Lesson 11: Using Equations to Solve Problems

Goals

- Interpret and coordinate tape diagrams, equations, and verbal descriptions for situations involving signed numbers.
- Solve an equation of the form $px + q = r$ or $p(x + q) = r$ to determine an unknown quantity in a situation, and present the solution method (orally, in writing, and through other representations).
- Write an equation of the form $px + q = r$ or $p(x + q) = r$ to represent a situation involving signed numbers.

Learning Targets

- I can solve story problems by drawing and reasoning about a tape diagram or by writing and solving an equation.

Lesson Narrative

This lesson brings together the skills and concepts that have been studied in the unit so far. Students solve problems that can be represented by equations of the form $p(x + q) = r$ and $px + q = r$. A bit of scaffolding is offered in the first activity to reactivate their understanding of tape diagrams, but after that no scaffolding is offered so that students can make sense of problems (MP1) and choose representations to use (MP5).

Alignments

Addressing

- 7.EE.B.3: Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. $
- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR7: Compare and Connect
- Think Pair Share

Required Materials

Sticky notes
Tools for creating a visual display
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

Required Preparation

Decide if students will conduct group presentations or a gallery walk for the last activity. If so, prepare tools for creating a visual display and around 3 sticky notes per student. If not, these materials are not necessary.

Student Learning Goals

Let's use tape diagrams, equations, and reasoning to solve problems.

11.1 Remember Tape Diagrams

Warm Up: 5 minutes
The purpose of this warm-up is to reactivate students' understanding of tape diagrams to make it more likely that tape diagrams are accessible as a tool for them to choose in this lesson. The diagram was deliberately constructed to encourage some students to write an equation like $24 = 3(a + 2)$ and others like $24 = 3a + 6$. Monitor for one student who writes each type of equation.

Addressing

- 7.EE.B.4.a

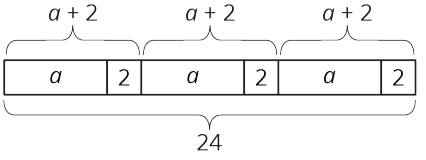
Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give 5 minutes of quiet think time and time to share their work with a partner followed by a whole-class discussion.

Student Task Statement



1. Write a story that could be represented by this tape diagram.
2. Write an equation that could be represented by this tape diagram.

Student Response

1. Answers vary. Sample story: A baker put a cookies in each of three boxes. Then, he put 2 more cookies in each box, and there were 24 total cookies in the 3 boxes.
2. $24 = 3(a + 2)$ or $24 = 3a + 6$ or equivalent

Activity Synthesis

After students have had a chance to share their work with their partner, select a few students to share their stories. Then, select one student to share each type of equation and explain its structure: $3(a + 2) = 24$ and $3a + 6 = 24$.

11.2 At the Fair

15 minutes

In this activity, students use a tape diagram to help them reason about a situation, write an equation that represents it, and solve the equation. Students can use both the diagram and the solution strategy of doing the same to each side and undoing that they saw in the past few lessons. The first two questions provide more scaffolding and the last question provides none.

When students work on the last question, monitor for students who

- reason numerically without any diagrams or representations.
- create a tape diagram and use it to reason numerically.
- write an equation like $6(x - 1.5) = 46.5$ and solve it by using the distributive property to find the total amount saved, $6 \cdot 1.50$.
- write an equation and solve it by first dividing by 6 to find the cost of each discounted ticket.

Addressing

- 7.EE.B.3
- 7.EE.B.4.a

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR7: Compare and Connect

Launch

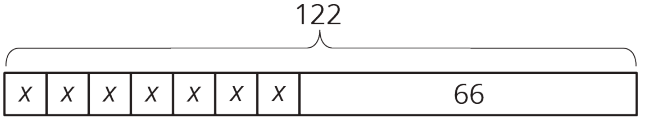
Keep students in the same groups. Give students 5–10 minutes of quiet work time and partner discussions followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

Student Task Statement

1. Tyler is making invitations to the fair. He has already made some of the invitations, and he wants to finish the rest of them within a week. He is trying to spread out the remaining work, to make the same number of invitations each day. Tyler draws a diagram to represent the situation.



   a. Explain how each part of the situation is represented in Tyler's diagram:

      How many total invitations Tyler is trying to make.

      How many invitations he has made already.

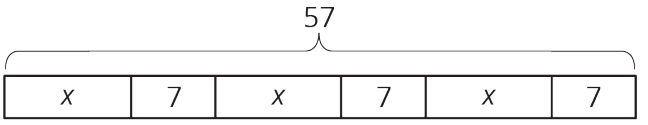
      How many days he has to finish the invitations.

   b. How many invitations should Tyler make each day to finish his goal within a week? Explain or show your reasoning.

   c. Use Tyler's diagram to write an equation that represents the situation. Explain how each part of the situation is represented in your equation.

   d. Show how to solve your equation.

