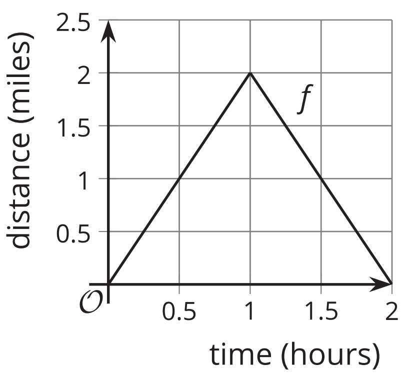
Lesson 9: Scaling the Inputs

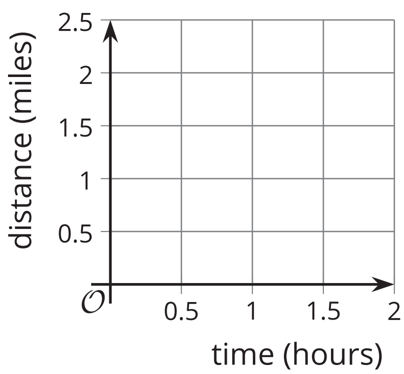
- Let’s use scale factors in different ways.

9.1: Out and Back

Every weekend, Elena takes a walk along the straight road in front of her house for 2 miles, then turns around and comes back home. Let’s assume Elena walks at a constant speed.

Here is a graph of the function f that gives her distance $f(t)$, in miles, from home as a function of time t if she walks 2 miles per hour.





1. Sketch a graph of the function g that gives her distance $g(t)$, in miles, from home as a function of time t if she walks 4 miles per hour.

2. Write an equation for g in terms of f. Be prepared to explain why your equation makes sense.

9.2: A New Set of Wheels

Remember Clare on the Ferris wheel? In the table, we have the function F which gives her height $F(t)$ above the ground, in feet, t seconds after starting her descent from the top. Today Clare tried out two new Ferris wheels.

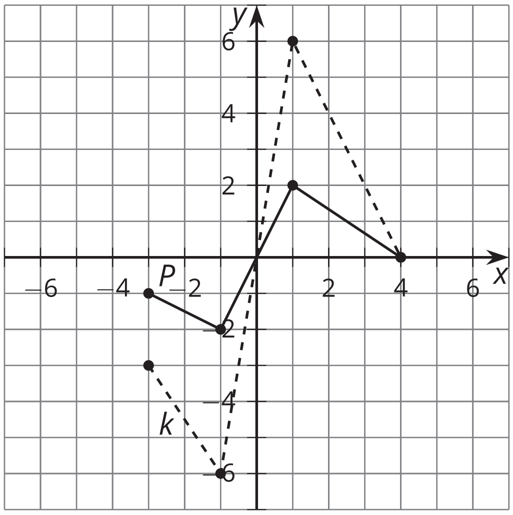
- The first wheel is twice the height of F and rotates at the same speed. The function g gives Clare's height $g(t)$, in feet, t seconds after starting her descent from the top.
- The second wheel is the same height as F but rotates at half the speed. The function h gives Clare's height $h(t)$, in feet, t seconds after starting her descent from the top.

t	$F(t)$	$g(t)$	$h(t)$
0	212		
20	181		
40	106		
60	31		
80	0		

1. Complete the table for the function g.
2. Explain why there is not enough information to find the exact values for $h(20)$ and $h(60)$.
3. Complete as much of the table as you can for the function h, modeling Claire's height on the second Ferris wheel.
4. Express g and h in terms of f. Be prepared to explain your reasoning.

9.3: The Many Transformations of a Function P

Function k is a transformation of function P due to a scale factor.



1. Write an equation for k in terms of P.

2. On the same axes, graph the function m where $m(x) = P(0.75x)$.

3. The highest point on the graph of P is $(1, 2)$. What is the highest point on the graph of a function n where $n(x) = P(5x)$? Explain or show your reasoning.

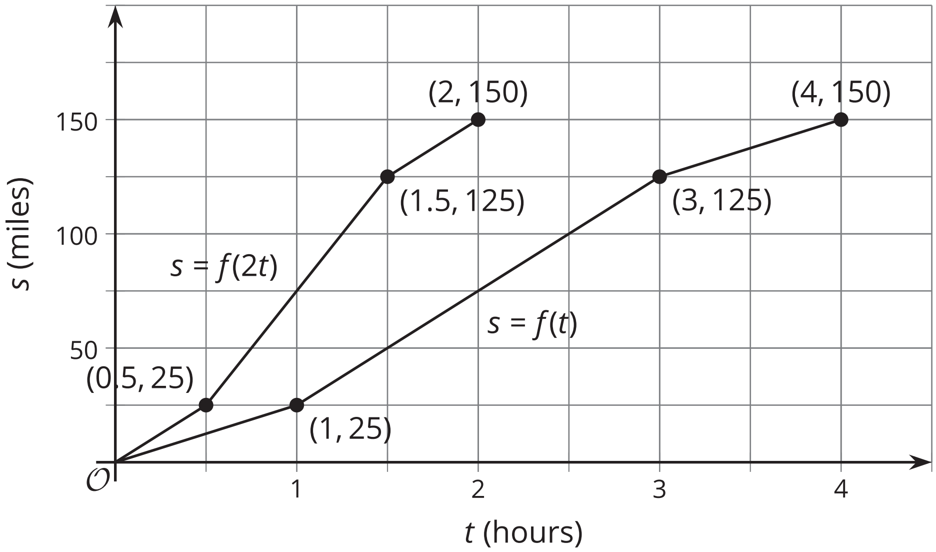
4. The point furthest to the right on the graph of P is $(4, 0)$. If the point furthest to the right on the graph of a function q is $(18, 0)$, write a possible equation for q in terms of P.

Are you ready for more?

What transformation takes $f(x) = 2x(x - 4)$ to $g(x) = 8x(x - 2)$?

Lesson 9 Summary

Here are two graphs showing the distance traveled by two trains t hours into their journeys. What do you notice?



Where Train A traveled 25 miles in 1 hour, Train B traveled 25 miles in half the time. Similarly, Train A traveled 150 miles in 4 hours while Train B traveled 150 miles in only 2 hours. Train B is traveling twice the speed of Train A.

A train travelling twice the speed gets to any particular point along the track in half the time, so the graph for Train B is compressed horizontally by a factor of $\frac{1}{2}$ when compared to the graph of Train A. If the function $f(t)$ represents the distance Train A travels in t hours, then $f(2t)$ represents the distance Train B travels in t hours, because Train B goes as far in t hours as Train A goes in $2t$ hours.

If a different Train C were going one fourth the speed of Train A, then its motion would be represented by $s = f(0.25t)$ and the graph would be stretched horizontally by a factor of 4 since it would take four times as long to travel the same distance.