# Lesson 10: Combining Functions

- Let’s make some new functions using other functions.

## 10.1: Notice and Wonder: Are Book Sales Improving?

What do you notice? What do you wonder?

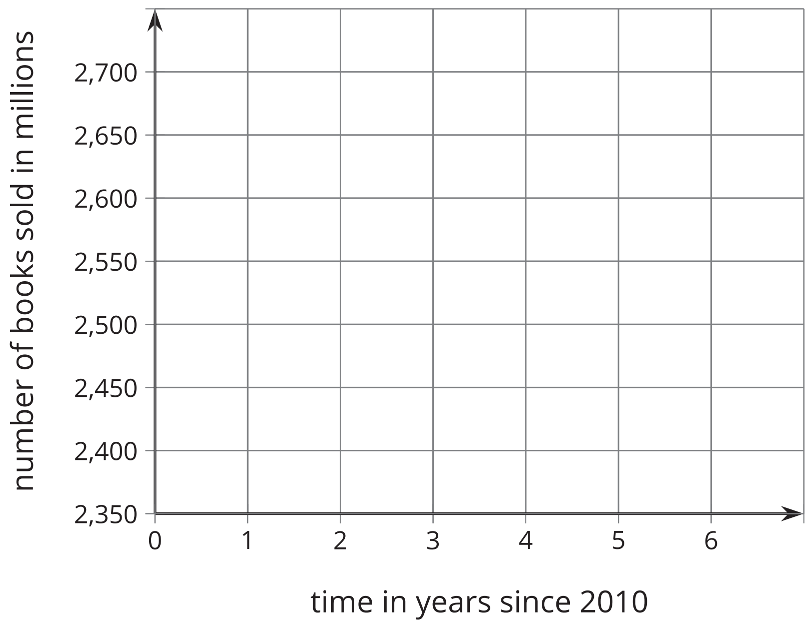
| $t$ (years since 2010) | number of books sold in the US (millions) | population of the US (millions) |
|---|---|---|
| 0 | 2,530 | 309.35 |
| 1 | 2,400 | 311.64 |
| 2 | 2,730 | 313.99 |
| 3 | 2,720 | 316.23 |
| 4 | 2,700 | 318.62 |
| 5 | 2,710 | 321.04 |
| 6 | 2,700 | 323.41 |

## 10.2: How Many Books Can One Person Have?

The table shows the values of two functions, $P$ and $B$, where $P(t)$ is the population of the US, in millions, $t$ years after 2010, and $B(t)$ is the number of books sold per year, in millions, $t$ years after 2010.

| $t$ (years since 2010) | $B(t)$ (millions) | $P(t)$ (millions) | $R(t)$ |
|---|---|---|---|
| 0 | 2,530 | 309.35 | |
| 1 | 2,400 | 311.64 | |
| 2 | 2,730 | 313.99 | |
| 3 | 2,720 | 316.23 | |
| 4 | 2,700 | 318.62 | |
| 5 | 2,710 | 321.04 | |
| 6 | 2,700 | 323.41 | |

