Lesson 9: Dealing with Negative Numbers

Goals

- Generalize (orally) that doing the same thing to each side of an equation generates an equivalent equation.
- Solve equations of the form $px + q = r$ or $p(x + q) = r$ that involve negative numbers, and explain (orally and in writing) the solution method.

Learning Targets

- I can use the idea of doing the same to each side to solve equations that have negative numbers or solutions.

Lesson Narrative

In the previous lessons, we used hangers to reason about ways to approach equations of the form $px + q = r$ or $p(x + q) = r$ (which can be summed up as "do the same thing to each side until the unknown equals a number"). Since the things we do to each side of an equation are just arithmetic operations, and the properties of operations extend to negative numbers, this method of solving equations also works when there are negative numbers, even though it doesn't make physical sense to think about weights on hangers representing negative numbers. After a warm-up designed to remind students about operating on rational numbers, students are asked to solve some straightforward equations involving negative numbers. "Doing the same thing to each side" is presented as a valid method, even though negative numbers are involved. In the last activity, students do the same thing to each side of an equation and their partner tries to guess what they did. The purpose of this is to communicate that doing the same thing to each side maintains equality even when the moves aren't intended to lead to the equation's solution.

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.
- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Which One Doesn’t Belong?

Student Learning Goals

Let’s show that doing the same to each side works for negative numbers too.

9.1 Which One Doesn’t Belong: Rational Number Arithmetic

Warm Up: 10 minutes

This warm-up prompts students to compare four equations. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear what they recall of arithmetic on signed numbers. To allow all students to access the activity, each equation has one obvious reason it does not belong. Encourage students to move past the obvious reasons and find reasons based on mathematical properties.

During the discussion, listen for strategies for evaluating expressions with rational numbers that will be helpful in the work of this lesson.

Building On

- 7.NS.A

Instructional Routines

- Which One Doesn’t Belong?

Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular equation does not belong and together find at least one reason each question doesn’t belong.

Student Task Statement

Which equation doesn't belong?

$15 = -5 \cdot -3$

$4 - -2 = 6$

$2 + -5 = -3$

$-3 \cdot -4 = -12$

Student Response

1. $15 = -5 \cdot -3$ doesn't belong because it's the only one with a number on the left and an operation on the right.
2. $2 + -5 = -3$ doesn't belong because it's the only one with addition.
3. $4 - -2 = 6$ doesn't belong because it's the only one with subtraction.
4. $-3 \cdot -4 = -12$ doesn't belong because it's the only one that is not true.

Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students' explanations and ensure the reasons given are correct. During the discussion, highlight any strategies for adding, subtracting, or multiplying signed numbers.

9.2 Old and New Ways to Solve

15 minutes

These are all solvable by thinking "what value would make the equation true." So, it's straightforward to figure out what the solution would be, but these equations present an opportunity to demonstrate that "doing the same thing to each side" still works when there are negative numbers. Monitor for students who reason about what value would make the equation true and those who reason by doing the same thing to each side.

Addressing

- 7.EE.B.4.a

Instructional Routines

- MLR7: Compare and Connect

Launch

Give 5–10 minutes of quiet work time followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge about strategies for performing operations on signed numbers, as well as different representations of equations. This will help students make connections between new problems and prior work. *Supports accessibility for: Memory; Conceptual processing*

Anticipated Misconceptions

Some students may need some additional support remembering and applying strategies for performing operations on signed numbers. Draw their attention to any anchor charts or notes that are available from the previous unit.

Student Task Statement

Solve each equation. Be prepared to explain your reasoning.

1. $x + 6 = 4$
2. $x - -4 = -6$
3. $2(x - 1) = -200$
4. $2x + -3 = -23$

Student Response

1. $x = -2$
2. $x = -10$
3. $x = -99$
4. $x = -10$

Activity Synthesis

For each equation, ask one student to explain how they know their solution is correct. If no students mention this approach, demonstrate solving each equation by doing the same thing to each side.

The purpose of this discussion is to make the claim that “do the same thing to each side” also works when subtraction or negative numbers are involved. Tell students that even though it doesn’t make sense to represent negative numbers using the hanger metaphor, we are going to take it as a fact that we can still do the same thing to each side of an equation even when we are working with negative numbers.

