Lesson 13: Tables and Double Number Line Diagrams

Goals

- Compare and contrast (orally) double number line diagrams and tables representing the same situation.
- Draw and label a table of equivalent ratios from scratch to solve problems about constant speed.

Learning Targets

- I can create a table that represents a set of equivalent ratios.
- I can explain why sometimes a table is easier to use than a double number line to solve problems involving equivalent ratios.
- I include column labels when I create a table, so that the meaning of the numbers is clear.

Lesson Narrative

In this lesson, students explicitly connect and contrast double number lines and tables. They also encounter a problem involving relatively small fractions, so the flexibility of a table makes it preferable to a double number line. Students have used tables in earlier grades to identify arithmetic patterns and record measurement equivalents. In grade 6, a new feature of working with tables is considering the relationship between values in different rows. Two features of tables make them more flexible than double number lines:

- On a double number line, differences between numbers are represented by lengths on each number line. While this feature can help support reasoning about relative sizes, it can be a limitation when large or small numbers are involved, which may consequently hinder problem solving. A table removes this limitation because differences between numbers are no longer represented by the geometry of a number line.
- A double number line dictates the ordering of the values on the line, but in a table, pairs of values can be written in any order. 5 pounds of coffee cost $40. How much does 8.5 pounds cost? You can see in the table below how being able to skip around makes for more nimble problem solving:

weight of coffee (pounds)	cost (dollars)
5	40
1	8
8.5	68

At this point in the unit, students should have a strong sense of what it means for two ratios to be equivalent, so they can fill in a table of equivalent ratios with understanding instead of just by following a procedure. Students can also always fall back to other representations if needed.

Alignments

Building On

- 5.NBT: Grade 5 - Number and Operations in Base Ten

Addressing

- 6.RP.A.3: Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.
- 6.RP.A.3.a: Make tables of equivalent ratios relating quantities with whole-number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Number Talk

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Make 1 copy of the The International Space Station blackline master for every 4 students, and cut them up ahead of time.

Student Learning Goals

Let’s contrast double number lines and tables.

13.1 Number Talk: Constant Dividend

Warm Up: 10 minutes

This number talk helps students think about what happens to a quotient when the divisor is doubled. In this lesson and in upcoming work on ratios and unit rates, students will be asked to find a fraction of a number and identify fractions on a number line.

Building On

- 5.NBT

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Tell students to give a signal when they have an answer and a strategy. After each problem, give students 1 minute of quiet think time followed by a whole-class discussion. Pause after discussing the third question and give students 1 minute of quiet think time to place the quotients on the number line.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

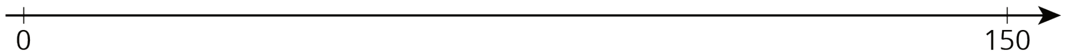
Find the quotients mentally.

$150 \div 2$

$150 \div 4$

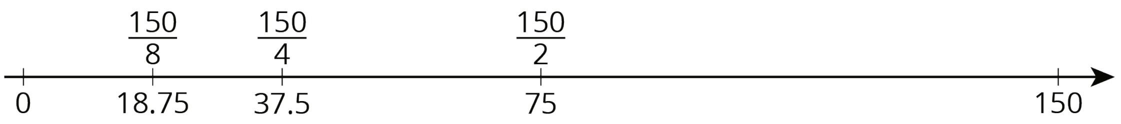
$150 \div 8$

Locate and label the quotients on the number line.



Student Response

-
    - $150 \div 2 = 75$. Possible strategies: $2 \cdot 75 = 150$, $(100 \div 2) + (50 \div 2) = 75$.
    - $150 \div 4 = 37.5$. Possible strategies: $75 \div 2 = 37.5$ from the previous question, $(148 \div 4) + (2 \div 4) = 37.5$.
    - $150 \div 8 = 18.75$. Possible strategies: $37.5 \div 2 = 18.75$ from the previous question, $144 \div 8 + 6 \div 8 = 18.75$.
- Here is the number line:



Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

For the fourth question, display the number line for all to see and invite a few students to share their reasoning about the location of each quotient on the number line. Discuss students’ observations from when they placed the numbers on the number line.

