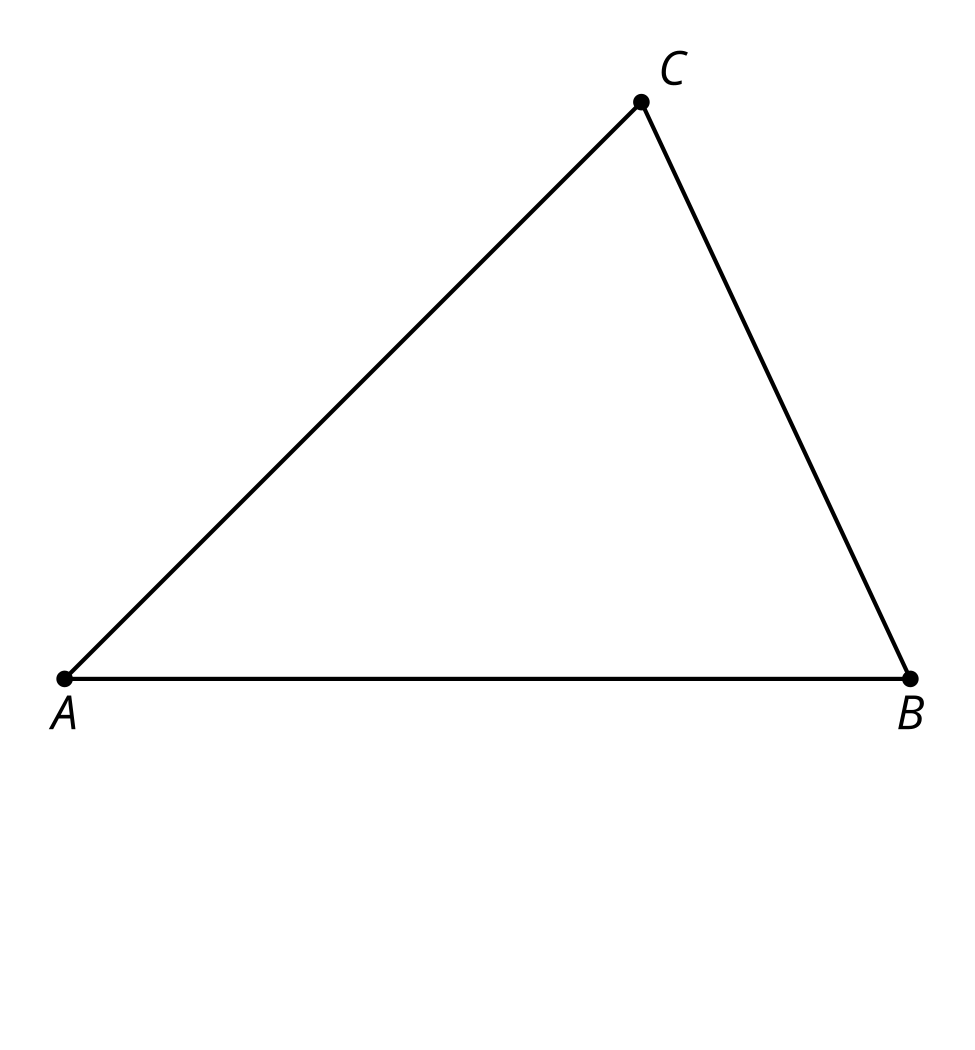
## Lesson 5: Triangles in Circles

* Let’s see how perpendicular bisectors relate to circumscribed circles.

### 5.1: One Perpendicular Bisector

The image shows a triangle.



