Lesson 2: Reasoning about Contexts with Tape Diagrams

Goals

- Draw and label a tape diagram to represent relationships between quantities in a situation.
- Explain (orally and in writing) how to use a tape diagram to determine the value of an unknown quantity in a situation.
- Interpret a tape diagram that represents a relationship of the form $px + q = r$ or $p(x + q) = r$.

Learning Targets

- I can explain how a tape diagram represents parts of a situation and relationships between them.
- I can use a tape diagram to find an unknown amount in a situation.

Lesson Narrative

In this lesson, students represent and reason about contexts using tape diagrams. Students may have had experience with tape diagrams in earlier grades, and have seen some examples of their use in prior units. For example, tape diagrams were used to represent percent increase and decrease situations. First, they interpret some given tape diagrams. Then, they interpret a story and create tape diagrams. While the contexts lead to equations of the forms $p(x + q) = r$ and $px + q = r$, this lesson is not about writing equations. Likewise, students are asked to find an unknown value in several story problems, but the intention is for them to use any reasoning that makes sense to them. It is not expected that they write and solve equations, or that any particular method is stressed.

Alignments

Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. $

Building Towards

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Notice and Wonder

Student Learning Goals

Let’s use tape diagrams to make sense of different kinds of stories.

2.1 Notice and Wonder: Remembering Tape Diagrams

Warm Up: 5 minutes

The purpose of this warm-up is to re-introduce students to these diagrams as a representation of relationships between quantities. As students use tape diagrams as a tool for reasoning, they understand that the length of a piece of the “tape” carries meaning. Two pieces drawn to be the same length are understood to represent the same value. These pieces can be labeled with values to clarify what is known about the diagram, so two pieces labeled with the same letter indicate that they have the same value, even if that value is not known. These diagrams will be helpful for reasoning about situations in activities in this lesson. When students choose to use a tape diagram to represent a relationship between values and reason about a problem, they are using appropriate tools strategically (MP5). Tasks like this one ensure that students understand how such a tool works so that they are more likely to choose to use it correctly and appropriately.

Building Towards

- 7.EE.B.4

Instructional Routines

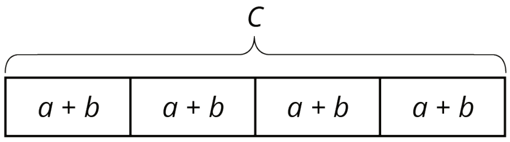
- Notice and Wonder

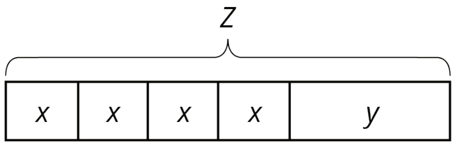
Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Give students 1 additional minute of quiet work time to complete the second question followed by a whole-class discussion.

Student Task Statement





1. What do you notice? What do you wonder?
2. What are some possible values for a, b, and c in the first diagram?

   For x, y, and z in the second diagram? How did you decide on those values?

Student Response

Answers vary. Sample responses:

1. Things students may notice or wonder:
   - There are two diagrams of rectangles with pieces labeled a, b, c, x, y, and z.
   - The c and z appear at the top of the diagrams.
   - Each diagrams consist of a large rectangle partitioned into smaller rectangles.
   - In the first diagram, the rectangle contains 4 $(a+b)$'s.
   - In the second diagram, the rectangle contains 4 x's and 1 y.
   - What do the diagrams represent?
   - What do the pieces of the diagrams represent?
   - Do all of the x's represent the same value? All the y's? All the z's?
   - Are longer pieces "worth" more?
2. In the first diagram, if $a=1$ and $b=4$, and we assume that c is the total, then $c=(1+4)\cdot 4=20$. In the second diagram, if $x=2$ and $y=1$, and we assume that z is the total, then $z=4\cdot 2+1=9$.

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image. After each response, ask the class whether they agree or disagree and to explain alternative ways of thinking, referring back to the images each time.

Ask students to share possible values for the variables in each diagram. Record and display their responses for all to see. If possible, record the values on the displayed diagram. If the idea that pieces labeled with the same variable represent the same value does not arise in the discussion, make that idea explicit. For example, students should assume that all the pieces labeled with y in one diagram have the same value. When they make tape diagrams, they know to draw rectangles of the same length to show the same value, but since quick diagrams are sometimes sloppy, it's also important to label pieces with numbers or letters to show known and relative values.