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

13.2 Moving 3,000 Meters

15 minutes

In this activity, students use tables of equivalent ratios to solve three problems, with decreasing scaffolding throughout the activity. For the first problem, students start by examining a table of equivalent ratios, noticing that descriptive column headers are important in helping you use a table of equivalent ratios to solve a problem. For the second problem, there is an empty table students can fill in. The third problem does not provide any scaffolding, allowing students to choose their own method of solving the problem.

Monitor for students solving the last problem in different ways.

Addressing

- 6.RP.A.3.a

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give 3 minutes of quiet work time and then have students work with their partner.

Student Task Statement

The other day, we saw that Han can run 100 meters in 20 seconds.

Han wonders how long it would take him to run 3,000 meters at this rate. He made a table of equivalent ratios.

1. Do you agree that this table represents the situation? Explain your reasoning.

20	100
10	50
1	5
3,000	

2. Complete the last row with the missing number.
3. What question about the situation does this number answer?
4. What could Han do to improve his table?
5. Priya can bike 150 meters in 20 seconds. At this rate, how long would it take her to bike 3,000 meters?

6. Priya’s neighbor has a dirt bike that can go 360 meters in 15 seconds. At this rate, how long would it take them to ride 3,000 meters?

Student Response

1. Answers vary. Sample response: I agree with the first three rows, but the last row would be for 3,000 seconds instead of 3,000 meters, so it wouldn't help Han answer the question.
2. The empty cell should contain 15,000.
3. How far he would go in 3,000 seconds.
4. He should label what quantities are in each column.
5. 400 seconds
6. 125 seconds

Activity Synthesis

Select students with different methods for the last question to explain their solutions to the class. Highlight the connections between different strategies, especially between tables and double number line diagrams.

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each method that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they heard to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.
Design Principle(s): Support sense-making

13.3 The International Space Station

15 minutes

This activity prompts students to compare and contrast two representations of equivalent ratios. Students work collaboratively to observe similarities and differences of using a double number line and using a table to express the same situation. Below are some key distinctions:

double number line	table
Distances between numbers and lengths of lines matter.	Distances and lengths do not matter because there are no lines.
The numbers on each line must be in order.	Rows of ratios can be out of order; within a column, numbers can go in any order that is convenient.
Each value of a ratio is shown on a line.	Each value of a ratio is shown in a column.
Pairs of values of a ratio are aligned vertically.	Pairs of values of a ratio appear in the same row.

You will need The International Space Station blackline master for this activity.

Addressing

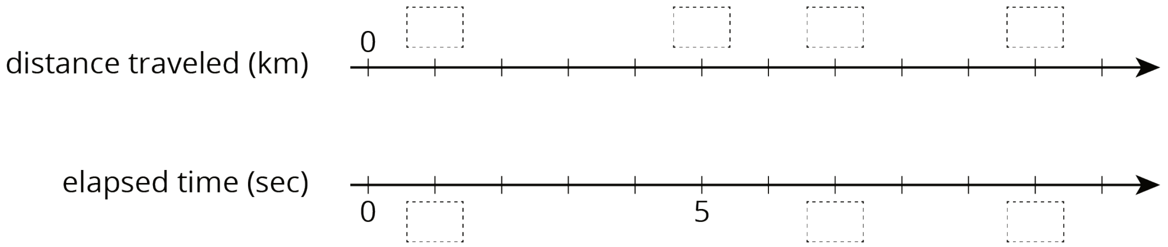
- 6.RP.A.3.a

Launch

To help students build some intuition about kilometers, begin by connecting it with contexts that are familiar to them. Tell students that “kilometer” is a unit used in the problem. Then ask a few guiding questions.

- “Can you name two things in our town (or city) that are about 1 kilometer apart?” (Consider finding some examples of 1-kilometer distances near your school ahead of time.)
- “How long do you think it would take you to walk 1 kilometer?” (Typical human walking speed is about 5 kilometers per hour, so it takes a person about 12 minutes to walk 1 kilometer.)
- “What might be a typical speed limit on a highway, in kilometers per hour?” (100 kilometers per hour is a typical highway speed limit. Students might be more familiar with a speed limit such as 65 miles per hour. Since there are about 1.6 kilometers in every mile, the same speed will be expressed as a higher number in kilometers per hour than in miles per hour.)