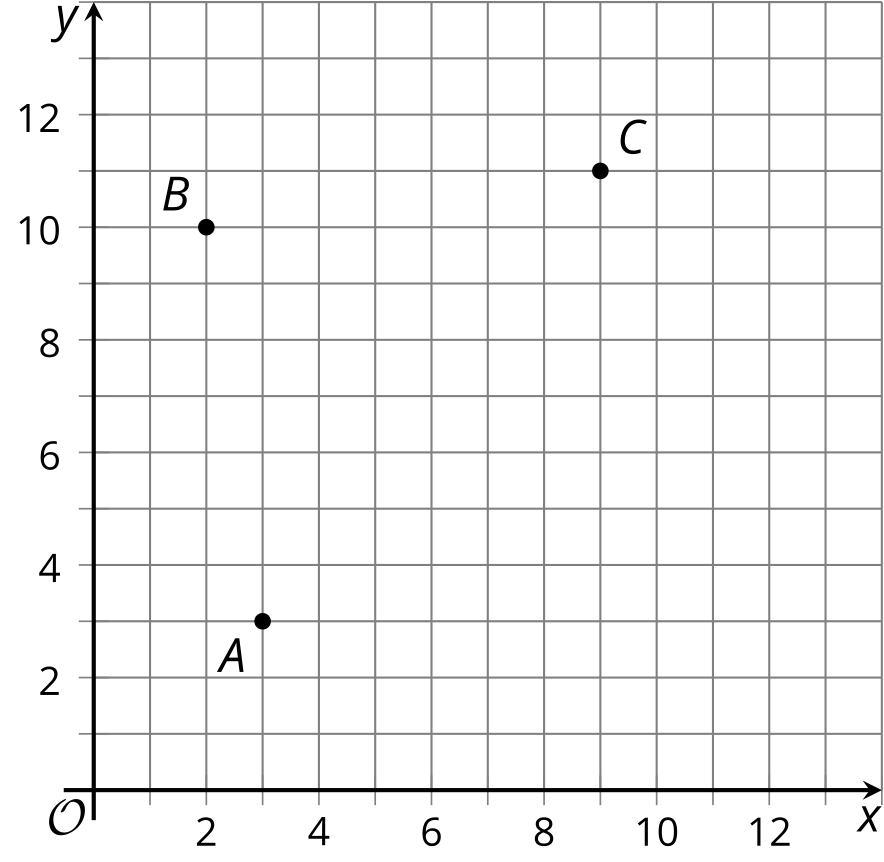
1. Construct the perpendicular bisector of segment .
2. Imagine a point placed anywhere on the perpendicular bisector you constructed. How would the distance from to compare to the distance from to ? Explain your reasoning.

### 5.2: Three Perpendicular Bisectors

1. Construct the perpendicular bisector of segment from the earlier activity. Label the point where the 2 perpendicular bisectors intersect as .
2. Use a colored pencil to draw segments and . How do the lengths of these segments compare? Explain your reasoning.
3. Imagine the perpendicular bisector of segment . Will it pass through point ? Explain your reasoning.
4. Construct the perpendicular bisector of segment .
5. Construct a circle centered at with radius .
6. Why does the circle also pass through points and ?

#### Are you ready for more?

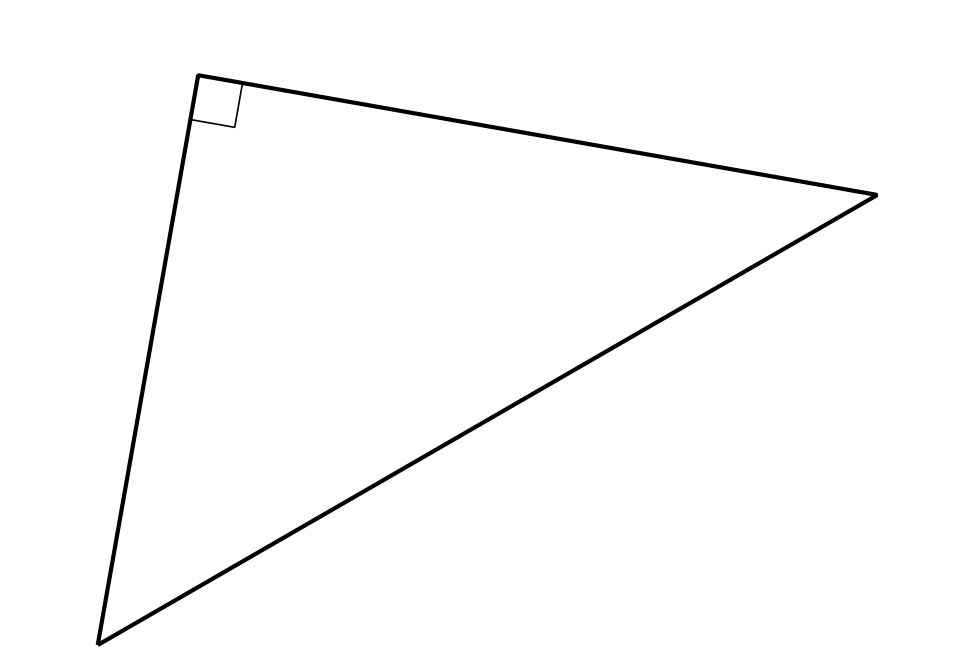
Points and are graphed. Find the coordinates of the circumcenter and the radius of the circumscribed circle for triangle .