2. Noah and his sister are making prize bags for a game at the fair. Noah is putting 7 pencil erasers in each bag. His sister is putting in some number of stickers. After filling 3 of the bags, they have used a total of 57 items.



a. Explain how the diagram represents the situation.

b. Noah writes the equation $3(x + 7) = 57$ to represent the situation. Do you agree with him? Explain your reasoning.

c. How many stickers is Noah's sister putting in each prize bag? Explain or show your reasoning.

3. A family of 6 is going to the fair. They have a coupon for $1.50 off each ticket. If they pay $46.50 for all their tickets, how much does a ticket cost without the coupon? Explain or show your reasoning. If you get stuck, consider drawing a diagram or writing an equation.

Student Response

1. a. 122 total invitations. 66 have been made already. 7 days to finish because there are 7 boxes with an equal, unknown amount x.

   b. 8. Sample reasoning: subtract 66 from 122 and divide the result, 56, by 7.

   c. $7x + 66 = 122$. He makes the same amount x each of 7 days, so $7x$ represents the number made in 7 days. He already made 66, so add those on, for a total of 122.

   d. $7x + 66 = 122$, $7x + 66 + (-66) = 122 + (-66)$, $7x = 56$, $7x \div 7 = 56 \div 7$, $x = 8$.

2. a. Answers vary. Sample response: There are 3 groups of x stickers and 3 groups of 7 erasers. All together there are 57 items.

   b. Answers vary. Sample response: Yes, Noah's equation says that 3 groups of $x + 7$ gives a total of 57 items.

   c. 12. Explanations vary. Sample responses: Each group of $x + 7$ represents $57 \cdot \frac{1}{3}$ or 19 items, so $x = 12$. Another way to find x is to subtract 21 from 57 and then $3x = 57 - 21 = 36$ so $x = 12$. The first strategy represents first multiplying each side of the equation by $\frac{1}{3}$, and the second represents using the distributive property to write $3(x + 7)$ as $3x + 21$.

3. \$9.25. Explanations vary. Sample response: Divide $46.50 \div 6 = 7.75$ to find what they paid for each ticket and then add \$1.50. Another way is to reason that they saved a total of $6 \cdot 1.50 = 9$, so add the 9 back to 46.50 to find the price of 6 tickets without the coupon, and then divide by 6. These two strategies connect to writing the equation $6(t - 1.50) = 46.50$, and solving it either by dividing by 6 first, or by using the distributive property to write $6t - 9 = 46.50$, $6t = 55.50$, $t = 9.25$.

Activity Synthesis

Invite selected students to share their strategies for the last problem, following the sequence of approaches in the Activity Narrative. As students present, display the different approaches side by side, and ask students to explain the meaning of the numbers they find.

Access for English Language Learners

Speaking, Representing: MLR7 Compare and Connect. Use this routine during the whole-group discussion as students compare approaches for solving the last problem. Comparisons should focus on different representations of the situation. Listen for phrases like: "I used the tape diagram because…." to highlight students' justifications for the representations they choose. Then ask students, "What is the same and what is different?" between the approaches. Amplify language that connect quantities between representations (e.g., "How is the 'family of 6' represented in your equation? In your diagram?"). This will help students produce language that describes their thinking about the connections between representations and approaches. *Design Principle(s): Maximize meta-awareness; Support sense-making*

11.3 Running Around

15 minutes

This activity offers four word problems. Depending on time constraints, you may have all students complete all four problems or assign a different problem to each group. The problems increase in difficulty. It is suggested that students create a visual display of one of the problems and do a gallery walk or presentation, but if time is short, you may choose to just have students work in their workbooks or devices.

Addressing

- 7.EE.B.3
- 7.EE.B.4
- 7.EE.B.4.a

Instructional Routines

- Group Presentations

Launch

Keep students in the same groups. Either instruct students to complete all four problems or assign one problem to each group. If opting to have students do presentations or a gallery walk, distribute tools for making a visual display.

Give students 5–6 minutes quiet work time and a partner discussion followed by a whole-class discussion or gallery walk.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems, and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

The phrases "9 times as far" and "9 times as many" may lead students to think about multiplying by 9 instead of dividing (or multiplying by $\frac{1}{9}$). Encourage students to act out the situations or draw diagrams to help reason about the relationship between the quantities. Remind them to pay careful attention to what or who a comparison refers to.

Student Task Statement

Priya, Han, and Elena, are members of the running club at school.