Arrange students in groups of 2. Give one person a slip with the table and the other a slip with a double number line (shown below). Ask students to first do what they can independently, and then to obtain information from their partners to fill in all the blanks. Explain that when the blanks are filled, the two representations will show the same information.



distance traveled (km)	elapsed time (sec)
0	0
80	10
	1

Anticipated Misconceptions

Students with the double number line representation may decide to label every tick mark instead of just the ones indicated with dotted rectangles. This is fine. Make sure they understand that the tick marks with dotted rectangles are the ones they are supposed to record in the table.

Student Task Statement

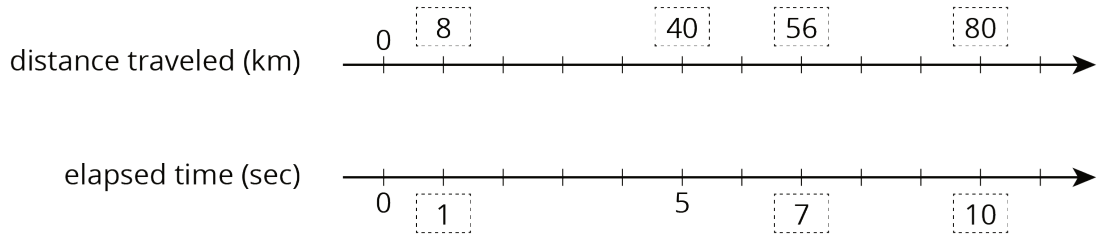
The International Space Station orbits around the Earth at a constant speed. Your teacher will give you either a double number line or a table that represents this situation. Your partner will get the other representation.

1. Complete the parts of your representation that you can figure out for sure.
2. Share information with your partner, and use the information that your partner shares to complete your representation.
3. What is the speed of the International Space Station?

4. Place the two completed representations side by side. Discuss with your partner some ways in which they are the same and some ways in which they are different.
5. Record at least one way that they are the same and one way they are different.

Student Response

distance traveled (kilometers)	elapsed time (seconds)
0	0
80	10
8	1
40	5
56	7



1. See table and double number line.
2. See table and double number line.
3. The ISS is traveling in its orbit at a speed of 8 kilometers per second. You can see this because both representations show the distance traveled in 1 second.
4. Answers vary; see Classroom Activity Synthesis.
5. See Classroom Activity Synthesis.

Are You Ready for More?

Earth's circumference is about 40,000 kilometers and the orbit of the International Space Station is just a bit more than this. About how long does it take for the International Space Station to orbit Earth?

Student Response

Between 80 and 100 minutes.

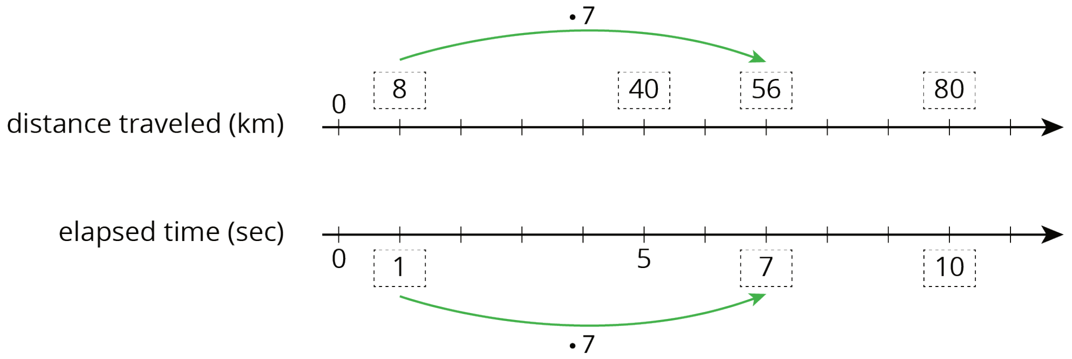
Earth’s circumference is about 40,000 kilometers. The orbit of the International Space Station is longer than this, but not a lot longer. The orbit will take a little more than $40{,}000 \div 8 = 5{,}000$ seconds. 80 minutes is 4,800 seconds, and 100 minutes is 6,000 seconds.

Activity Synthesis

Display completed versions of both representations for all to see. Invite students to share the ways the representations are alike and different. Consider writing some of these on the board, or this could just be a verbal discussion. Highlight the distinctions in terms of distances between numbers, order of numbers, and the vertical or horizontal orientations of the representations.