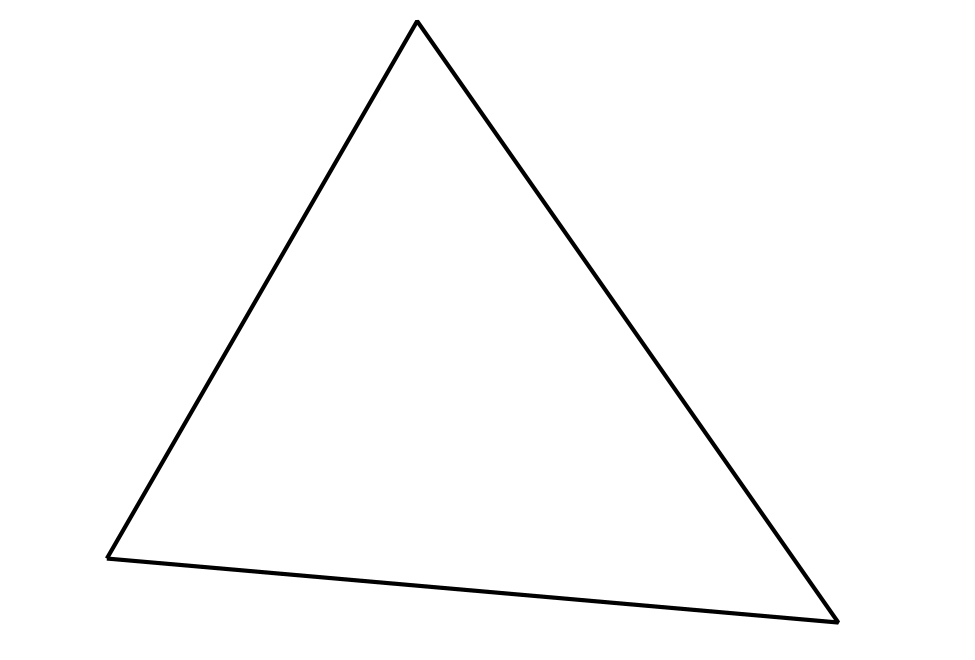


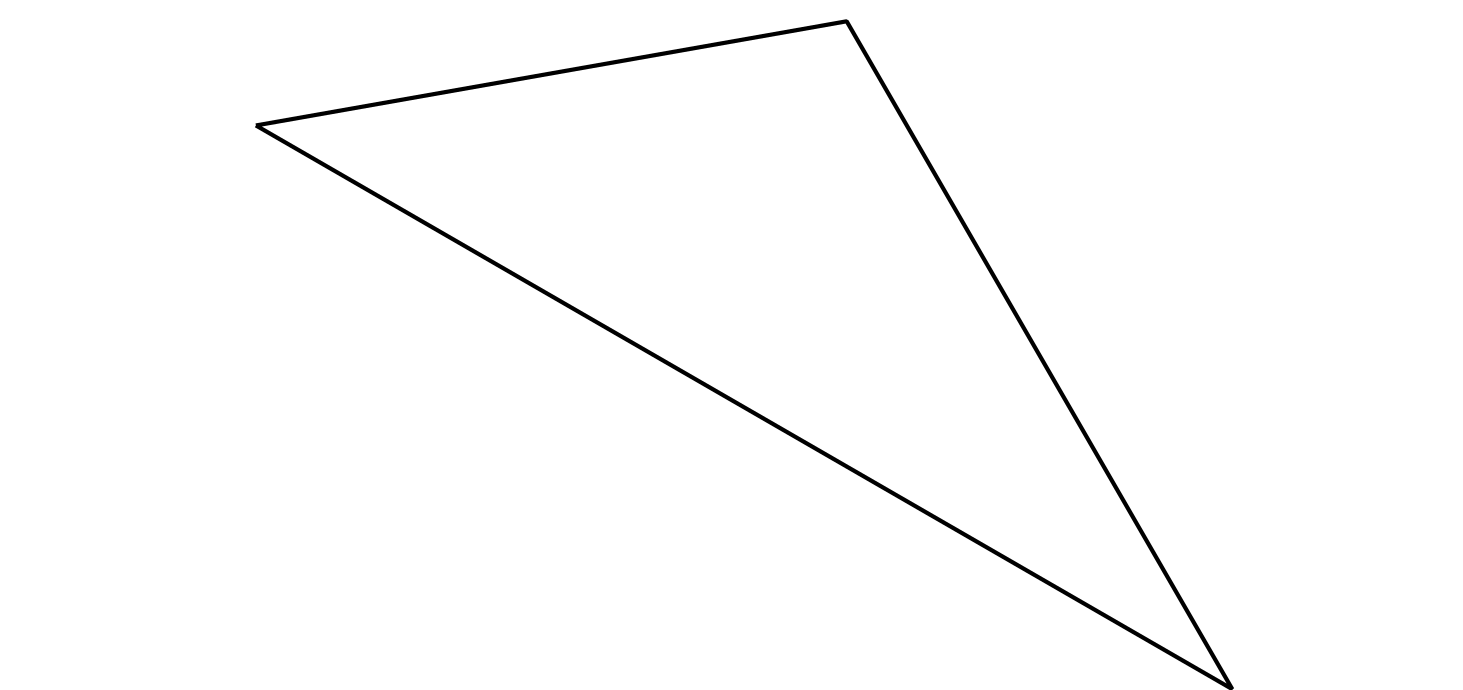
### 5.3: Wandering Centers

Each student in your group should choose 1 triangle. It’s okay for 2 students to choose the same triangle as long as all 3 are chosen by at least 1 student.

1. Construct the circumscribed circle of your triangle.
2. After you finish, compare your results. What do you notice about the location of the **circumcenter** in each triangle?