2.2 Every Picture Tells a Story

15 minutes

In this activity, students explain how a tape diagram represents a situation. They also use the tape diagram to reason about the value of the unknown quantity. Students are not expected to write and solve equations here; any method they can explain for finding values for x and y is acceptable. While some students might come up with equations to describe the diagram and solve for the unknown, there is no need to focus on developing those ideas at this time.

Addressing

- 7.EE.B.3

Building Towards

- 7.EE.B.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 3. (Some groups of 2 are okay, if needed.)

Ask students if they know what a "flyer" is. If any students do not know, explain or ask a student to explain. If possible, reference some examples of flyers hanging in school.

Ensure students understand they should take turns speaking and listening, and that there are two things to do for each diagram: explain why it represents the story, and also figure out any unknown values in the story.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Consider pausing after the first question for a brief class discussion before moving on. *Supports accessibility for: Organization; Attention*

Anticipated Misconceptions

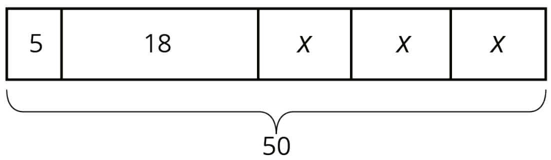
Students may not realize that when a variable is assigned to represent a quantity in a situation, it has the same value each time it appears. Revisit what x and y represent in these problems and why each occurrence of a variable must represent the same value.

In the second situation, students might argue that a more accurate representation would be 5 boxes with y to show the first distribution of stickers, and then five boxes with 2 to show the second distribution. Tell students that such a representation would indeed correctly describe the actions in the situation, but that the work of the task is to understand *this* diagram to set us up for success later.

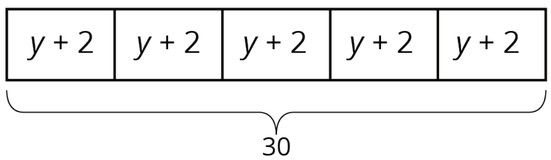
Student Task Statement

Here are three stories with a diagram that represents it. With your group, decide who will go first. That person explains why the diagram represents the story. Work together to find any unknown amounts in the story. Then, switch roles for the second diagram and switch again for the third.

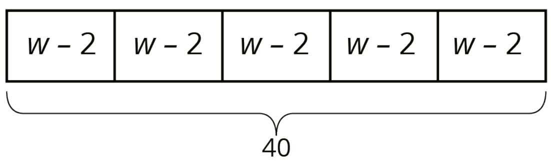
1. Mai made 50 flyers for five volunteers in her club to hang up around school. She gave 5 flyers to the first volunteer, 18 flyers to the second volunteer, and divided the remaining flyers equally among the three remaining volunteers.



2. To thank her five volunteers, Mai gave each of them the same number of stickers. Then she gave them each two more stickers. Altogether, she gave them a total of 30 stickers.



3. Mai distributed another group of flyers equally among the five volunteers. Then she remembered that she needed some flyers to give to teachers, so she took 2 flyers from each volunteer. Then, the volunteers had a total of 40 flyers to hang up.



Student Response

1. Answers vary. Unknown amounts that students may find include the remaining number of flyers (27) and the number of flyers given to each of the 3 remaining volunteers (9). The whole rectangle represents the 50 flyers that Mai made. She split them up into five parts: 5, 18, and 3 equal parts for the rest. The 3 equal parts are shown by 3 same-sized boxes. x represents the number of flyers for each of the 3 remaining volunteers. $3 \cdot x$ is the number of flyers remaining after Mai gave out 5 and 18. That part has to represent 27 flyers, since 23 of them $(5 + 18)$ have already been given out. So each x represents 9 flyers.

2. Answers vary. Unknown amounts that students may find include the total number of stickers each student receives (6) and the number they received at first (4). The whole rectangle represents the 30 stickers. They are divided into 5 equal parts since the 5 volunteers each got the same number of stickers. They each got some (y) and then each got 2 more, so each one got $y + 2$ stickers. We can find y by thinking that 30 divides into 5 groups of 6. If each volunteer received 6 stickers in total, they got 4 before the extra 2 were added. Another way to think about y is to first take away the 10 extra stickers that were given out. Then 5 groups of 4 would make up the remaining 20 stickers. So y represents 4 stickers.

