.



Student Learning Goals

Let’s use ratios to work with how fast things move.

9.1 Number Talk: Dividing by Powers of 10

Warm Up: 10 minutes
This number talk encourages students to use the structure of base ten numbers to find the quotient of a base ten number and 10. The goal is to get students to see how understanding each quotient helps them find the next quotient. Reasoning about this computation will be important in both this lesson and future lessons where students are working with the metric system and percentages.

Building On

- 5.NBT.A.1

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find the quotient mentally.

$30 \div 10$

$34 \div 10$

$3.4 \div 10$

$34 \div 100$

Student Response

- $30 \div 10 = 3$ Possible strategy: $3 \cdot 10 = 30$, or students may say 10 goes into 30 three times.
- $34 \div 10 = 3.4$ Possible strategy: Students may say 10 goes into 34 three times with four tenths (written as a fraction or decimal) left over.
- $3.4 \div 10 = 0.34$ Possible strategy: Using the previous problem, since 34 was divided by 10, students may divide their previous answer by 10. $3.4 \div 10 = 0.34$
- $34 \div 100 = 0.34$ Possible strategy: Students may notice both the dividend and divisor were multiplied by 10 to get this problem, so the quotient is the same.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Emphasize student strategies based in place value to explain methods students may have learned about “moving the decimal point” left or right or “crossing out zeros.” To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

9.2 Moving 10 Meters

25 minutes
This activity gives students first-hand experience in relating ratios of time and distance to speed. Students time one another as they move 10 meters at a constant speed—first slowly and then quickly—and then reason about the distance traveled in 1 second.

Double number lines play a key role in helping students see how time and distance relate to constant speed, allowing us to compare how quickly two objects are moving in two ways. We can look at how long it takes to move 10 meters (a shorter time needed to move 10 meters means faster movement), or at how far one travels in 1 second (a longer distance in one second means faster movement).

Along the way, students see that the language of “per” and “at this rate,” which was previously used to talk about unit price, is also relevant in the context of constant speed. They begin to use “meters per second” to express measurements of speed.

As students work, notice the different ways they use double number lines or other means to reason about distance traveled in one second.

Addressing

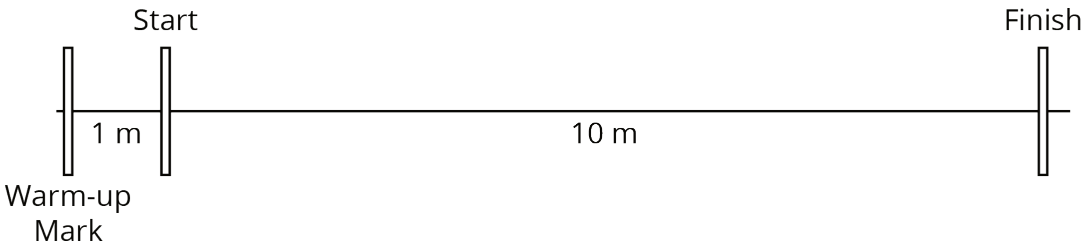
- 6.RP.A.3.b

Instructional Routines

- MLR7: Compare and Connect

Launch

Before class, set up 4 paths with a 1-meter warm-up zone and a 10-meter measuring zone.



Arrange students into 4 groups, with one for each path. Provide a stopwatch. Explain that they will gather some data on the time it takes to move 10 meters. Select a student to be your partner and demonstrate the activity for the class.

- Share that the experiment involves timing how long it takes to move the distance from the start line to the finish line.
- Explain that each person in the pair will play two roles: “the mover” and “the timer.” Each mover will go twice—once slowly and once quickly—starting at the warm-up mark each time. The initial 1-meter-long stretch is there so the mover can accelerate to a constant speed before the timing begins.
- Demonstrate the timing protocol as shown in the task statement.

Stress the importance of the mover moving at a constant speed while being timed. The warm-up segment is intended to help them reach a steady speed. To encourage students to move slowly, consider asking them to move as if they are balancing something on their head or carrying a full cup of water, trying not to spill it.

Alternatively, set up one path and ask for two student volunteers to demonstrate while the rest of the class watches.