Access for English Language Learners

Representing: MLR7 Compare and Connect. Use this routine when students explain how they solved their equations. Ask students, “What is the same and what is different?” about the strategies. Draw students’ attention to the connection between the approaches of ‘finding the value that makes the equation true’ and ‘doing the same to each side.’ These exchanges strengthen students’ mathematical language use and reasoning based on ways to solve equations that involve negative numbers.
Design Principle(s): Support sense-making; Maximize meta-awareness

9.3 Keeping It True

15 minutes
The purpose of this activity is to note that doing the same thing to each side of an equation keeps it in balance, even if those moves don’t get us closer to solving the equation. Students first explain how each equation in a sequence follows logically from the previous one. Then, they start with the equation $-5 = x$ and repeatedly do the same thing to each side to create a new equation. Their partner tries to guess which moves they made.

Building Towards

- 7.EE.B.4.a

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Give students 2 minutes of quiet work time on the first question, pause for a discussion, and then time to complete the task with their partner.

After students have a chance to work on the first question, pause for a discussion. Ask students what different types of moves could we do to $x = -6$? List the different kinds of things in the board, so when students do their own, you can say you have to use different combinations of things on the list. The purpose of this is to prevent students from going wild and generating equations that are far out of the scope of the work in this unit.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. Consider pausing after the first question for a brief class discussion before moving on. *Supports accessibility for: Organization; Attention*

Access for English Language Learners

Speaking: MLR8 Discussion Supports. To support students to produce statements about how their partner transformed the equation $-5 = x$, ask students to revisit the moves that were used earlier in the lesson to transform $x = -6$. Students should use different combinations of the moves on the list in conjunction with sentence frames such as “I noticed ___ so I . . .” or “I know ___ because . . . .” This will help students explain their thinking. *Design Principle(s): Optimize output (for explanation)*

Anticipated Misconceptions

Some students may need some additional support remembering and applying strategies for performing operations on signed numbers. Draw their attention to any anchor charts or notes that are available from the previous unit.

Student Task Statement

Here are some equations that all have the same solution.

$$\begin{aligned} x &= -6 \\ x - 3 &= -9 \\ -9 &= x - 3 \\ 900 &= -100(x - 3) \\ 900 &= (x - 3) \cdot (-100) \\ 900 &= -100x + 300 \end{aligned}$$

1. Explain how you know that each equation has the same solution as the previous equation. Pause for discussion before moving to the next question.

2. Keep your work secret from your partner. Start with the equation $-5 = x$. Do the same thing to each side at least three times to create an equation that has the same solution as the starting equation. Write the equation you ended up with on a slip of paper, and trade equations with your partner.

3. See if you can figure out what steps they used to transform $-5 = x$ into their equation. When you think you know, check with them to see if you are right.

Student Response

1. Answers vary. Sample responses:
    - subtract 3 from each side
    - swap the two sides of the equation
    - multiply each side by -100
    - swap the factors -100 and $x - 3$ (the commutative property of multiplication)
    - apply the distributive property
2. Answers vary.
3. Answers vary.

Activity Synthesis

Much of the discussion will take place in small groups. Questions for discussion:

- "Did you have any disagreements, and how did you resolve them?"
- "Did anything surprise you? Explain."
- "What are some important things to keep in mind when working with negative numbers?"

Lesson Synthesis

Ask students to think of one or two important things they learned in this lesson, and share them with a partner. Points to highlight include:

- Doing the same thing to each side of an equation still keeps the equation balanced, even when there are negative numbers.
- Doing the same thing to each side of an equation still keeps the equation balanced, even when the moves don't get you closer to a solution.

9.4 Solve Two More Equations

Cool Down: 5 minutes

Addressing

- 7.EE.B.4

Student Task Statement

Solve each equation. Show your work, or explain your reasoning.

1. $-3x - 5 = 16$
2. $-4(y - 2) = 12$

Student Response

1. $x = -7$

2. $y = -1$

Student Lesson Summary

When we want to find a solution to an equation, sometimes we just think about what value in place of the variable would make the equation true. Sometimes we perform the same operation on each side (for example, subtract the same amount from each side). The balanced hangers helped us to understand that doing the same thing to each side of an equation keeps the equation true.

Since negative numbers are just numbers, then doing the same thing to each side of an equation works for negative numbers as well. Here are some examples of equations that have negative numbers and steps you could take to solve them.