1. Priya was busy studying this week and ran 7 fewer miles than last week. She ran 9 times as far as Elena ran this week. Elena only had time to run 4 miles this week.
   a. How many miles did Priya run last week?
   b. Elena wrote the equation $\frac{1}{9}(x - 7) = 4$ to describe the situation. She solved the equation by multiplying each side by 9 and then adding 7 to each side. How does her solution compare to the way you found Priya's miles?
2. One day last week, 6 teachers joined $\frac{5}{7}$ of the members of the running club in an after-school run. Priya counted a total of 31 people running that day. How many members does the running club have?
3. Priya and Han plan a fundraiser for the running club. They begin with a balance of -80 because of expenses. In the first hour of the fundraiser they collect equal donations from 9 family members, which brings their balance to -44. How much did each parent give?
4. The running club uses the money they raised to pay for a trip to a canyon. At one point during a run in the canyon, the students are at an elevation of 128 feet. After descending at a rate of 50 feet per minute, they reach an elevation of -472 feet. How long did the descent take?

Student Response

1. a. 43 miles

b. Answers vary. Sample response: Since Priya ran 9 times as far as Elena, I multiplied 9 by 4 to get 36 miles. This is the number of miles Priya ran this week, so 36 is 7 less than what she ran last week. I added 36 and 7 to get 43 miles. Elena solved an equation and I worked backwards with the information in the problem (or used a diagram). But we both took the same steps, multiply by 9 and add 7.

2. 35 members (one way is to solve the equation $\frac{5}{7}x + 6 = 31$)

3. \$4 (one way is to solve $-80 + 9x = -44$)

4. 12 minutes (one way is to solve $128 - 50x = -472$)

Are You Ready for More?

A musician performed at three local fairs. At the first he doubled his money and spent \$30. At the second he tripled his money and spent \$54. At the third, he quadrupled his money and spent \$72. In the end he had \$48 left. How much did he have before performing at the fairs?

Student Response

\$29.

Arithmetic solution: Work backwards: $48 + 72 = 120$, $120 \div 4 = 30$, $30 + 54 = 84$, $84 \div 3 = 28$, $28 + 30 = 58$, $58 \div 2 = 29$.

Algebraic solution: Let m represent original amount of money. After first fair: $2x - 30$. After second fair: $3(2x - 30) - 54 = 6x - 144$. After third fair, $4(6x - 144) - 72 = 24x - 648$. Write equation $24x - 648 = 48$ to show how much money is left. Solution is $696 \div 24 = 29$.

Activity Synthesis

If students created a visual display and you opt to conduct a gallery walk, ask students to post their solutions. Distribute sticky notes and ask students to read others' solutions, using the sticky notes to leave questions or comments. Give students a moment to review any questions or comments left on their display.

Invite any students who chose to draw a diagram to share; have the class agree or disagree with their diagrams and suggest any revisions. Next, invite students who did not try to draw a diagram to share strategies. Ask students about any difficulties they had creating the expressions and equations. Did the phrase "9 times as many" suggest an incorrect expression? If yes, how did they catch and correct for this error?

Lesson Synthesis

Ask students to reflect on the work done in this unit so far. What strategies have they learned? What kinds of problems can they solve that they weren't able to, previously? Ask them to write down or share with a partner one new thing they have learned and one thing they still have questions or confusion about.

11.4 The Basketball Game

Cool Down: 5 minutes

Addressing

- 7.EE.B.3
- 7.EE.B.4.a

Student Task Statement

Diego scored 9 points less than Andre in the basketball game. Noah scored twice as many points as Diego. If Noah scored 10 points, how many points did Andre score?

Student Response

14 points. Explanations vary. Sample responses:

- Equation: $2(x - 9) = 10$, where x is the number of points scored by Andre. $x - 9 = 5$, $x = 14$.
- Reasoning: Diego scored half as many points as Noah, so he scored 5 points. Andre scored 9 points more than Diego, or 14 points.
- Diagram: One possibility is two boxes each with $x - 9$ showing a total of 10. Each box represents 5 points, so x is 14.

Student Lesson Summary

Many problems can be solved by writing and solving an equation. Here is an example:

Clare ran 4 miles on Monday. Then for the next six days, she ran the same distance each day. She ran a total of 22 miles during the week. How many miles did she run on each of the 6 days?

One way to solve the problem is to represent the situation with an equation, $4 + 6x = 22$, where x represents the distance, in miles, she ran on each of the 6 days. Solving the equation gives the solution to this problem.

$$\begin{aligned} 4 + 6x &= 22 \\ 6x &= 18 \\ x &= 3 \end{aligned}$$

Lesson 11 Practice Problems

Problem 1

Statement

Find the value of each variable.

a. $a \cdot 3 = -30$

b. $-9 \cdot b = 45$

c. $-89 \cdot 12 = c$

d. $d \cdot 88 = -88{,}000$

Solution

a. $a = -10$

b. $b = -5$

c. $c = -1{,}068$

d. $d = -1{,}000$

(From Unit 5, Lesson 9.)