Access for Students with Disabilities

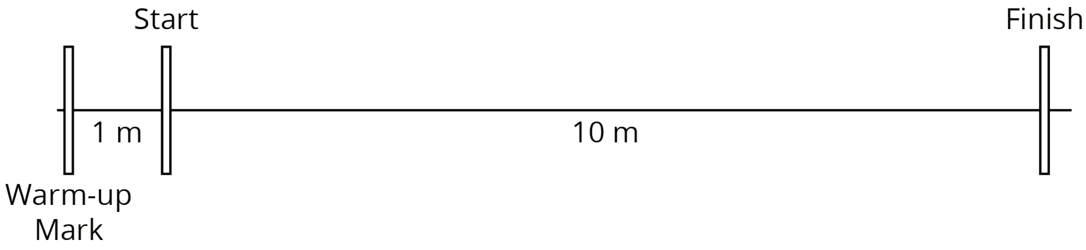
Representation: Internalize Comprehension. Begin with a physical demonstration of the activity. Highlight connections between prior understandings about using number lines to show how time and distance relate to constant speed.
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Students may have difficulty estimating the distance traveled in 1 second. Encourage them to mark the double number line to help. For example, marking 5 meters halfway between 0 and 10 and determining the elapsed time as half the recorded total may cue them to use division.

Student Task Statement

Your teacher will set up a straight path with a 1-meter warm-up zone and a 10-meter measuring zone. Follow the following instructions to collect the data.



1. a. The person with the stopwatch (the "timer") stands at the finish line. The person being timed (the "mover") stands at the warm-up line.

   b. On the first round, the mover starts moving *at a slow, steady speed* along the path. When the mover reaches the start line, they say, "Start!" and the timer starts the stopwatch.

   c. The mover keeps moving steadily along the path. When they reach the finish line, the timer stops the stopwatch and records the time, rounded to the nearest second, in the table.

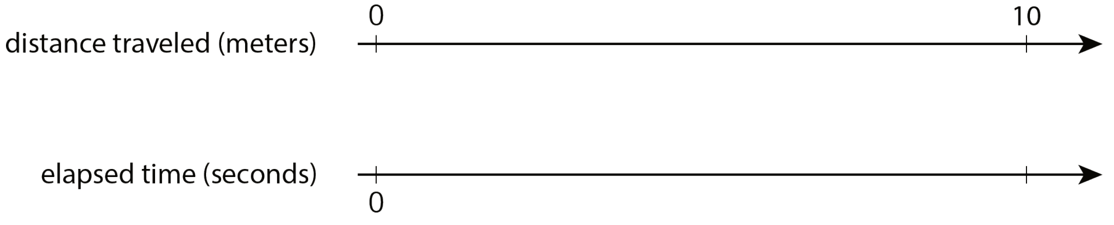
   d. On the second round, the mover follows the same instructions, but this time, moving *at a quick, steady speed*. The timer records the time the same way.

   e. Repeat these steps until each person in the group has gone twice: once at a slow, steady speed, and once at a quick, steady speed.

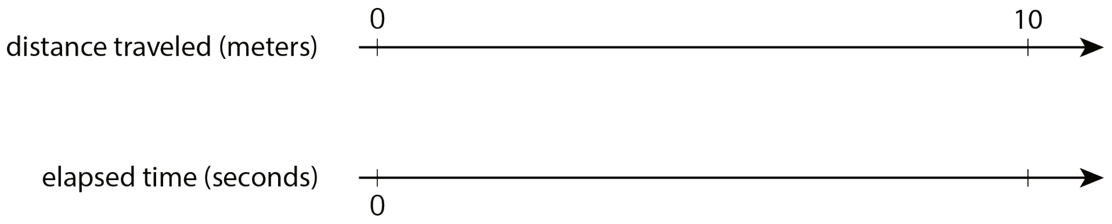
your slow moving time (seconds)	your fast moving time (seconds)

2. After you finish collecting the data, use the double number line diagrams to answer the questions. Use the times your partner collected while you were moving.

   Moving slowly:



   Moving quickly:



a. Estimate the distance in meters you traveled in 1 second when moving slowly.

b. Estimate the distance in meters you traveled in 1 second when moving quickly.

c. Trade diagrams with someone who is not your partner. How is the diagram representing someone moving slowly different from the diagram representing someone moving quickly?