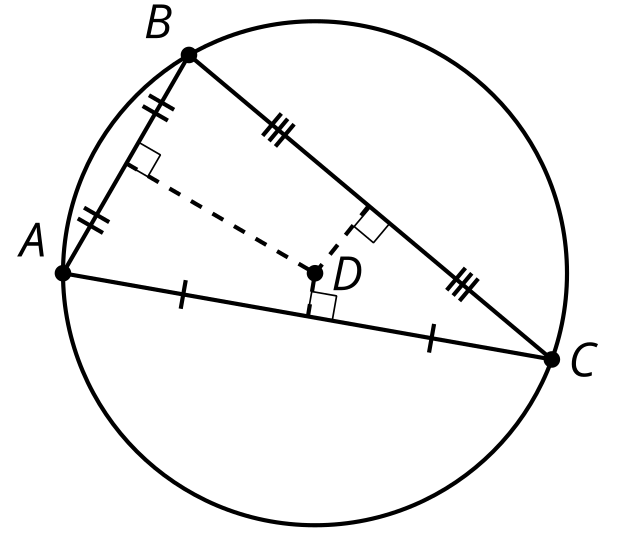




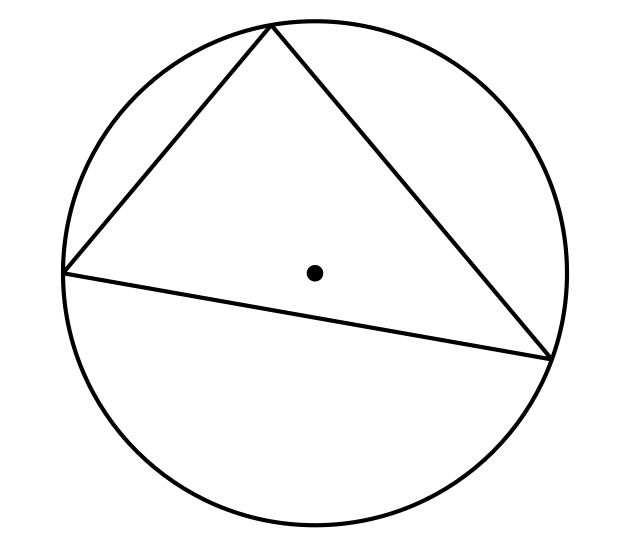
### Lesson 5 Summary

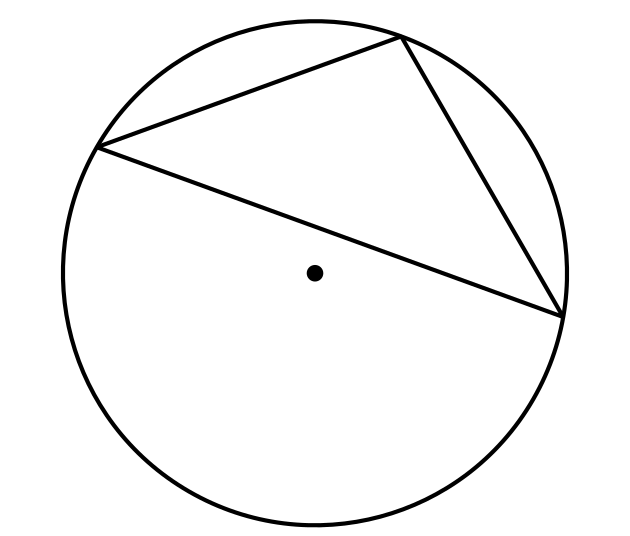
We saw that some quadrilaterals have circumscribed circles. Is the same true for triangles? In fact, *all* triangles have circumscribed circles. The key fact is that all points on the perpendicular bisector of a segment are equidistant from the endpoints of the segment.

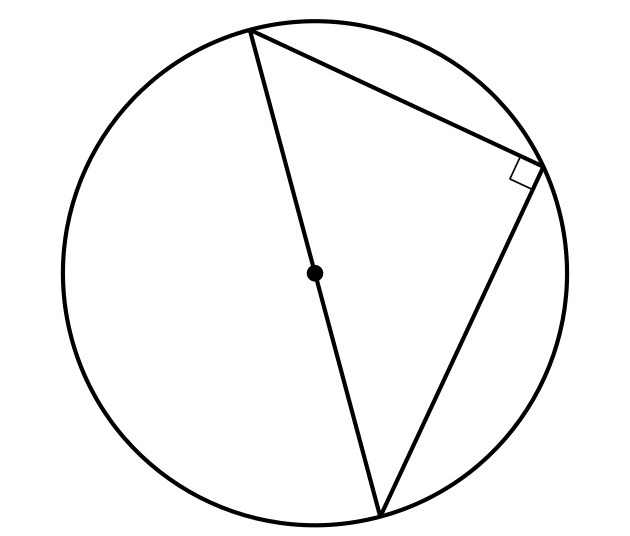
Suppose we have triangle and we construct the perpendicular bisectors of all 3 sides. These perpendicular bisectors will all meet at a single point called the **circumcenter** of the triangle (label it ). This point is on the perpendicular bisector of , so it’s equidistant from and . It’s also on the perpendicular bisector of , so it’s equidistant from and . So, it is actually the same distance from *and* . We can draw a circle centered at with radius . The circle will pass through and too because the distances and are the same as the radius of the circle.



In this case, the circumcenter ​​​happened to fall inside triangle , but that does not need to happen. The images show cases where the circumcenter is inside a triangle, outside a triangle, and on one of the sides of a triangle.









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