3. Answers vary. Unknown amounts that students may find include the number of flyers each student has in the end (8), the number they received at first (10), and the total number of flyers (50). The whole rectangle represents 40 flyers. They are divided into 5 equal parts since the 5 volunteers each got the same number of flyers. They each got some (w) and then each gave back 2, so each one has $w - 2$ flyers. We can find w by thinking that 40 divides into 5 groups of 8. If each volunteer has 8 flyers, they got 10 before the 2 were taken away. So there were originally 50 flyers, which is the 40 that the volunteers have, plus the 10 that Mai took back.

Activity Synthesis

Tape diagrams represent relationships between quantities in stories. The goals here are to make sure students understand how parts of the diagram match the information about the story, and for them to begin to reason about how the diagrams connect to the operations that can help find unknown amounts.

Invite one group to provide an explanation for each diagram—both how the diagram represents the story, and how they reasoned about the unknown amounts. After each, ask the class if anyone thought about it a different way. (One additional line of reasoning for each diagram is probably sufficient.)

Here are some questions you might ask to encourage students to be more specific:

- “What question could you ask about the story?”
- “Where in the diagrams do you see equal parts? How do you know they are equal?”
- “What quantity does the variable represent in the story? How do you know?”
- “In the first story, where in the diagram do we see the ‘remaining flyers’?”

- “Why don’t we see the number 3 in the first diagram to show the 3 remaining volunteers?”
- “In the second diagram, where are the five volunteers represented?”
- “How did the diagrams help you find the value of the unknown quantities?”

Access for English Language Learners

Conversing: MLR3 Clarify, Critique, Correct. Present an incorrect statement for the second situation that reflects a possible misunderstanding from the class. For example, “Mai gave 6 stickers to each of the volunteers because 30 divided by 5 is 6. So y is 6.” Prompt students to identify the error, and then write a correct statement. This helps students evaluate, and improve on, the written mathematical arguments of others and to understand the importance of defining the variable in context of the situation.
Design Principle(s): Maximize meta-awareness

2.3 Every Story Needs a Picture

15 minutes (there is a digital version of this activity)
In the previous activity, students interpreted given tape diagrams and explained how they represented a story. Here, they have a chance to draw tape diagrams to represent a story. The first story is a bit more scaffolded because it specifies what x represents. In the other two stories, students need to decide which quantity to represent with a variable and choose a letter to use. As with all activities in this lesson, students are *not* expected to write and solve an equation. This preliminary work supports the understanding needed to be able to represent such situations with equations.

Addressing

- 7.EE.B.3

Building Towards

- 7.EE.B.4

Launch

Keep students in the same groups. You might have *each* student draw *all three* diagrams and compare them with their groups, working together to resolve any discrepancies. Or if time is short, you might assign *each* student in the group *a different story*—ask each student to explain their diagram to their group to see if their group members agree with their interpretation.

For classrooms using the digital version of the materials, take a minute to demonstrate how the controls work in the applet. Some students may prefer to draw the diagrams in their notebooks or on scratch paper.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a blank template of a tape diagram to represent each story.
Supports accessibility for: Language; Organization

Student Task Statement

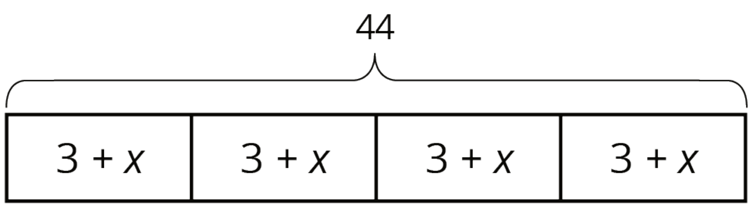
Here are three more stories. Draw a tape diagram to represent each story. Then describe how you would find any unknown amounts in the stories.

1. Noah and his sister are making gift bags for a birthday party. Noah puts 3 pencil erasers in each bag. His sister puts x stickers in each bag. After filling 4 bags, they have used a total of 44 items.

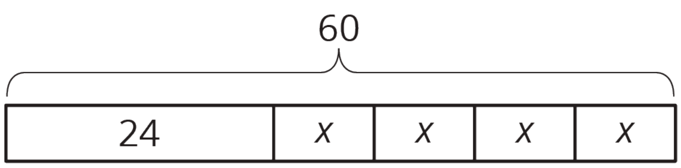
2. Noah's family also wants to blow up a total of 60 balloons for the party. Yesterday they blew up 24 balloons. Today they want to split the remaining balloons equally between four family members.