Student Response

1. Diagrams vary.
2. Answers vary. Sample response: The diagram that represents moving quickly will have a higher number of meters per second, or a lower number of seconds at 10 meters.

Activity Synthesis

Select students to share who used different methods to reason about the distance traveled in 1 second. It may be helpful to discuss the appropriate amount of precision for their answers. Dividing the distance by the elapsed time can result in a quotient with many decimal places; however, the nature of this activity leads to reporting an approximate answer.

During the discussion, demonstrate the use of the phrase **meters per second** or emphasize it, if it comes up naturally in students' explanations. Discuss how we can use double number lines to distinguish faster movement from slower movement. If it hasn't already surfaced in discussion, help students see we can compare the time it takes to travel the same distance (in this case, 10 meters) as well as the distance traveled in the same amount of time (say, 1 second).

Explain to students that when we represent time and distance on a double number line, we are saying the object is traveling at a constant speed or a constant rate. This means that the ratios of meters traveled to seconds elapsed (or miles traveled to hours elapsed) are equivalent the entire time the object is traveling. The object does not move faster or slower at any time. The equal intervals on the double number line show this steady rate.

Access for English Language Learners

Representing: MLR7 Compare and Connect. As students share different approaches for reasoning about distance traveled in 1 second, ask students to identify "what is the same and what is different?" about the approaches. Help students connect approaches by asking "Where do you see the measurement of speed '____ meters per second' in each approach?" This helps students connect the concept of rate and a visual representation of that rate.
Design Principle(s): Maximize meta-awareness

9.3 Moving for 10 Seconds

10 minutes (there is a digital version of this activity)
In the previous activity, students traveled the same distance in differing amounts of time. In this activity, students analyze a situation where two people travel for the same amount of time, each at a constant speed, but go different distances. The use of double number lines is suggested, but not required.

Monitor students' work and notice different ways students compare speeds. For the first question, students may use meters per second or compare the distance traveled in the same number of seconds. For the last question, they may draw a double number line for each of the scenarios being compared, or calculate each speed in meters per second.

Addressing

- 6.RP.A.3.b

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR5: Co-Craft Questions
- Think Pair Share

Launch

Keep students with the same partners. Give students quiet think time, and then time to share their responses with their partners.

Tell students that, in the last activity, everyone traveled the same distance but in different times. Now they will analyze a situation in which two people travel for the same amount of time but cover different distances.

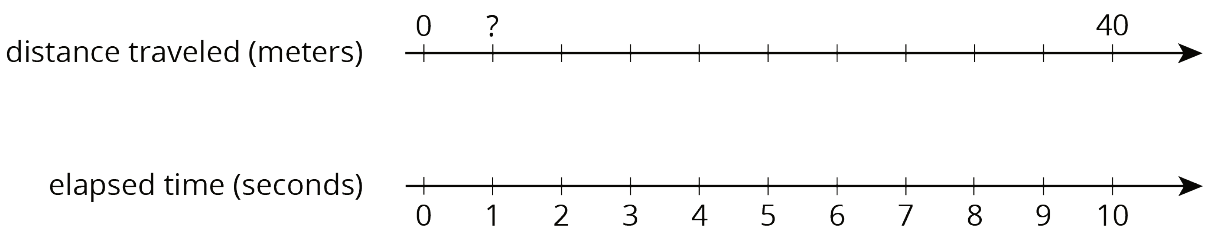
If students have digital access, they can use an applet to explore the problem and solidify their thinking. This applet is similar to the one used in the "grocery shopping" lessons. To maximize the effectiveness of the applet, encourage students to have their data from the previous activities organized.

Access for English Language Learners

Writing: MLR5 Co-Craft Questions. Present students with the situation "Lin and Diego both ran for 10 seconds, each at their own constant speed. Lin ran 40 meters and Diego ran 55 meters" without revealing the questions that follow, and ask students to write possible mathematical questions about the situation. Then, invite students to share their questions with a partner before sharing with the whole class. This helps students produce the language of mathematical questions and talk about the relationships between the two speeds in this task. Ask students to use the phrase "at a constant speed" so that they must reason about its mathematical meaning.
Design Principle(s): Maximize meta-awareness

Anticipated Misconceptions

Instead of dividing 40 by 10, some students may instead calculate $10 \div 40$. Ask them to articulate what the resulting number means (0.25 seconds to travel 1 meter) and contrast that meaning with what the problem is asking (how many meters in one second). Another approach would be to encourage them to draw a double number line and think about how they can figure out what value for distance corresponds to 1 second on the line for elapsed time.