1. Plot the values of $B$ as a function of $t$. What does the plot tell you about book sales?

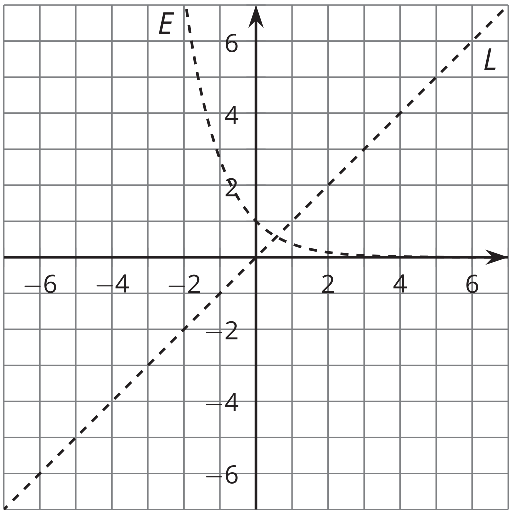


2. How many books were sold per person in 2010 and 2016? What do these values tell you about book sales?

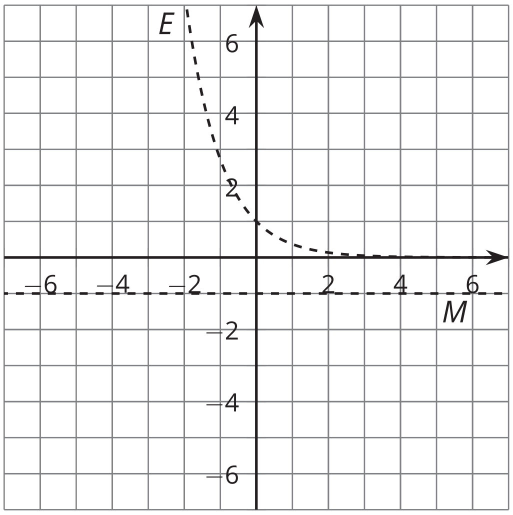
3. Define a new function $R$ by $R(t) = \frac{B(t)}{P(t)}$. Complete the table and then graph the values of $R(t)$. What do the values of $R$ tell you?

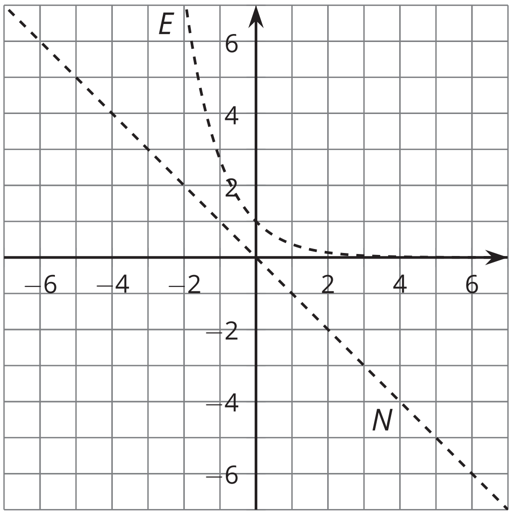
## 10.3: Adding Functions

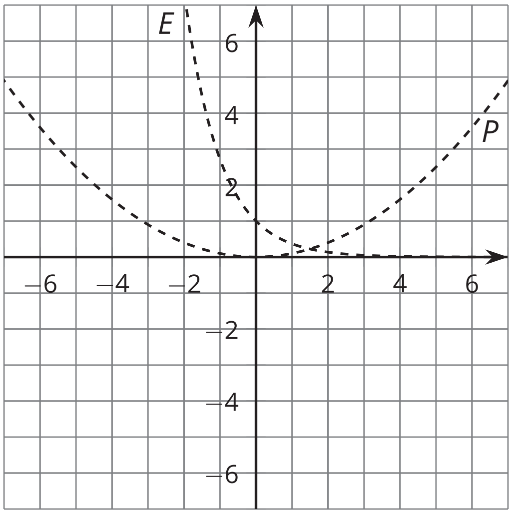
1. Here are the graphs of two functions, $E$ and $L$. Define a new function $S$ by adding $E$ and $L$, so $S(x) = E(x) + L(x)$. On the same axes, sketch what you think the graph of $S$ looks like.

