Lesson 12: Navigating a Table of Equivalent Ratios

Goals

- Choose multipliers strategically while solving multi-step problems involving equivalent ratios.
- Describe (orally and in writing) how a table of equivalent ratios was used to solve a problem about prices and quantities.
- Remember that dividing by a whole number is the same as multiplying by an associated unit fraction.

Learning Targets

- I can solve problems about situations happening at the same rate by using a table and finding a “1” row.
- I can use a table of equivalent ratios to solve problems about unit price.

Lesson Narrative

The purpose of this lesson is to develop students’ ability to work with a table of equivalent ratios. It also provides opportunities to compare and contrast different ways of solving equivalent ratio problems.

Students see that a table accommodates different ways of reasoning about equivalent ratios, with some being more direct than others. They notice (MP8) that to find an unknown quantity, they can:

- Find the multiplier that relates two corresponding values in different rows (e.g., “What times 5 equals 8?”) and use that multiplier to find unknown values. (This follows the multiplicative thinking developed in previous lessons.)
- Find an equivalent ratio with one quantity having a value of 1 and use that ratio to find missing values.

amount earned ($)	time worked (hours)
90	5
18	1
144	8

All tasks in the lesson aim to strengthen students’ understanding of the multiplicative relationships between equivalent ratios—that given a ratio $a : b$, an equivalent ratio may be found by multiplying both a and b by the same factor. They also aim to build students’ awareness of how a table can facilitate this reasoning to varying degrees of efficiency, depending on one’s approach.

Ultimately, the goal of this unit is to prepare students to make sense of situations involving equivalent ratios and solve problems flexibly and strategically, rather than to rely on a procedure (such as "set up a proportion and cross multiply") without an understanding of the underlying mathematics.

To reason using ratios in which one of the quantities is 1, students are likely to use division. In the example above, they are likely to divide the 90 by 5 to obtain the amount earned per hour. Remind students that dividing by a whole number is the same as multiplying by its reciprocal (a unit fraction) and encourage the use of multiplication (as shown in the activity about hourly wages) whenever possible. Doing so will better prepare students to: 1) scale down, i.e., to find equivalent ratios involving values that are smaller than the given ones, 2) relate fractions to percentages later in the course, and 3) understand division of fractions (including the "invert and multiply" rule) in a later unit.

Alignments

Building On

- 5.NF: Grade 5 - Number and Operations---Fractions

Addressing

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.a: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Instructional Routines

- MLR5: Co-Craft Questions
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

> **Student Learning Goals**
>
> Let's use a table of equivalent ratios like a pro.

12.1 Number Talk: Multiplying by a Unit Fraction

Warm Up: 10 minutes

The purpose of this number talk is to encourage students to use the meaning of fractions and the properties of operations to find the product of fractions and decimals.

In grade 4, students multiplied a fraction by a whole number, reasoning about these problems based on their understandings of multiplication as groups of a number. In grade 5, students multiply fractions by whole numbers, reasoning in terms of taking a part of a part, whether that be by using division or partitioning a whole. In both grade levels, the context of the problem played a significant role in how students reasoned and notated the problem and solution. Based on these understandings, two ideas will be relevant to future work in the unit and are important to emphasize during discussions:

- Dividing by a number is the same as multiplying by its reciprocal.
- The commutative property of multiplication can help us solve a problem regardless of the context.

Building On

- 5.NF

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Tell students to give a signal when they have an answer and a strategy. After each problem, give students 1 minute of quiet think time and follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find the product mentally.

$\frac{1}{3} \cdot 21$

$\frac{1}{6} \cdot 21$

$(5.6) \cdot \frac{1}{8}$

$\frac{1}{4} \cdot (5.6)$

Student Response

- $\frac{1}{3} \cdot 21 = 7$. Possible strategies: $21 \div 3$ or $3 \cdot 7$.
- $\frac{1}{6} \cdot 21 = 3.5$. Possible strategies: $21 \div 6$, or divide the product from the first question by 2 because $\frac{1}{6}$ is half of $\frac{1}{3}$.
- $(5.6) \cdot \frac{1}{8} = 0.7$. Possible strategies: $5.6 \div 8$ or $8 \cdot (0.7)$.
- $\frac{1}{4} \cdot (5.6) = 1.4$. Possible strategies: $5.6 \div 4$, or multiply the product from the third question by 2 because $\frac{1}{4}$ is twice as much as $\frac{1}{8}$.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted their strategy choice. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

12.2 Comparing Taco Prices

10 minutes

The purpose of this activity is to encourage students to use a table to find the price for one taco for two different situations. Students are likely to divide the cost of the tacos by the number of tacos to find the cost for one taco, which is appropriate. Use the opportunity to remind students that dividing by a whole number is the same as multiplying by its reciprocal (a unit fraction). This insight will come in handy in future activities and lessons.