3. Noah's family bought some fruit bars to put in the gift bags. They bought one box each of four flavors: apple, strawberry, blueberry, and peach. The boxes all had the same number of bars. Noah wanted to taste the flavors and ate one bar from each box. There were 28 bars left for the gift bags.

Student Response



1. or equivalent.
   Answers vary. Unknown amounts that students may find include the number of stickers in each bag (8) and the total number of items in each bag (11). Dividing 44 into 4 equal parts gives 11 items for each bag, which means 3 erasers and 8 stickers in each. Another way is to subtract the 12 erasers from 44, giving 32 items left. 32 stickers split evenly among 4 bags is 8 in each bag.



2. or equivalent.

Answers vary. Unknown amounts that students may find include the number of balloons they need to blow up today (36) and the number that each of the four family members blows up (9). The number of balloons left to blow up is found by $60 - 24$ or 36. Splitting those up equally among four people is $36 \div 4$ or 9 each.

$$\underbrace{\boxed{x - 1}\boxed{x - 1}\boxed{x - 1}\boxed{x - 1}}_{28}$$

3. or equivalent.

   Answers vary. Unknown amounts that students may find include the number of bars left in each box (7), the number of bars originally in each box (8), and the total number of bars there were in the four boxes (32). Dividing 28 into 4 equal parts gives 7 bars left in each box. Adding 1 to each gives 8 in each box originally for a total of $8 \cdot 4$ or 32 bars.

Are You Ready for More?

Design a tiling that uses a repeating pattern consisting of 2 kinds of shapes (e.g., 1 hexagon with 3 triangles forming a triangle). How many times did you repeat the pattern in your picture? How many individual shapes did you use?

Student Response

Answers vary.

Activity Synthesis

Much of the discussion will take place in groups. Here are some ideas for synthesizing students' learning about creating tape diagrams:

- Ask students if they had any disagreements in their groups and how they resolved them.
- Ask students how they decided which unknown quantity to find in the story. The first story specifies x stickers, but the other stories do not define a variable.
- Display one diagram for each story and ask students to explain how they are alike and how they are different.

Lesson Synthesis

Display one or more of the tape diagrams students encountered or created during the lesson. Ask, "What are some ways that tape diagrams give information about a story?" Responses to highlight:

- A total amount is indicated.
- Pieces that represent equal amounts are the same length (or roughly the same length, if sketching by hand).

- Pieces that represent different amounts are not the same length.
- Pieces are labeled with either their amounts, a variable representing an unknown amount, or an expression like $x + 1$ to mean “1 more than the unknown amount.”

2.4 Red and Yellow Apples

Cool Down: 5 minutes

Addressing

- 7.EE.B.3

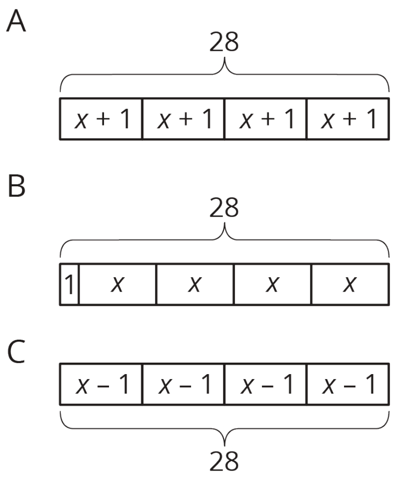
Building Towards

- 7.EE.B.4

Student Task Statement

Here is a story: Lin bought 4 bags of apples. Each bag had the same number of apples. After eating 1 apple from each bag, she had 28 apples left.

1. Which diagram best represents the story? Explain why the diagram represents it.



2. What part of the story does x represent?
3. Describe how you would find the unknown amount in the story.

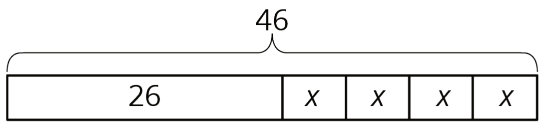
Student Response

1. C. When she ate 1 apple from each bag, there were $x - 1$ apples left in each bag.
2. x represents the number of apples in 1 bag before Lin ate any apples.
3. Each of the 4 pieces of the diagram represents 7 apples, because $28 \div 4 = 7$. If $x - 1 = 7$, then x is 8.