Student Task Statement

Lin and Diego both ran for 10 seconds, each at their own constant speed. Lin ran 40 meters and Diego ran 55 meters.

1. Who was moving faster? Explain your reasoning.

2. How far did each person move in 1 second? If you get stuck, consider drawing double number line diagrams to represent the situations.

3. Use your data from the previous activity to find how far *you* could travel in 10 seconds at your quicker speed.

4. Han ran 100 meters in 20 seconds at a constant speed. Is this speed faster, slower, or the same as Lin's? Diego's? Yours?

Student Response

1. Diego ran faster, covering a greater distance in the same amount of time.
2. Lin ran 4 meters per second, and Diego ran 5.5 meters per second.
3. Answers vary, but 10 times the distance traveled in 1 second.
4. Han ran 5 meters per second, which is faster than Lin's speed, but slower than Diego's.

Are You Ready for More?

Lin and Diego want to run a race in which they will both finish when the timer reads exactly 30 seconds. Who should get a head start, and how long should the head start be?

Student Response

Lin needs a 45-meter head start. Lin will travel 120 meters in 30 seconds, and Diego will travel 165 meters in 30 seconds.

Activity Synthesis

Select students who reasoned differently to share. Some students will know that Diego ran faster, simply because he ran further, but this reasoning is not always correct. Han runs further, but is slower than Diego because he had more time. Be sure to attend to both distance and time when making the comparison. Help students draw connections between the different ways they represented and reasoned about the problem.

During the discussion, keep as much emphasis as possible on the concept of *meters per second*.

Lesson Synthesis

The work in this lesson parallels the work in the previous lesson. Knowing a speed in **meters per second** gives the same kind of information as knowing a unit price in dollars per item.

The overall objective is for students to see consistencies in the underlying mathematical structure of these contexts.

Time permitting, discuss the ways in which this work was similar to the work on unit prices, asking students to state in their own words which actions and methods felt consistent.

9.4 Train Speeds

Cool Down: 5 minutes

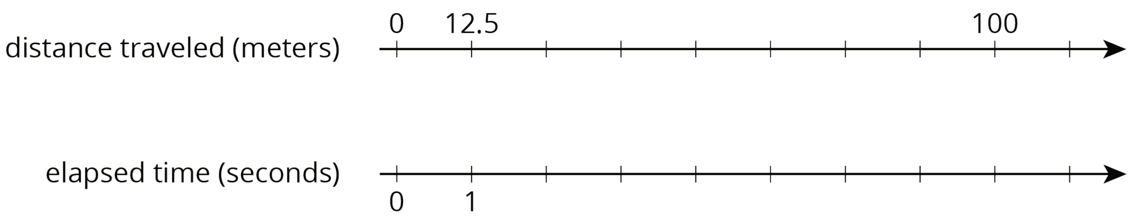
Addressing

- 6.RP.A.3.b

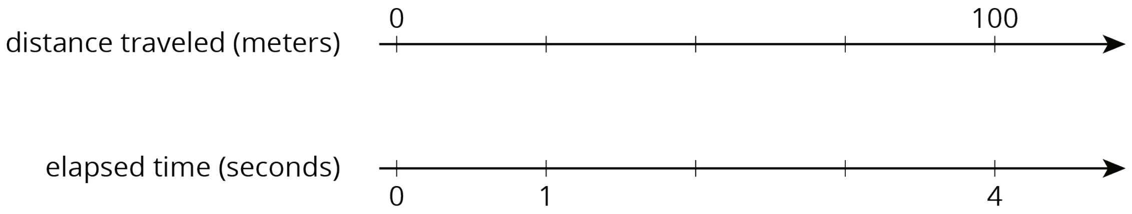
Student Task Statement

Two trains are traveling at constant speeds on different tracks.

Train A:



Train B:



Which train is traveling faster? Explain your reasoning.