Example:

$$\begin{aligned} 2(x-5) &= \text{-}6 \\ \tfrac{1}{2} \cdot 2(x-5) &= \tfrac{1}{2} \cdot (\text{-}6) && \text{multiply each side by } \tfrac{1}{2} \\ x-5 &= \text{-}3 \\ x-5+5 &= \text{-}3+5 && \text{add 5 to each side} \\ x &= 2 \end{aligned}$$

Example:

$$\begin{aligned} \text{-}2x + \text{-}5 &= 6 \\ \text{-}2x + \text{-}5 - \text{-}5 &= 6 - \text{-}5 && \text{subtract -5 from each side} \\ \text{-}2x &= 11 \\ \text{-}2x \div \text{-}2 &= 11 \div \text{-}2 && \text{divide each side by -2} \\ x &= \text{-}\tfrac{11}{2} \end{aligned}$$

Doing the same thing to each side maintains equality even if it is not helpful to solving for the unknown amount. For example, we could take the equation $\text{-}3x + 7 = \text{-}8$ and add -2 to each side:

$$\begin{aligned} \text{-}3x + 7 &= \text{-}8 \\ \text{-}3x + 7 + \text{-}2 &= \text{-}8 + \text{-}2 && \text{add -2 to each side} \\ \text{-}3x + 5 &= \text{-}10 \end{aligned}$$

If $\text{-}3x + 7 = \text{-}8$ is true then $\text{-}3x + 5 = \text{-}10$ is also true, but we are no closer to a solution than we were before adding -2. We can use moves that maintain equality to make new equations that all have the same solution. Helpful combinations of moves will eventually lead to an equation like $x = 5$, which gives the solution to the original equation (and every equation we wrote in the process of solving).

Lesson 9 Practice Problems

Problem 1

Statement

Solve each equation.

a. $4x = \text{-}28$

b. $x - \text{-}6 = \text{-}2$

c. $\text{-}x + 4 = \text{-}9$

d. $\text{-}3x + 7 = 1$

e. $25x + \text{-}11 = \text{-}86$

Solution

a. -7

b. -8

c. 13

d. 2

e. -3

Problem 2

Statement

Here is an equation $2x + 9 = \text{-}15$. Write three different equations that have the same solution as $2x + 9 = \text{-}15$. Show or explain how you found them.

Solution

Equations vary. Sample equations: $24 + 2x = 0$, $4x + 10 = \text{-}38$, $85 = 2x + 109$

Sample explanation:

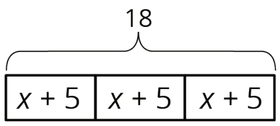
- Start with: $2x + 9 = \text{-}15$.
- Add 20 to each side: $2x + 29 = 5$.
- Use the commutative property of addition: $29 + 2x = 5$.

- Subtract 5 from each side: $24 + 2x = 0$.

Problem 3

Statement

Select **all** the equations that match the diagram.



A. $x + 5 = 18$

B. $18 \div 3 = x + 5$

C. $3(x + 5) = 18$

D. $x + 5 = \frac{1}{3} \cdot 18$

E. $3x + 5 = 18$

Solution

["B", "C", "D"]

(From Unit 6, Lesson 3.)

Problem 4

Statement

There are 88 seats in a theater. The seating in the theater is split into 4 identical sections. Each section has 14 red seats and some blue seats.

a. Draw a tape diagram to represent the situation.

b. What unknown amounts can be found by by using the diagram or reasoning about the situation?

Solution

Answers vary. Sample responses:

a. A tape diagram with 4 equal parts, each labeled $x + 14$, for a total of 88.

b. Each section has 22 seats, of which 8 are blue. There are 32 blue seats and 56 red seats in the theater.

(From Unit 6, Lesson 2.)

Problem 5

Statement

Match each story to an equation.

A. A stack of nested paper cups is 8 inches tall. The first cup is 4 inches tall and each of the rest of the cups in the stack adds $\frac{1}{4}$ inch to the height of the stack.

B. A baker uses 4 cups of flour. She uses $\frac{1}{4}$ cup to flour the counters and the rest to make 8 identical muffins.

C. Elena has an 8-foot piece of ribbon. She cuts off a piece that is $\frac{1}{4}$ of a foot long and cuts the remainder into four pieces of equal length.

1. $\frac{1}{4} + 4x = 8$
2. $4 + \frac{1}{4}x = 8$
3. $8x + \frac{1}{4} = 4$

Solution

- A: 2
- B: 3
- C: 1

(From Unit 6, Lesson 4.)