## Lesson 5: Triangles in Circles

* Let’s see how perpendicular bisectors relate to circumscribed circles.

### 5.1: One Perpendicular Bisector

The image shows a triangle.



1. Construct the perpendicular bisector of segment $AB$.
2. Imagine a point $D$ placed anywhere on the perpendicular bisector you constructed. How would the distance from $D$ to $A$ compare to the distance from $D$ to $B$? Explain your reasoning.

### 5.2: Three Perpendicular Bisectors

1. Construct the perpendicular bisector of segment $BC$ from the earlier activity. Label the point where the 2 perpendicular bisectors intersect as $P$.
2. Use a colored pencil to draw segments $PA,PB,$ and $PC$. How do the lengths of these segments compare? Explain your reasoning.
3. Imagine the perpendicular bisector of segment $AC$. Will it pass through point $P$? Explain your reasoning.
4. Construct the perpendicular bisector of segment $AC$.
5. Construct a circle centered at $P$ with radius $PA$.
6. Why does the circle also pass through points $B$ and $C$?

#### Are you ready for more?

Points $A,B,$ and $C$ are graphed. Find the coordinates of the circumcenter and the radius of the circumscribed circle for triangle $ABC$.



### 5.3: Wandering Centers

Each student in your group should choose 1 triangle. It’s okay for 2 students to choose the same triangle as long as all 3 are chosen by at least 1 student.

1. Construct the circumscribed circle of your triangle.
2. After you finish, compare your results. What do you notice about the location of the **circumcenter** in each triangle?







### Lesson 5 Summary

We saw that some quadrilaterals have circumscribed circles. Is the same true for triangles? In fact, *all* triangles have circumscribed circles. The key fact is that all points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment.

Suppose we have triangle $ABC$ and we construct the perpendicular bisectors of all 3 sides. These perpendicular bisectors will all meet at a single point called the **circumcenter** of the triangle (label it $D$). This point is on the perpendicular bisector of $AB$, so it’s equidistant from $A$ and $B$. It’s also on the perpendicular bisector of $BC$, so it’s equidistant from $B$ and $C$. So, it is actually the same distance from $A,B,$ *and* $C$. We can draw a circle centered at $D$ with radius $AD$. The circle will pass through $B$ and $C$ too because the distances $BD$ and $CD$ are the same as the radius of the circle.



In this case, the circumcenter ​​​happened to fall inside triangle $ABC$, but that does not need to happen. The images show cases where the circumcenter is inside a triangle, outside a triangle, and on one of the sides of a triangle.









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