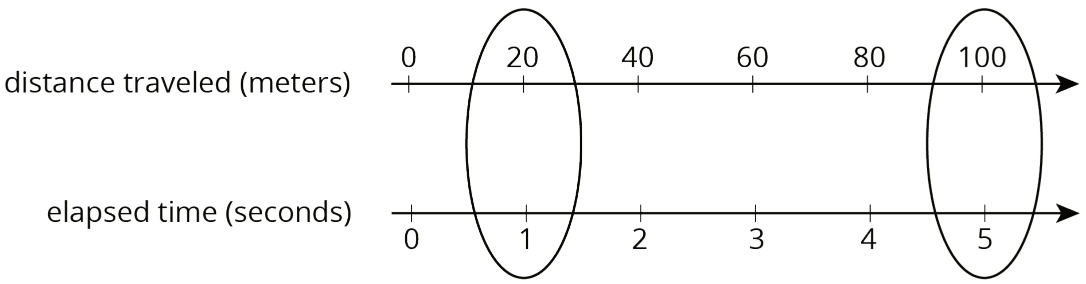
Student Response

Train B travels faster because it only took 4 seconds to travel 100 meters, while it took Train A 8 seconds to go the same distance.

Train B travels faster because its speed is 25 meters per second. Train A's speed is 12.5 meters per second.

Student Lesson Summary

Suppose a train traveled 100 meters in 5 seconds at a constant speed. To find its speed in **meters per second,** we can create a double number line:



The double number line shows that the train's speed was 20 meters per second. We can also find the speed by dividing: $100 \div 5 = 20$.

Once we know the speed in meters per second, many questions about the situation become simpler to answer because we can multiply the amount of time an object travels by the speed to get the distance. For example, at this rate, how far would the train go in 30 seconds? Since $20 \cdot 30 = 600$, the train would go 600 meters in 30 seconds.

Glossary

- meters per second

Lesson 9 Practice Problems

Problem 1

Statement

Han ran 10 meters in 2.7 seconds. Priya ran 10 meters in 2.4 seconds.

a. Who ran faster? Explain how you know.

b. At this rate, how long would it take each person to run 50 meters? Explain or show your reasoning.

Solution

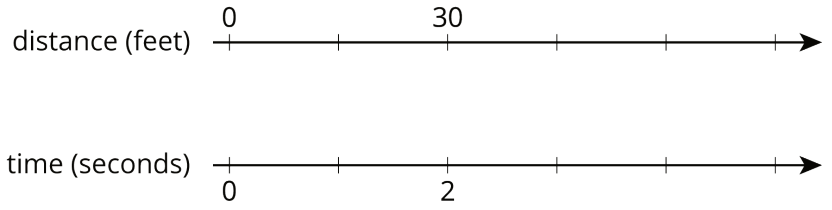
a. Priya ran faster. Sample explanation: Priya ran the same distance (10 meters) in *less* time than Han. This means she was running faster.

b. At this rate, it would take Han13.5 seconds to run 50 meters. Since 50 meters is 5 times 10 meters, the time it would take is 5 times 2.7 seconds. It would take Priya 12 seconds, which is 5 times 2.4 seconds, to run 50 meters.

Problem 2

Statement

A scooter travels 30 feet in 2 seconds at a constant speed.



a. What is the speed of the scooter in feet per second?

b. Complete the double number line to show the distance the scooter travels after 1, 3, 4, and 5 seconds.

c. A skateboard travels 55 feet in 4 seconds. Is the skateboard going faster, slower, or the same speed as the scooter?

Solution

a. 15 feet per second

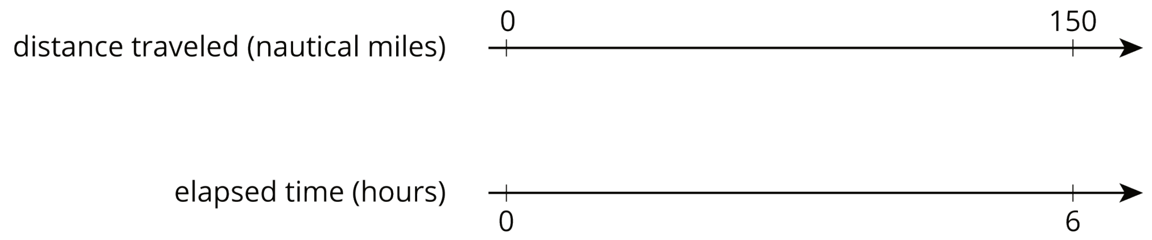
b. Distance: 0, 15, 30, 45, 60, 75. Time: 0, 1, 2, 3, 4, 5.

c. Slower. The scooter travels 60 feet in 4 seconds, so it is going faster than the skateboard, which travels 55 feet in 4 seconds.