Addressing

- 6.RP.A.3.a

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Tell students that we usually use tables to show equivalent ratios, but since we do not know in advance whether the ratios of number of tacos to price will be the same, we might want to keep track of them in separate tables.

Arrange students in groups of 2. Give students 3 minutes of quiet think time, and then time to discuss their responses and reasoning with their partner.

Access for English Language Learners

Conversing: MLR5 Co-Craft Questions. Display the problem statement, "Noah bought 4 tacos and paid $6.", without revealing the questions that follow. Invite students to discuss possible mathematical questions they could ask about this situation. Listen for questions that ask about the price for one taco, or the price of multiple tacos, and select these students to share their questions with the class. This will help draw students attention to the relationships between the two quantities in this task (number of tacos and price in dollars) prior to being asked to calculate any values.
Design Principle(s): Cultivate conversation; Support sense-making

Student Task Statement

number of tacos	price in dollars

Use the table to help you solve these problems. Explain or show your reasoning.

1. Noah bought 4 tacos and paid $6. At this rate, how many tacos could he buy for $15?
2. Jada's family bought 50 tacos for a party and paid $72. Were Jada's tacos the same price as Noah's tacos?

Student Response

1. 10 tacos
2. No. Noah's tacos cost $1.50 each. Jada's cost $1.44 each.

Activity Synthesis

The focus should be on how students found the cost of a single taco in each situation. Be sure to remind students that dividing by a whole number is the same as multiplying by its reciprocal (a unit fraction).

12.3 Hourly Wages

10 minutes
This task introduces students to the strategy of using an equivalent ratio with one quantity having a value of 1 to find other equivalent ratios. Students look at a worked-out example of the strategy, make sense of how it works, and later apply it to solve other problems.

There are a couple of key insights to uncover here:

- The ratios we deal with do not always have corresponding quantities that are multiples of each other (e.g., in the task, 5 is not a multiple of 8, or vice versa).
- In those situations, finding an equivalent ratio where one of the quantities is 1 can be a helpful intermediate step.

Also highlighted and reinforced here is an idea students learned in Grade 5, that dividing by a whole number is equivalent to multiplying by its reciprocal (e.g., dividing by 5 is the same as multiplying by $\frac{1}{5}$).

Expect some students to initially overlook the benefit of using a ratio involving a "1," to rely on methods from previous work, and to potentially get stuck (especially when dealing with a decimal value in the last row). For example, since the table shows an arrow and a multiplication from the first to the second row and from the second to third, students may try to do the same to find the missing value in the fourth row. While finding a factor that can be multiplied to 8 to obtain 3 is valid, encourage students to consider an alternative, given what they already know about the situation (i.e., how much the person earned in 1 hour). If needed, scaffold their thinking by asking how much Lin would earn in 2 hours and *then* in 3 hours.

Identify a student or two who can articulate why $\frac{1}{5}$ is used as a multiplier. Also notice those who can correctly reason why using a ratio with one of the values being 1 helps to find other equivalent ratios and students who reason differently. Invite these students to share later.

Addressing

- 6.RP.A.3

Instructional Routines

- Think Pair Share

Launch

This may be some students' first time reasoning about money earned by the hour. Take a minute to ensure everyone understands the concept. Ask if anyone has earned money based on the number of hours doing a job. Some students may have experience being paid by the hour for helping with house cleaning, a family business, babysitting, dog walking, or doing other jobs.

Give students quiet think time to complete the activity and a minute to share their responses (especially to the last two questions) with a partner before discussing as a class.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate background knowledge about finding equivalent ratios. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

Lin is paid $90 for 5 hours of work. She used the table to calculate how much she would be paid at this rate for 8 hours of work.

amount earned ($)	time worked (hours)
90	5
18	1
144	8

1. What is the meaning of the 18 that appears in the table?
2. Why was the number $\frac{1}{5}$ used as a multiplier?
3. Explain how Lin used this table to solve the problem.
4. At this rate, how much would Lin be paid for 3 hours of work? For 2.1 hours of work?

Student Response

1. Lin earned $18 for 1 hour of work or for every hour of work.

2. We wanted to turn the 5 into a 1, so the 1 could be multiplied by 8. $\frac{1}{5}$ was chosen because $5 \cdot \frac{1}{5} = 1$.
3. First, they found how much Lin made in 1 hour by multiplying both the 90 and the 5 by $\frac{1}{5}$ (or dividing them both by 5). Then, they multiplied both the 18 and the 1 by 8 to find that she earned $144 in 8 hours.
4. Lin would be paid $54 for 3 hours of work and $37.80 for 2.1 hours of work.