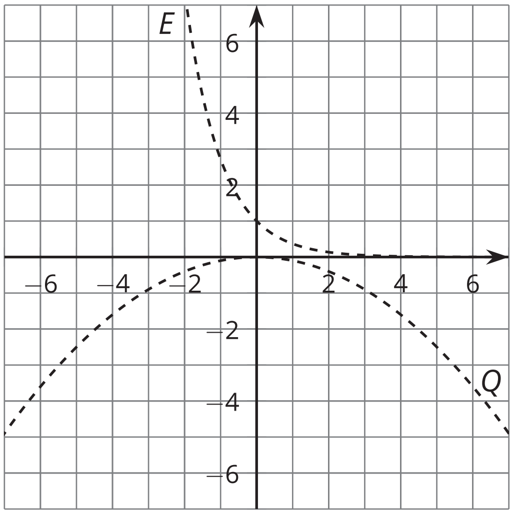



2. Sketch the graph of the sum of $E$ and each of the following functions.



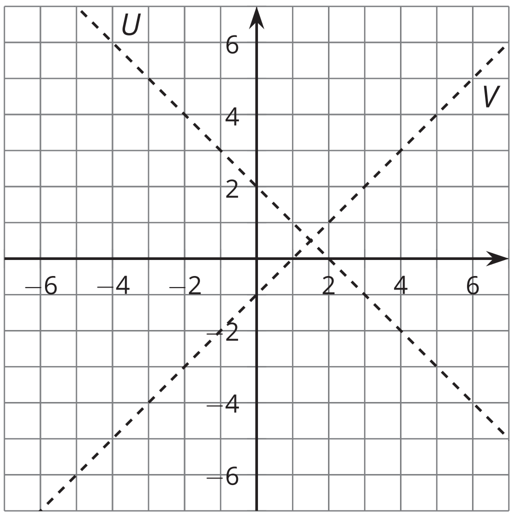






## Are you ready for more?

Here are the graphs of two functions, $U$ and $V$. Define a new function $W$ by multiplying $U$ and $V$, so $W(x) = U(x)V(x)$. On the same axes, sketch what you think the graph of $W$ looks like.

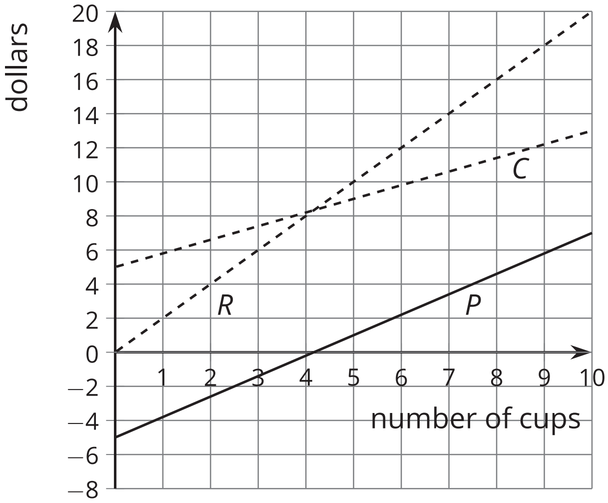


## Lesson 10 Summary

We can add, subtract, multiply, and divide functions to get new functions. For example, the cost in dollars of producing $n$ cups of lemonade at a lemonade stand is $C(n) = 5 + 0.8n$. The revenue (amount of money collected) from selling $n$ cups is $R(n) = 2n$ dollars. The profit $P(n)$ from selling $n$ cups is the revenue minus the cost, so

$$P(n) = R(n) - C(n) = 2n - (5 + 0.8n) = 1.2n - 5$$

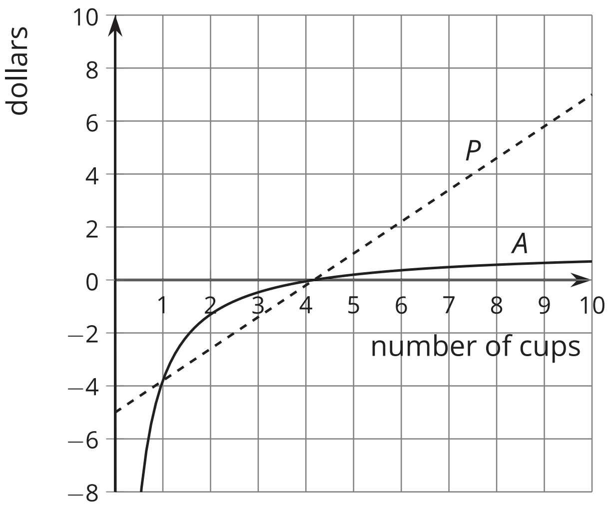
Here are the graphs of $C$, $R$, and $P$. Can you see how each value on $P$ is the result of the difference between the corresponding points on $R$ and $C$?




The average profit per cup, $A(n)$, from selling $n$ cups, is the quotient of the profit and the number of cups, so

$$A(n) = \frac{P(n)}{n} = \frac{1.2n - 5}{n} = 1.2 - \frac{5}{n}$$

Here are the graphs of $P$ and $A$. Can you see how the value of $A(n)$ is the result of the quotient of $P(n)$ and $n$? Why does it make sense that both functions are negative when $n < 4\frac{1}{6}$ and positive when $n > 4\frac{1}{6}$?




Since $n$ can only be positive, $P(n)$ and $A(n)$ always have the same sign for a given $n$ value. Notice that for the average profit to be positive, the seller has to sell at least 5 cups (since $4\frac{1}{6}$ is not in the domain, we must round up). It is also true that for a large number of cups, the average profit is close to \$1.20 per cup.