## Unit 7 Lesson 21: Sums and Products of Rational and Irrational Numbers

### 1 Operations on Integers (Warm up)

#### Student Task Statement

Here are some examples of integers (positive or negative whole numbers):

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| -25 | -10 | -2 | -1 | 0 | 5 | 9 | 40 |

1. Experiment with adding any two numbers from the list (or other integers of your choice). Try to find one or more examples of two integers that:
	1. add up to another integer
	2. add up to a number that is *not* an integer
2. Experiment with multiplying any two numbers from the list (or other integers of your choice). Try to find one or more examples of two integers that:
	1. multiply to make another integer
	2. multiply to make a number that is *not* an integer

### 2 Sums and Products of Rational Numbers

#### Student Task Statement

1. Here are a few examples of adding two rational numbers. Is each sum a rational number? Be prepared to explain how you know.
	1. $4+0.175=4.175$
	2. $\frac{1}{2}+\frac{4}{5}=\frac{5}{10}+\frac{8}{10}=\frac{13}{10}$
	3. $-0.75+\frac{14}{8}=\frac{-6}{8}+\frac{14}{8}=\frac{8}{8}=1$
	4. $a$ is an integer: $\frac{2}{3}+\frac{a}{15}=\frac{10}{15}+\frac{a}{15}=\frac{10+a}{15}$
2. Here is a way to explain why the sum of two rational numbers is rational.
* Suppose $\frac{a}{b}$ and $\frac{c}{d}$ are fractions. That means that $a,b,c,$ and $d$ are integers, and $b$ and $d$ are not 0.
	1. Find the sum of $\frac{a}{b}$ and $\frac{c}{d}$. Show your reasoning.
	2. In the sum, are the numerator and the denominator integers? How do you know?
	3. Use your responses to explain why the sum of $\frac{a}{b}+\frac{c}{d}$ is a rational number.
1. Use the same reasoning as in the previous question to explain why the product of two rational numbers, $\frac{a}{b}⋅\frac{c}{d}$, must be rational.

### 3 Sums and Products of Rational and Irrational Numbers

#### Student Task Statement

1. Here is a way to explain why $\sqrt{2}+\frac{1}{9}$ is irrational.
	* Let $s$ be the sum of $\sqrt{2}$ and $\frac{1}{9}$, or $s=\sqrt{2}+\frac{1}{9}$.
	* Suppose $s$ is rational.
	1. Would $s+-\frac{1}{9}$ be rational or irrational? Explain how you know.
	2. Evaluate $s+-\frac{1}{9}$. Is the sum rational or irrational?
	3. Use your responses so far to explain why $s$ cannot be a rational number, and therefore $\sqrt{2}+\frac{1}{9}$ cannot be rational.
2. Use the same reasoning as in the earlier question to explain why $\sqrt{2}⋅\frac{1}{9}$ is irrational.

### 4 Equations with Different Kinds of Solutions (Optional)

#### Student Task Statement

1. Consider the equation $4x^{2}+bx+9=0$. Find a value of $b$ so that the equation has:
	1. 2 rational solutions
	2. 2 irrational solutions
	3. 1 solution
	4. no solutions
2. Describe all the values of $b$ that produce 2, 1, and no solutions.
3. Write a new quadratic equation with each type of solution. Be prepared to explain how you know that your equation has the specified type and number of solutions.
	1. no solutions
	2. 2 irrational solutions
	3. 2 rational solutions
	4. 1 solution



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