Activity Synthesis

Select a few students to share about the use of $\frac{1}{5}$ as a multiplier and to explain the reasoning process shown in the table. If different approaches are used, take the opportunity to compare and contrast the efficacy of each.

If students had trouble reasoning to find the pay for 2.1 hours of work, help them articulate what they have done in each preceding case and urge them to think about the 2.1 the same way. If they are unsure whether multiplying 18 by 2.1 would work, encourage them to check whether the answer makes sense. (For two hours of work, Lin would earn $36, so it stands to reason that she would earn a bit more than $36 for 2.1 hours.) In doing so, students practice decontextualizing and contextualizing their reasoning and solutions (MP2).

12.4 Zeno's Memory Card

Optional: 15 minutes
Previously, students explored the limitation of a double number line when dealing with greatly scaled-up ratios; they saw that extending the number lines can be impractical. Here, they encounter a situation involving significantly scaled-down ratios, in which a double number line is likewise impractical (i.e., there is not enough room to fit relevant information) and see that a table is clearly preferable.

The given table deliberately includes more rows than necessary to answer the question. Some students may realize that it is not necessary to fill in all the rows if they use a different factor in finding equivalent ratios. Notice students who take such shortcuts so they can share later. Their reasoning can further highlight the flexibility of a table.

Addressing

- 6.RP.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Open the task with a request for two volunteers and a question. Have the volunteers stand at different distances from a wall but with a clear path toward it.

Ask: “Can either student reach the wall if every time they make a move toward it, they only move half the distance between them and the wall?”

Give students a moment to think and share their predictions with a partner. Without further explanations, ask the two volunteers to begin their halfway-at-a-time journey toward the wall. When the wall is within an arm’s reach, ask the volunteers to stop. Select two students who made different predictions—one who thought it was impossible to reach the wall and one who thought otherwise—to share their reasoning. If one opinion is not represented, share the reasoning for it yourself.

Explain that the situation at hand is a famous paradox, credited to an ancient Greek philosopher, Zeno of Elea (c. ~450 BCE). Tell students: “A paradox is a situation that both cannot be true and must be true at the same time. Going halfway toward a destination is one of Zeno’s paradoxes.”

Explain that it is both impossible and possible to reach the wall by following this go-halfway procedure. Because some distance, although increasingly small, will always be left by constantly going halfway, we say it is impossible to reach the wall. But it is also obvious that the volunteers can get close enough to touch the wall, thereby “reaching” the wall.

Tell students that the next task also uses a “go halfway” process, but in a different context.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate background knowledge about using double number lines to represent ratios. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

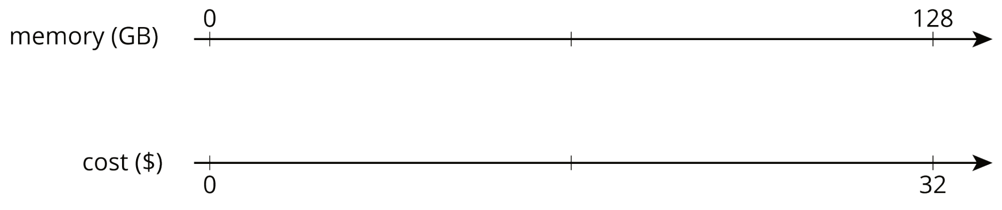
Anticipated Misconceptions

Watch out for students being overly precise or wildly imprecise with drawing tick marks on their double number line diagram. We want them to eyeball approximately half the distance, but it would be too time-consuming to measure precisely.

Student Task Statement

In 2016, 128 gigabytes (GB) of portable computer memory cost $32.

1. Here is a double number line that represents the situation:



One set of tick marks has already been drawn to show the result of multiplying 128 and 32 each by $\frac{1}{2}$. Label the amount of memory and the cost for these tick marks.

Next, keep multiplying by $\frac{1}{2}$ and drawing and labeling new tick marks, until you can no longer clearly label each new tick mark with a number.

