Lesson 23: Applications of Expressions

Goals

- Determine which order for applying multiple coupons gives the better discount and explain (orally and in writing) the reasoning.
- Justify (orally, in writing, and using other representations) that two different sequences of calculations give the same result.

Learning Targets

- I can write algebraic expressions to understand and justify a choice between two options.

Lesson Narrative

In this culminating lesson, students look at several real-world situations that can be represented by an expression with a variable. In the warm-up, students decide whether each of four expressions is equivalent to a given expression, recalling what it means for expressions to be equivalent and relevant terminology. In the following activity, students write expressions corresponding to two ways of doing a real-world calculation, and explain why the two ways are equivalent. Finally, students are presented with two coupons to a store (a 20% off coupon and a $30 off coupon), and use their skills to decide in which order the coupons should be applied to save more money on a purchase. In this lesson, students write expressions to represent calculation methods, which allows them to use familiar properties to decide whether two methods are equivalent. This is an example of decontextualizing and recontextualizing (MP2) and creating a mathematical model to understand a situation (MP4).

Alignments

Building On

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.EE.B: Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

Instructional Routines

- Algebra Talk
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

Student Learning Goals

- Let’s use expressions to solve problems.

23.1 Algebra Talk: Equivalent to $0.75t - 21$

Warm Up: 5 minutes

The purpose of this algebra talk is to remind students that expressions can be written in different, equivalent ways, and to give them an opportunity to recall relevant terminology like "distributive property."

Building On

- 7.EE.A.1

Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Decide whether each expression is equivalent to $0.75t - 21$. Be prepared to explain how you know.

$\frac{3}{4}t - 21$

$\frac{3}{4}(t - 21)$

$0.75(t - 28)$

$t - 0.25t - 21$

Student Response

- $\frac{3}{4}t - 21$ is equivalent because $0.75 = \frac{3}{4}$.
- $\frac{3}{4}(t - 21)$ is not equivalent because by the distributive property, it is equivalent to $\frac{3}{4}t - 15.75$.
- $0.75(t - 28)$ is equivalent because of the distributive property.

- $t - 0.25t - 21$ is equivalent because if you combine $t - 0.25t$, you get $0.75t$.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- "Who can restate _____'s reasoning in a different way?"
- "Did anyone have the same strategy but would explain it differently?"
- "Did anyone solve the problem in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

23.2 Two Ways to Calculate

15 minutes
The purpose of this task is to encourage students to represent each situation using an expression with a variable, so that the calculation methods can be shown to be equivalent in a straightforward way using familiar properties. This approach will be useful for tackling the next activity. Students may find other ways of explaining why the calculation methods are equivalent, but the synthesis should highlight explaining why two expressions that use a variable are equivalent.

Building On

- 7.EE.A.1
- 7.EE.B

Instructional Routines

- MLR7: Compare and Connect

Launch

Depending on the time available, ask each student to respond to 1, 2, or all of the situations. Note that the last situation is a bit more difficult than the first two.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

For students who have trouble getting started, ask them to first calculate using a few specific values. For example, calculate the temperature in Fahrenheit if the temperature in celsius is 0 degrees, 5 degrees, 10 degrees, and then use the same operations to write an expression for x degrees. Another option would be to provide a bank of expressions for students to choose from, rather than asking them to generate the expressions.

Student Task Statement

Usually when you want to calculate something, there is more than one way to do it. For one or more of these situations, show how the two different ways of calculating are equivalent to each other.

1. Estimating the temperature in Fahrenheit when you know the temperature in Celsius
   a. Double the temperature in Celsius, then add 30.
   b. Add 15 to the temperature in Celsius, then double the result.
2. Calculating a 15% tip on a restaurant bill
   a. Take 10% of the bill amount, take 5% of the bill amount, and add those two values together.
   b. Multiply the bill amount by 3, divide the result by 2, and then take $\frac{1}{10}$ of that result.
3. Changing a distance in miles to a distance in kilometers
   a. Take the number of miles, double it, then decrease the result by 20%.
   b. Divide the number of miles by 5, then multiply the result by 8.

Student Response

1. Celsius to Fahrenheit:
   a. If c represents the temperature in Celsius, this way of calculating can be expressed with $2c + 30$.
   b. This way of calculating can be represented with $2(c + 15)$. These are equivalent because of the distributive property.
2. 15% tip:

a. If b represents the bill amount, this way of calculating can be expressed with $0.1b + 0.05b$, which is equivalent to $0.15b$ by combining like terms.

b. This way of calculating can be represented with $\frac{3b}{2} \cdot \frac{1}{10}$. This is equivalent to $1.5b \cdot 0.1$ because $\frac{3}{2} = 1.5$ and $\frac{1}{10} = 0.1$. This is equivalent to $0.1 \cdot 1.5b$ because multiplication is commutative, and $0.15b$ because $0.1 \cdot 1.5 = 0.15$.