Although it is not a structural distinction, students might describe the direction in which multiplying happens as a difference between the two representations. They might say that we “multiply up or down” to find equivalent ratios in a table, and we “multiply across” to do the same on a double number line. You could draw arrows to illustrate this fact:

distance traveled (km)	elapsed time (sec)
0	0
80	10
8	1
40	5
56	7



The vertical orientation of tables and the horizontal orientation of double number lines are conventions we decided to consistently use in these materials. Mathematically, there is nothing wrong with orienting each representation the other way. Students may encounter tables oriented horizontally in a later course. Later in this course, they will encounter number lines oriented vertically.

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, highlight the distinctions in terms of distances between numbers, order of numbers, and the vertical or horizontal orientations of the representations.
Supports accessibility for: Conceptual processing; Organization

Lesson Synthesis

Briefly revisit the two tasks, displaying the representations for all to see, and pointing out ways in which tables and double number lines are the same and different. Emphasize that tables are sometimes easier to work with.

- In one task, we looked at the distance the ISS travels in its orbit and the time it takes to orbit Earth. How are the table and the double number line similar to each other? How are they different?
- Why is it important to include descriptive column headers on tables?

13.4 Bicycle Sprint

Cool Down: 5 minutes

Addressing

- 6.RP.A.3

Student Task Statement

In a sprint to the finish, a professional cyclist travels 380 meters in 20 seconds. At that rate, how far does the cyclist travel in 3 seconds?

Student Response

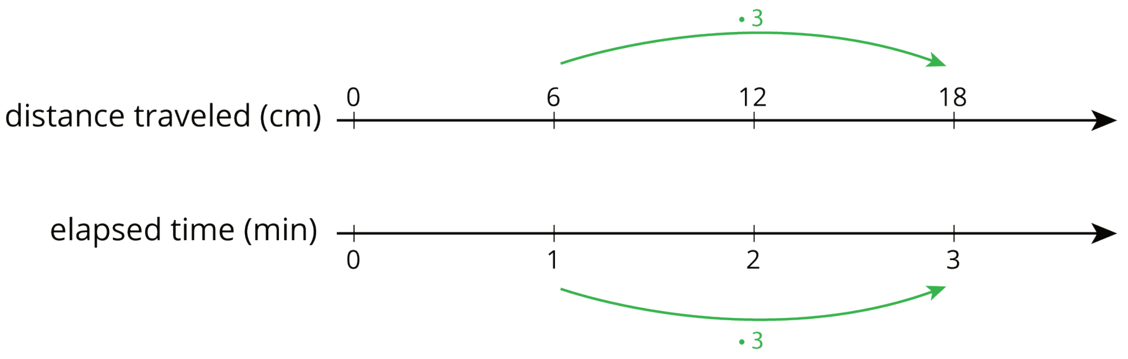
They travel 57 meters in 3 seconds. Possible strategy:

distance traveled (meters)	elapsed time (seconds)
380	20
19	1
57	3

Student Lesson Summary

On a double number line diagram, we put labels in front of each line to tell what the numbers represent. On a table, we put labels at the top of each column to tell what the numbers represent.

Here are two different ways we can represent the situation: “A snail is moving at a constant speed down a sidewalk, traveling 6 centimeters per minute.”



distance traveled (cm)	elapsed time (min)
12	2
6	1
60	10
18	3

Both double number lines and tables can help us use multiplication to make equivalent ratios, but there is an important difference between the two representations.

On a double number line, the numbers on each line are listed in order. With a table, you can write the ratios in any order. For this reason, sometimes a table is easier to use to solve a problem.

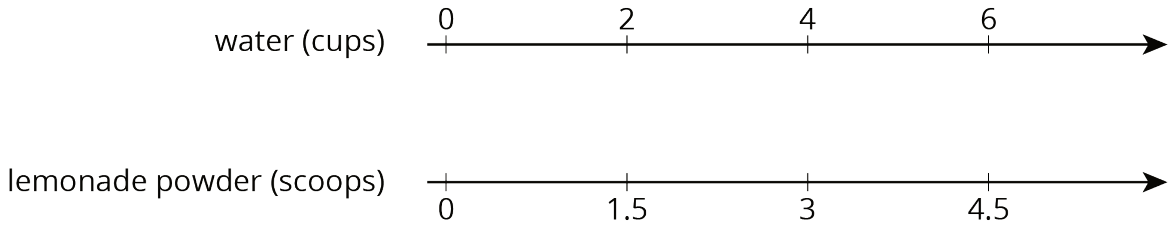
For example, what if we wanted to know how far the snail travels in 10 minutes? Notice that 60 centimeters in 10 minutes is shown on the table, but there is not enough room for this information on the double number line.

