## Lesson 8: Equivalent Quadratic Expressions

* Let’s use diagrams to help us rewrite quadratic expressions.

### 8.1: Diagrams of Products



1. Explain why the diagram shows that $6\left(3+4\right)=6⋅3+6⋅4$.
2. Draw a diagram to show that $5\left(x+2\right)=5x+10$.

### 8.2: Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand, $4\left(x+2\right)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $\left(x+2\right)$ and 4 to illustrate the multiplication.



1. Draw a diagram to show that $n\left(2n+5\right)$ and $2n^{2}+5n$ are equivalent expressions.
2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.
* a. $6\left(\frac{1}{3}n+2\right)$
* b. $p\left(4p+9\right)$
* c. $5r\left(r+\frac{3}{5}\right)$
* d. $\left(0.5w+7\right)w$

### 8.3: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x+1$ and $x+3$. Use this diagram to show that $\left(x+1\right)\left(x+3\right)$ and $x^{2}+4x+3$ are equivalent expressions.
* 
1. Draw diagrams to help you write an equivalent expression for each of the following:
	1. $\left(x+5\right)^{2}$
	2. $2x\left(x+4\right)$
	3. $\left(2x+1\right)\left(x+3\right)$
	4. $\left(x+m\right)\left(x+n\right)$
2. Write an equivalent expression for each expression without drawing a diagram:
	1. $\left(x+2\right)\left(x+6\right)$
	2. $\left(x+5\right)\left(2x+10\right)$

#### Are you ready for more?



1. Is it possible to arrange an $x$ by $x$ square, five $x$ by 1 rectangles and six 1 by 1 squares into a single large rectangle?  Explain or show your reasoning.
2. What does this tell you about an equivalent expression for $x^{2}+5x+6$?
3. Is there a different non-zero number of 1 by 1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

### Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at $x$ dollars can be expressed with $x\left(18−x\right)$, which can also be written as $18x−x^{2}$. The former is a product of $x$ and $18−x$, and the latter is a difference of $18x$ and $x^{2}$, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $\left(x+2\right)\left(x+3\right)$. We can write an equivalent expression by thinking about each factor, the $\left(x+2\right)$ and $\left(x+3\right)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $\left(x+2\right)$ and $\left(x+3\right)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $\left(x+2\right)\left(x+3\right)$ is equivalent to $x^{2}+2x+3x+6$, or to $x^{2}+5x+6$.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the $x$ and the 2 in $x+2$) is multiplied by every term in the other factor (the $x$ and the 3 in $x+3$).



In general, when a quadratic expression is written in the form of $\left(x+p\right)\left(x+q\right)$, we can apply the distributive property to rewrite it as $x^{2}+px+qx+pq$ or $x^{2}+\left(p+q\right)x+pq$.



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