Problem 2

Statement

Match each equation to its solution and to the story it describes.

Equations:

a. $5x - 7 = 3$

b. $7 = 3(5 + x)$

c. $3x + 5 = -7$

d. $\frac{1}{3}(x + 7) = 5$

Solutions:

a. -4

b. $\frac{-8}{3}$

c. 2

d. 8

Stories:

- The temperature is -7. Since midnight the temperature tripled and then rose 5 degrees. What was temperature at midnight?
- Jada has 7 pink roses and some white roses. She gives all of them away: 5 roses to each of her 3 favorite teachers. How many white roses did she give away?
- A musical instrument company reduced the time it takes for a worker to build a guitar. Before the reduction it took 5 hours. Now in 7 hours they can build 3 guitars. By how much did they reduce the time it takes to build each guitar?
- A club puts its members into 5 groups for an activity. After 7 students have to leave early, there are only 3 students left to finish the activity. How many students were in each group?

Solution

a. 3, club activity story

b. 2, building guitars story

c. 1, temperature story

d. 4, roses story

Problem 3

Statement

The baby giraffe weighed 132 pounds at birth. He gained weight at a steady rate for the first 7 months until his weight reached 538 pounds. How much did he gain each month?

Solution

58 pounds. He gained $538 - 132$, or 406 pounds, over 7 months. $406 \div 7 = 58$. (Or solve $132 + 7x = 538$.)

Problem 4

Statement

Six teams are out on the field playing soccer. The teams all have the same number of players. The head coach asks for 2 players from each team to come help him move some equipment. Now there are 78 players on the field. Write and solve an equation whose solution is the number of players on each team.

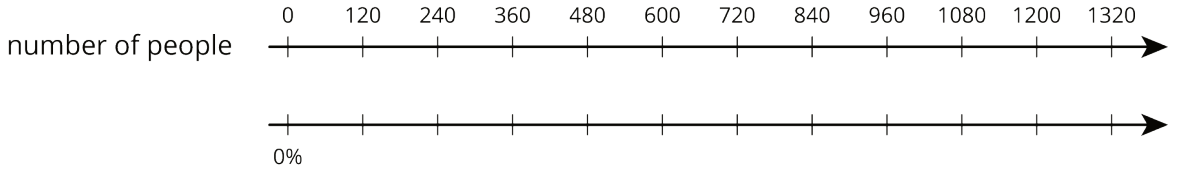
Solution

$6(x - 2) = 78$ (or $6x - 12 = 78$), $x = 15$

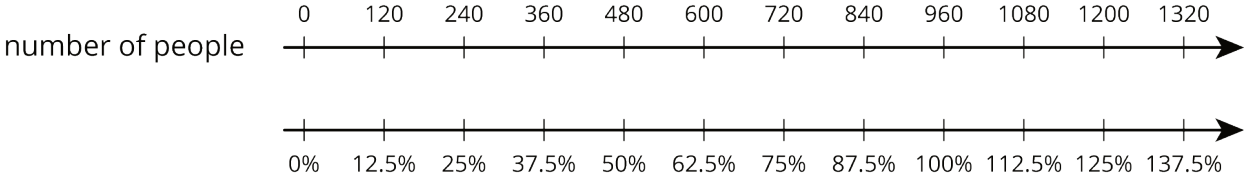
Problem 5

Statement

A small town had a population of 960 people last year. The population grew to 1200 people this year. By what percentage did the population grow?



Solution



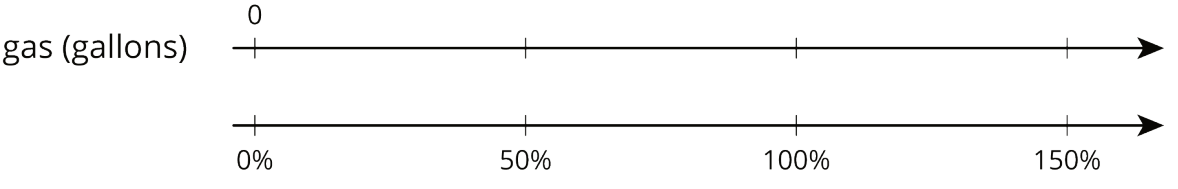
The town has grown by 25%.

(From Unit 4, Lesson 7.)

Problem 6

Statement

The gas tank of a truck holds 30 gallons. The gas tank of a passenger car holds 50% less. How many gallons does it hold?



Solution

15 gallons because 50% less than 30 is 15. (If the double number line is used, the tick marks on the top are labeled 0, 15, 30, 45.)

(From Unit 4, Lesson 7.)