Problem 3

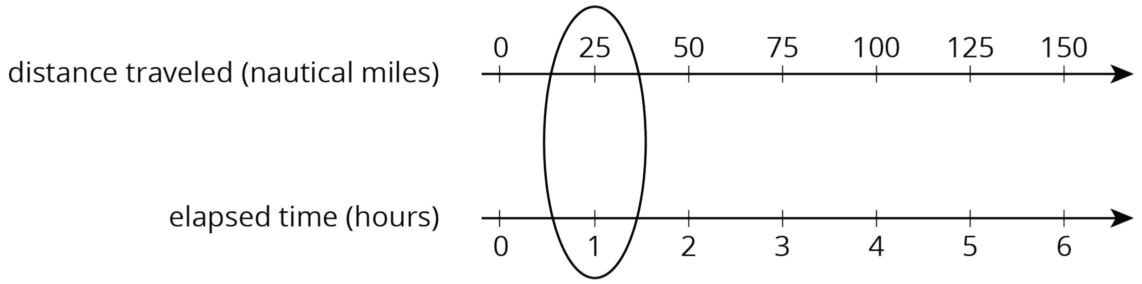
Statement

A cargo ship traveled 150 nautical miles in 6 hours at a constant speed. How far did the cargo ship travel in one hour?



Solution

The ship travels 25 nautical miles in 1 hour. Possible strategy:



Problem 4

Statement

A recipe for pasta dough says, “Use 150 grams of flour per large egg.”

a. How much flour is needed if 6 large eggs are used?

b. How many eggs are needed if 450 grams of flour are used?

Solution

a. 900 grams

b. 3 eggs

(From Unit 2, Lesson 3.)

Problem 5

Statement

The grocery store is having a sale on frozen vegetables. 4 bags are being sold for \$11.96. At this rate, what is the cost of:

a. 1 bag

b. 9 bags

Solution

a. \$2.99

b. \$26.91

(From Unit 2, Lesson 8.)

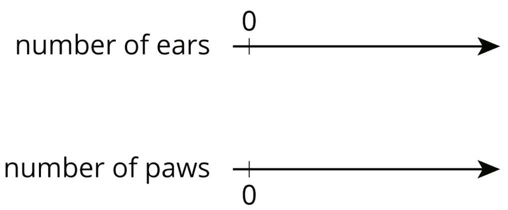
Problem 6

Statement

A pet owner has 5 cats. Each cat has 2 ears and 4 paws.

a. Complete the double number line to show the numbers of ears and paws for 1, 2, 3, 4, and 5 cats.

b. If there are 3 cats in the room, what is the ratio of ears to paws?



a. If there are 4 cats in the room, what is the ratio of paws to ears?

b. If all 5 cats are in the room, how many more paws are there than ears?

Solution

a. Ears: 2, 4, 6, 8, 10; Paws: 4, 8, 12, 16, 20

b. $6 : 12$

c. $16 : 8$

d. 10

(From Unit 2, Lesson 7.)

Problem 7

Statement

Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a representation that shows why they are equivalent ratios.

a. $5 : 1$ and $15 : 3$

b. $25 : 5$ and $10 : 2$

c. $198 : 1{,}287$ and $2 : 13$

Solution

Answers vary. Sample response:

a. Multiplying the numbers in the first ratio by 3 gives the numbers in the second ratio.

b. Multiplying the numbers in the second ratio by 10 gives the numbers in the first ratio.

c. Multiply 2 by 99 (or $100 - 1$), to get 198 (or $200 - 2$), and multiply 13 by 99, to get 1,287 $(1,300 - 13)$.

(From Unit 2, Lesson 5.)