Lesson 10: Measuring Long Distances Over Uneven Terrain

Goals

- Choose a method and appropriate measuring tools to measure the length of a path.
- Compare and contrast (orally and in writing) methods for measuring distances.
- Compare measurements of a path and represent (in writing) the difference between measurements as a percentage.

Lesson Narrative

This lesson is optional. It is the first of four lessons where students explore ways of measuring long distances. Over the course of these lessons:

- Students first think about different ways to measure distances of various lengths and in which situations different methods might work better.
- They then read about and build a trundle wheel (also known as a surveyor wheel or measuring wheel) that is commonly used to measure walking distances.
- They design a walking course for a 5K race on their school campus. (The course should be one lap of about 500 m. The actual race would go around the course multiple times.)
- They use their trundle wheel to measure the path of the walking course and make a scale drawing of the course on a map or satellite image of the school grounds.

In this first lesson, students brainstorm ideas about how to measure long distances, possibly over uneven terrain. Students work in groups to try out the accuracy and effectiveness of different methods. Some of the methods involve proportional reasoning. Students engage in many aspects of mathematical modeling (MP4) and will use appropriate tools (MP5) when they are planning and trying out methods of measurement.

As with all lessons in this unit, all related standards have been addressed in prior units. This lesson provides an optional opportunity to go deeper and make connections between domains.

Alignments

Building On

- 2.MD.A.1: Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.

Addressing

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Building Towards

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Measuring tapes **Yardsticks**
Meter sticks

Required Preparation

Choose a path outside of the classroom that students can measure the length of. The length should be between 50 and 100 meters (long enough that it cannot be measured directly with a tape measure. Ideally it should include some curves or elevation changes (but not stairs). A long hallway would also be okay, especially if it goes around corners. This is not part of the 5K course, rather it is just a path to test measuring methods.

Student Learning Goals

Let's measure long distances over uneven terrain.

10.1 How Far Is It?

Optional: 5 minutes
Students have experienced measuring short distances with a ruler or a measuring tape. In this activity, students start to think about how they can measure longer distances over uneven terrain. This activity is intended to set the stage for the upcoming activities, not to completely resolve the question. Students have an opportunity to think about the limitations of methods that may work for short distances but not for long distances. They also consider real-world situations that involve the measurement of long distances.

Building On

- 2.MD.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 3–4. They will stay in these groups throughout this 4-lesson unit. Ask students how they have measured the length of objects in school (with a ruler, yardstick, or measuring tape). Where else in real life do people measure distances, especially longer ones?

Brainstorm some situations together (distance driven in a car, length of a garden fence, length of a hiking trail, etc.). Give students 2–3 minutes of quiet work time, followed by small-group discussion.

Anticipated Misconceptions

Some students may get stuck thinking about classroom situations. Prompt them to think about other situations outside of school, such as driving in a car, measuring the distance between cities, measuring the length of a fence around a yard, etc.

Student Task Statement

How do people measure distances in different situations? What tools do they use? Come up with at least three different methods and situations where those methods are used.

Student Response

Answers vary. Sample responses:

- Use a yardstick, a measuring tape or ruler repeatedly, if necessary.
- When driving use the odometer to measure distance between departure and destination location.
- Count your steps and estimate how long one step is, for example, to measure the distance across the room.
- Estimate an inch with your fingers and iterate across the width of your table to measure table width.
- Count the number of ceiling tiles or the number of windows across the room, estimate the width of tiles or windows and multiply the number of objects times the width of the object.
- GPS
- Use “rate times time,” if you know your speed and how long it takes to get somewhere.
- Distance between stars.

Activity Synthesis

Invite students to share some ideas of how to measure with their group.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Give students additional time to make sure that everyone in their group can explain all three different methods and situations they created. Prompt groups to rehearse what they will say when they share with the whole class. Rehearsing provides students with additional opportunities to speak and clarify their thinking. This will also help students improve the quality of their explanations during the whole-class discussion.
Design Principle(s): Optimize output (for explanation)

10.2 Planning a 5K Course

Optional: 10 minutes
In the previous activity, students started to think about how to measure distances in different situations. The activity introduces the context of designing a course for a 5K fundraising walk. Students will continue working with this context in future lessons. In this activity, they come up with a method for measuring the walking distance of a path that is too long to measure with a measuring tape. Students will try out their method in the next activity.