3. miles to kilometers:
   a. If m represents the distance in miles, this way of calculating can be expressed with $0.8(2m)$, and then $1.6m$ after multiplying. Alternatively, it can be expressed with $2m - 0.2(2m)$, which is equivalent to $1.6m$ after multiplying to get $2m - 0.4m$ and then combining like terms.

   b. This way of calculating can be expressed with $\frac{m}{5} \cdot 8$, which is equivalent to $0.2m \cdot 8$ because $\frac{1}{5} = 0.2$. After rearranging and multiplying, this is also equivalent to $1.6m$.

Activity Synthesis

For each situation, invite at least one student to share their reasoning for why the two calculation methods are equivalent. Be sure to include, for each situation, an approach that involves writing expressions that contain a variable and using properties of operations to show that they are equivalent.

Spend a little time on the last situation, emphasizing that 20% off an amount is the same as 80% of the amount. That is, if you want to compute x decreased by 20%, you can write $0.8x$, because $x - 0.2x$ is equivalent to $0.8x$. This insight will help students write less-complicated expressions in the next activity.

Access for English Language Learners

Representing, Conversing: MLR 7 Compare and Connect. Use this routine to prepare students for the whole-class discussion. Give students quiet think time to consider what is the same and what is different about the two different ways of calculating the last situation. Next, ask students to share what they noticed with a partner. Listen for and amplify mathematical language students use to explain why the two ways are equivalent.
Design Principle(s): Cultivate conversation

23.3 Which Way?

15 minutes

The purpose of this activity is to use expressions with a variable to represent applying coupons in a different order. By analyzing these expression, you learn that no matter how much you spend, you always pay $6 less if the 20% off coupon is applied first.

Monitor for how students are approaching the problem. If, once students have an answer, they have not written expressions with a variable to show the best way to apply the coupons, consider asking them to try writing an expression using a variable to represent the purchase amount.

Students should be familiar with the idea of coupons and discounts from their work in an earlier unit. If this is not the case, more time may be needed for the launch to familiarize students with the context.

Students may notice that if you spend less than $30, the store probably won't let you take $30 off. This is a possible constraint, and students who include this constraint are engaging in aspects of mathematical modeling (MP4).

Building On

- 7.EE.B

Instructional Routines

- Notice and Wonder

Launch

Display the image of two coupons for all to see. Ask students to think of some things they notice and some things they wonder. Things students might notice:

- There are two coupons to the same store.
- One coupon is for 20% off and one coupon is for $30 off.

Things students might wonder:

- Can you use both coupons on the same purchase?
- Do the coupons expire?
- Is there a minimum or maximum amount you have to spend?

If no students wonder this, ask, "What if you could use both coupons on the same purchase? Should you deduct 20%, and then $30 from the result? Or the other way around? Does it matter?"

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

If students have trouble getting started, suggest that they calculate which order is better for some specific purchase amounts.

Student Task Statement

You have two coupons to the same store: one for 20% off and one for $30 off. The cashier will let you use them both, and will let you decide in which order to use them.

- Mai says that it doesn't matter in which order you use them. You will get the same discount either way.
- Jada says that you should apply the 20% off coupon first, and then the $30 off coupon.
- Han says that you should apply the $30 off coupon first, and then the 20% off coupon.
- Kiran says that it depends on how much you are spending.

Do you agree with any of them? Explain your reasoning.

Student Response

It is always better to use the 20% off coupon first.

- Let x represent the amount of the purchase.
- 20% off is $0.8x$ or equivalent. $30 off that is $0.8x - 30$.
- $30 off is $x - 30$. 20% off that is $0.8(x - 30)$. By the distributive property, this is equivalent to $0.8x - 24$.
- Comparing $0.8x - 30$ to $0.8x - 24$, your resulting bill will always be $6 less if you use the 20% off coupon first.

Activity Synthesis

If any students only computed their resulting bill using a specific dollar amount, ask them to present their solution first. For example, on a $100 purchase, this might look like:

- 20% off is $80. Then $30 off of $80 is $50.
- $30 off is $70. Then 20% off of $70 is $56.

So if your purchase was \$100, it's better to apply the 20% off coupon first. What about other purchase amounts? (If students tried other purchase amounts, consider also having them demonstrate. It is helpful if students see a few different examples that always result in a \$6 difference.)

Select a student to present who wrote an expression using a variable for the purchase amount. If no students did so, demonstrate this approach.

- Let x represent the amount of the purchase.
- 20% off is $0.8x$ or equivalent. \$30 off that is $0.8x - 30$.
- \$30 off is $x - 30$. 20% off that is $0.8(x - 30)$. By the distributive property, this is equivalent to $0.8x - 24$.
- Comparing $0.8x - 30$ to $0.8x - 24$, your resulting bill will always be \$6 less if you use the 20% off coupon first.