Lesson 13 Practice Problems

Problem 1

Statement

The double number line shows how much water and how much lemonade powder to mix to make different amounts of lemonade.



Make a table that represents the same situation.

Solution

water (cups)	lemonade powder (scoops)
0	0
2	1.5
4	3
6	4.5

Problem 2

Statement

A bread recipe uses 3 tablespoons of olive oil for every 2 cloves of crushed garlic.

a. Complete the table to show different-sized batches of bread that taste the same as the recipe.

b. Draw a double number line that represents the same situation.

c. Which representation do you think works better in this situation? Explain why.

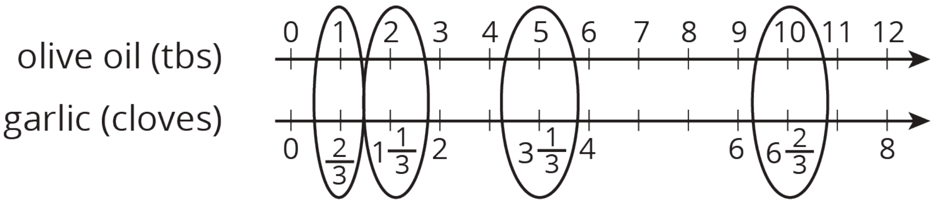
olive oil (tablespoons)	crushed garlic (cloves)
3	2
1	
2	
5	
10	

Solution

a.

olive oil (tablespoons)	crushed garlic (cloves)
3	2
1	$\frac{2}{3}$
2	$1\frac{1}{3}$
5	$3\frac{1}{3}$
10	$6\frac{2}{3}$

b.

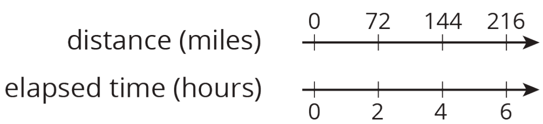


c. Answers vary. Sample response: The table is more convenient because the rows of the table can be listed in any order and without worrying about placing numbers accurately on the number line.

Problem 3

Statement

Clare travels at a constant speed, as shown on the double number line.



At this rate, how far does she travel in each of these intervals of time? Explain or show your reasoning. If you get stuck, consider using a table.

a. 1 hour

b. 3 hours

c. 6.5 hours

Solution

Explanations vary. Sample responses:

a. 36 miles. 1 hour is half of 2 hours, so half of 72 is 36. She traveled 36 miles in 1 hour.

b. 108 miles. Since the rate is 36 miles per hour, to find her distance in 3 hours, multiply 36 by 3. She traveled 108 miles in 3 hours.

c. 234 miles. Multiply the rate by 6.5. She traveled 234 miles in 6.5 hours.

distance (miles)	elapsed time (hours)
72	2
36	1
108	3
234	6.5

Problem 4

Statement

Lin and Diego travel in cars on the highway at constant speeds. In each case, decide who was traveling faster and explain how you know.

a. During the first half hour, Lin travels 23 miles while Diego travels 25 miles.

b. After stopping for lunch, they travel at different speeds. To travel the next 60 miles, it takes Lin 65 minutes and it takes Diego 70 minutes.

Solution

Explanations vary. Sample response:

a. Diego traveled faster because he covered more distance than Lin in the same amount of time.

b. Lin traveled faster because she covered the same distance as Diego but in less time.

(From Unit 2, Lesson 9.)

Problem 5

Statement

A sports drink recipe calls for $\frac{5}{3}$ tablespoons of powdered drink mix for every 12 ounces of water. How many batches can you make with 5 tablespoons of drink mix and 36 ounces of water? Explain your reasoning.

Solution

3 batches. Each batch has $\frac{5}{3}$ tablespoons of drink mix, so 3 batches will have 5 tablespoons of drink mix, since $3 \cdot \frac{5}{3} = 5$. Similarly, we can make 3 batches with 36 ounces of water, since $3 \cdot 12 = 36$.

(From Unit 2, Lesson 3.)

Problem 6

Statement

In this cube, each small square has side length 1 unit.

a. What is the surface area of this cube?

b. What is the volume of this cube?

Solution

a. 54 square units

b. 27 cubic units

(From Unit 1, Lesson 18.)