Student Lesson Summary

Tape diagrams are useful for representing how quantities are related and can help us answer questions about a situation.

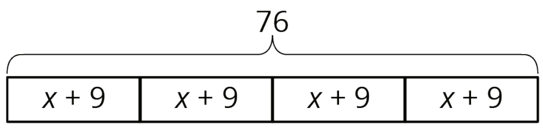
Suppose a school receives 46 copies of a popular book. The library takes 26 copies and the remainder are split evenly among 4 teachers. How many books does each teacher receive? This situation involves 4 equal parts and one other part. We can represent the situation with a rectangle labeled 26 (books given to the library) along with 4 equal-sized parts (books split among 4 teachers). We label the total, 46, to show how many the rectangle represents in all. We use a letter to show the unknown amount, which represents the number of books each teacher receives. Using the same letter, x, means that the same number is represented four times.



Some situations have parts that are all equal, but each part has been increased from an original amount:

A company manufactures a special type of sensor, and packs them in boxes of 4 for shipment. Then a new design increases the weight of each sensor by 9 grams. The new package of 4 sensors weighs 76 grams. How much did each sensor weigh originally?

We can describe this situation with a rectangle representing a total of 76 split into 4 equal parts. Each part shows that the new weight, $x + 9$, is 9 more than the original weight, x.



Lesson 2 Practice Problems

Problem 1

Statement

The table shows the number of apples and the total weight of the apples.

number of apples	weight of apples (grams)
2	511
5	1200
8	2016

Estimate the weight of 6 apples.

Solution

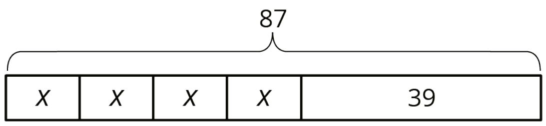
About 1500 grams.

(From Unit 3, Lesson 1.)

Problem 2

Statement

Select **all** stories that the tape diagram can represent.



A. There are 87 children and 39 adults at a show. The seating in the theater is split into 4 equal sections.

B. There are 87 first graders in after-care. After 39 students are picked up, the teacher put the remaining students into 4 groups for an activity.

C. Lin buys a pack of 87 pencils. She gives 39 to her teacher and shared the remaining pencils between herself and 3 friends.

D. Andre buys 4 packs of paper clips with 39 paper clips in each. Then he gives 87 paper clips to his teacher.

E. Diego's family spends $87 on 4 tickets to the fair and a $39 dinner.

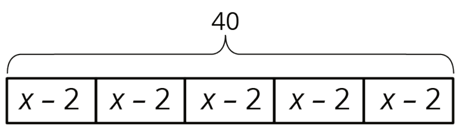
Solution

["B", "C", "E"]

Problem 3

Statement

Andre wants to save $40 to buy a gift for his dad. Andre's neighbor will pay him weekly to mow the lawn, but Andre always gives a $2 donation to the food bank in weeks when he earns money. Andre calculates that it will take him 5 weeks to earn the money for his dad's gift. He draws a tape diagram to represent the situation.



a. Explain how the parts of the tape diagram represent the story.

b. How much does Andre's neighbor pay him each week to mow the lawn?

Solution

a. Answers vary. Sample response: The 5 equal parts represent the 5 weeks. In each week, Andre will earn x dollars for mowing his neighbor's lawn and give $2 to the food bank, so he will save $x - 2$ dollars. In five weeks, he will save a total of $40.

b. $10

Problem 4

Statement

Without evaluating each expression, determine which value is the greatest. Explain how you know.

a. $7\frac{5}{6} - 9\frac{3}{4}$

b. $(-7\frac{5}{6}) + (-9\frac{3}{4})$

c. $(-7\frac{5}{6}) \cdot 9\frac{3}{4}$

d. $(-7\frac{5}{6}) \div (-9\frac{3}{4})$

Solution

$(-7\frac{5}{6}) \div (-9\frac{3}{4})$ is the greatest because it is the only expression with a positive value.

(From Unit 5, Lesson 13.)

Problem 5

Statement

Solve each equation.

a. $(8.5) \cdot (-3) = a$

b. $(-7) + b = (-11)$

c. $c - (-3) = 15$

d. $d \cdot (-4) = 32$

Solution

a. -25.5

b. -4

c. 12

d. -8

(From Unit 5, Lesson 15.)