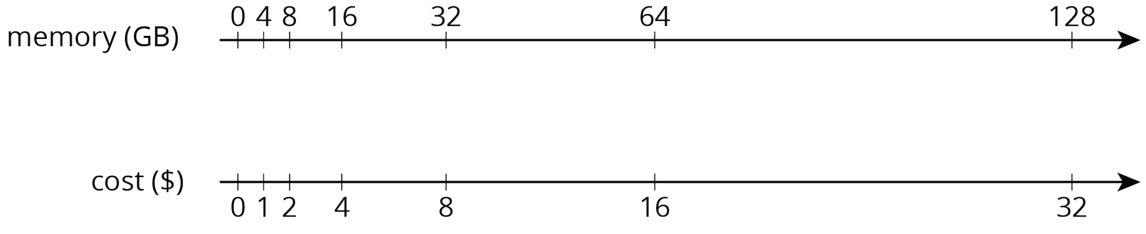
2. Here is a table that represents the situation. Find the cost of 1 gigabyte. You can use as many rows as you need.

memory (gigabytes)	cost (dollars)
128	32

3. Did you prefer the double number line or the table for solving this problem? Why?

Student Response

1. Using the double number line:



2. Using the table. Note: it’s not actually necessary to write all of these rows. Bigger jumps could be made if you multiply by a number other than $\frac{1}{2}$.

memory (gigabytes)	cost (dollars)
128	32
64	16
32	8
16	4
8	2
4	1
2	0.5
1	0.25

3. Answers vary. The purpose of this question is to give students a chance to prepare for the discussion that follows.

Are You Ready for More?

A kilometer is 1,000 meters because *kilo* is a prefix that means 1,000. The prefix *mega* means 1,000,000 and *giga* (as in gigabyte) means 1,000,000,000. One byte is the amount of memory needed to store one letter of the alphabet. About how many of each of the following would fit on a 1-gigabyte flash drive?

1. letters
2. pages
3. books
4. movies
5. songs

Student Response

1. 1,000,000,000 The rest of these are estimates and are based on assumptions and averages.
2. At around 1,500 letters per page, so 1,500 bytes per page. $1{,}000{,}000{,}000 \div 1{,}500 \approx 666{,}667$ pages.
3. At around 250 pages per book, 375,000 bytes or 375 kilobytes. $1{,}000{,}000{,}000 \div 375{,}000 \approx 2{,}667$ books.

4. According to Google, 1 gb of data is enough for about 1 hour of video. So the flash drive would not be able to hold an entire typical movie (which is longer than 1 hour).
5. A song file is about 5 megabytes, or 5,000,000 bytes. $1{,}000{,}000{,}000 \div 5{,}000{,}000 = 200$ songs. (Approximately 200 songs, since the actual size of song files varies.)

Activity Synthesis

The discussion should center around why the table was easier to use for this problem: the numbers we started with were so large that there wasn't enough room to locate 1 gigabyte on the number line.

If any students multiplied the ratios by a fraction other than $\frac{1}{2}$ so that they did not have to fill all the rows, consider highlighting this shortcut. (They could even divide 128 and 32 by 128 to arrive at an answer directly, using what they have learned about unit price.) It shows how the table enables reasoning with numbers (rather than with lengths) and is more flexible.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Before students discuss which representation was easier to use to find the cost of 1 gigabyte, ask students to describe how they used the double number line compared to how they used the table with a partner. If needed, provide sentence frames such as, "_____ was easier to use because . . ." or "I prefer _____ because . . . ." Encourage students to challenge each other when they disagree or to press for additional details when the reasoning is unclear. This will give students an opportunity to produce mathematical language to describe their reasoning about the usefulness of each representation before sharing with the whole class.
Design Principle(s): Support sense-making

Lesson Synthesis

This lesson is about using a table of equivalent ratios in an efficient way. To wrap up, highlight a few important points:

- In problems with equivalent ratios, finding an equivalent ratio containing a "1" is often a good strategy.
- To create a new row in a table of equivalent ratios, take an existing row and multiply both values by the same number.
- Remember that we can multiply whole numbers by unit fractions to get smaller numbers.

12.5 Price of Bagels

Cool Down: 5 minutes

Addressing

- 6.RP.A.3

Student Task Statement

A shop sells bagels for $5 per dozen. You can use the table to answer the questions. Explain your reasoning.

number of bagels	price in dollars
12	5

1. At this rate, how much would 6 bagels cost?

2. How many bagels can you buy for $50?

Student Response

1. $2.50

2. 120 bagels

The table might look like this:

number of bagels	price in dollars
12	5
6	2.5
120	50

Student Lesson Summary

Finding a row containing a "1" is often a good way to work with tables of equivalent ratios. For example, the price for 4 lbs of granola is $5. At that rate, what would be the price for 62 lbs of granola?

Here are tables showing two different approaches to solving this problem. Both of these approaches are correct. However, one approach is more efficient.