Students get a chance to engage in many aspects of mathematical modeling (MP4). The modeling cycle starts with formulating the question that we want to answer, clarifying which quantities are involved, and how to measure them. In many problems, this step is done for students. Here, we are giving them the opportunity to think about how to set up the problem and what tools are appropriate to measure distances.

Building On

- 2.MD.A.1

Building Towards

- 7.RP.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups. Provide access to measuring tools, such as yardsticks, meter sticks, and tape measures. Ask students if they have ever participated in or watched a walk-a-thon or race. Explain that sometimes a race is done by repeating a shorter course several times, e.g. a mile is about 4 laps around a track. For this activity, they should plan for a course that is about 500 meters long that walkers can go around multiple times.

Give students 5–6 minutes to work with their group.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. When preparing a plan for measuring the course, invite students to use a sentence frame such as: "One method for measuring the course is . . . ." Encourage students to consider what details are important to share and to think about how they will explain their reasoning using mathematical language. This will help students to converse while they design the course and decide how to measure its distance.
Design Principle(s): Optimize output (for description)

Student Task Statement

The school is considering holding a 5K fundraising walk on the school grounds. Your class is supposed to design the course for the walk.

1. What will you need to do to design the course for the walk?
2. Come up with a method to measure the course. Pause here so your teacher can review your plan.

Student Response

1. Answers vary. Sample response: We need to find a course for one lap of the race, decide where the start and end is, measure it, and then figure out how often you have to go around it to complete 5 km.
2. Answers vary. Sample responses:
   a. Use a measuring tape over and over again.
   b. Measure your stride length, and then count the number of steps.
   c. Find a map of the campus, and use the scale on the map to compute the length of the course.

Activity Synthesis

Students check with the teacher about their method of measurement and then move on to the next activity.

10.3 Comparing Methods

Optional: 20 minutes
In this activity, students use the method they came up with in the previous activity to measure the length of a path chosen by the teacher. Each group can begin working on this activity as soon as they have finished the previous activity and checked in with the teacher.

It is not important that students' results are very accurate. They will measure the distance again with a trundle wheel in a later lesson. The main point of this activity is to think about measurement methods and to discuss the advantages and disadvantages of different methods.

Addressing

- 7.RP.A

Launch

Keep students in the same groups. Provide access to measuring tools. Show students the path they should measure. Give students time to measure with their group.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer for data collection and organizing information about methods, lengths and average between two measurements.
Supports accessibility for: Language; Organization

Anticipated Misconceptions

Some students may need to be reminded how to use the measuring tools accurately, such as starting at the 0 mark and keeping the measuring tool going in a straight line.

Student Task Statement

Let's see how close different measuring methods are to each other. Your teacher will show you a path to measure.

1. Use your method to measure the length of the path at least two times.
2. Decide what distance you will report to the class.
3. Compare your results with those of two other groups. Express the differences between the measurements in terms of percentages.
4. Discuss the advantages and disadvantages of each group's method.

Student Response

1. Answers vary.
2. Answers vary. Sample response: Report the average between the two measurements.
3. Answers vary. Sample responses: If group A's measurement is 50 m and group B's measurement is 51 m, then group B's measurement is 2% larger than group A's since $51 \div 50 = 1.02$.
4. Answers vary. For the two methods given in previous task:

a. Use a measuring tape over and over again. Advantages: Can be very accurate. Disadvantage: It takes two people and is quite cumbersome. If not done carefully, each time the tape moves, an error is introduced. So this is not very practical for long distances and if there are a lot of corners to go around.

b. Measure stride length, and then count the number of strides. Advantages: Very easy to do and very quick. Disadvantage: Not all strides are equal. The longer the distance, the more chances for errors there are.

Activity Synthesis

Invite the different groups to share their solutions. Ask them to:

- Compare how close their answers are.
- Compute the approximate relative error (difference/total length).
- Discuss the advantages and disadvantages of their methods and sources of discrepancies in their measurements, and how a small error can propagate.

The takeaway should include:

- We can use proportional reasoning to find longer distances. If we know it takes 10 steps to walk 8 meters, then it will take 20 steps to walk 16 meters.
- Small errors can magnify over longer distances.
- Methods were either not very precise (prone to introduce error), or they were precise but cumbersome to implement.

Access for English Language Learners

Representing: MLR3 Clarify, Critique, Correct. Display an incorrect statement about the percentage difference between measurements such as: "One group compared their measurement of 1000 meters to another group's measurement of 992 meters and calculated the percent difference as 8%.". Prompt students to clarify and critique the error, and then write a correct version. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Cultivate conversation; Maximize meta-awareness