- Less efficient

	granola (lbs)	price ($)	
	4	5	
•2	8	10	•2
•2	16	20	•2
•2	32	40	•2
•2	64	80	•2
- 2 lbs	62	77.50	- $2.50

- More efficient

	granola (lbs)	price ($)	
	4	5	
$\cdot \frac{1}{4}$	1	1.25	$\cdot \frac{1}{4}$
•62	62	77.50	•62

Notice how the more efficient approach starts by finding the price for 1 lb of granola.

Remember that dividing by a whole number is the same as multiplying by a unit fraction. In this example, we can divide by 4 or multiply by $\frac{1}{4}$ to find the unit price.

Lesson 12 Practice Problems

Problem 1

Statement

Priya collected 2,400 grams of pennies in a fundraiser. Each penny has a mass of 2.5 grams. How much money did Priya raise? If you get stuck, consider using the table.

number of pennies	mass in grams
1	2.5
	2,400

Solution

$9.60. Possible strategy:

number of pennies	mass in grams
1	2.5
1,000	2,500
4	10
40	100
960	2,400

Problem 2

Statement

Kiran reads 5 pages in 20 minutes. He spends the same amount of time per page. How long will it take him to read 11 pages? If you get stuck, consider using the table.

time in minutes	number of pages
20	5
	1
	11

Solution

44 minutes

time in minutes	number of pages
20	5
4	1
44	11

Problem 3

Statement

Mai is making personal pizzas. For 4 pizzas, she uses 10 ounces of cheese.

number of pizzas	ounces of cheese
4	10

a. How much cheese does Mai use per pizza?

b. At this rate, how much cheese will she need to make 15 pizzas?

Solution

Mai uses 2.5 ounces of cheese per pizza, because $10 \div 4 = 2.5$. She will need 37.5 ounces of cheese for 15 pizzas, because $2.5 \cdot 15 = 37.5$.

Problem 4

Statement

Clare is paid $90 for 5 hours of work. At this rate, how many seconds does it take for her to earn 25 cents?

Solution

Clare earns 25 cents every 50 seconds. She earns $18 per hour, and an hour has 3,600 seconds. $18 is 72 quarters, and $3,600 \div 72 = 50$.

Problem 5

Statement

A car that travels 20 miles in $\frac{1}{2}$ hour at constant speed is traveling at the same speed as a car that travels 30 miles in $\frac{3}{4}$ hour at a constant speed. Explain or show why.

Solution

Answers vary. Sample responses:

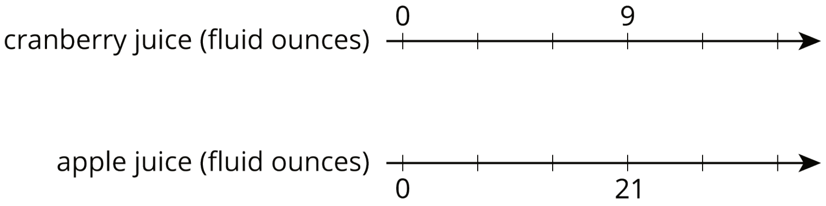
- Both cars go 10 miles in $\frac{1}{4}$ of an hour so they are traveling at the same speed.
- In 1 hour, both cars travel 40 miles so they are both traveling at the same speed.

(From Unit 2, Lesson 10.)

Problem 6

Statement

Lin makes her favorite juice blend by mixing cranberry juice with apple juice in the ratio shown on the double number line. Complete the diagram to show smaller and larger batches that would taste the same as Lin's favorite blend.



Solution

Cranberry (cups): 0, 3, 6, 9, 12, 15. Apple (cups): 0,7,14, 21, 28, 35

(From Unit 2, Lesson 6.)

Problem 7

Statement

Each of these is a pair of equivalent ratios. For each pair, explain why they are equivalent ratios or draw a representation that shows why they are equivalent ratios.

a. $600 : 450$ and $60 : 45$

b. $60 : 45$ and $4 : 3$

c. $600 : 450$ and $4 : 3$

Solution

Answers vary. Sample response:

a. $60 \cdot 10 = 600$ and $45 \cdot 10 = 450$.

b. Multiplying 4 and 3 by 15 gives 60 and 45.

c. Multiply 4 by 150 to get 600 and multiply 3 by 150 to get 450. Or use problems 4 and 5 together: problem 4 shows that $600 : 450$ is equivalent to $60 : 45$ and problem 5 shows that $60 : 45$ is equivalent to $4 : 3$. This means that $600 : 450$ is equivalent to $4 : 3$.

(From Unit 2, Lesson 5.)