# Lesson 18: Subtraction in Equivalent Expressions

## Goals

- Explain (orally, in writing, and using other representations) how the distributive and commutative properties apply to expressions with negative coefficients.
- Justify (orally and in writing) whether expressions are equivalent, including rewriting subtraction as adding the opposite.

## Learning Targets

- I can organize my work when I use the distributive property.
- I can re-write subtraction as adding the opposite and then rearrange terms in an expression.

## Lesson Narrative

Previously in this unit, students solved equations of the form $px + q = r$ and $p(x + q) = r$. Sometimes, work has to be done on a more complicated expression to get an equation into one of these forms. And sometimes, it is desirable to rewrite an expression in an equivalent form to understand how the quantities it represents are related. This work has some pitfalls when the expression has negative numbers or subtraction. For example, it is common for people to rewrite $6x - 5 + 2x$ as $4x + 5$ by reading "$6x$ minus" and so subtracting the $2x$ from the $6x$. Another example is rewriting an expression like $5x - 2(x + 3)$ as $5x - 2x + 6$. Students do not see expressions as complicated as these in this lesson (they are coming in the next few lessons), but this lesson is meant to inoculate students against errors like these by reminding them that while subtraction is not commutative, addition is, and subtraction can be rewritten as adding the opposite. So in our example, $6x - 5 + 2x$ can be rewritten $6x + \text{-}5 + 2x$ and then rearranged $6x + 2x + \text{-}5$.  Likewise, $5x - 2(x + 3)$ can be rewritten $5x + \text{-}2(x + 3)$ before distributing -2.

### Alignments

### Building On

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

### Addressing

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.
- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

### Building Towards

- 7.EE.A.1: Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

### Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Number Talk

> **Student Learning Goals**
>
> Let's find ways to work with subtraction in expressions.

# 18.1 Number Talk: Additive Inverses

**Warm Up: 5 minutes**

The purpose of this Number Talk is to elicit strategies and understandings that students have for adding and subtracting signed numbers. These understandings help students develop fluency and will be helpful later in this lesson when students will need to be able to rewrite subtraction as adding the opposite. While four problems are given, it may not be possible to share every strategy. Consider gathering only two or three different strategies per problem, saving most of the time for the final question.

### Addressing

- 7.NS.A.1

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

### Access for Students with Disabilities

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.
*Supports accessibility for: Memory; Organization*

## Student Task Statement

Find each sum or difference mentally.

$-30 + -10$

$-10 + -30$

$-30 - 10$

$10 - -30$

## Student Response

Answers vary. Possible responses:

- $-30 + -10$ is -40, because I can represent -30 as an arrow pointing left from 0 to -30 on the number line. Adding -10 tacks on an additional 10 to the left, arriving at -40.
- $-10 + -30$ is -40, because addition is commutative.
- $-30 - 10$ is -40, because subtracting 10 is the same as adding -10, and $-30 + -10 = -40$.
- $10 - -30$ is 40, because subtracting -30 is the same as adding 30, and $10 + 30 = 40$.

## Activity Synthesis

When it comes up, emphasize that “subtract 10” can be rewritten “add negative 10.” Also that addition is commutative but subtraction is not. Mention these points even if students do not bring them up.

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
*Design Principle(s): Optimize output (for explanation)*

# 18.2 A Helpful Observation

**10 minutes**
Students recall that subtracting a number (or expression) is the same as adding its additive inverse. This concept is applied to get students used to the idea that the subtraction sign has to stay with the term it is in front of. Making this concept explicit through a numeric example will help students see its usefulness and help them avoid common errors in working with expressions that involve subtraction.

## Building On

- 7.NS.A.1.c

## Building Towards

- 7.EE.A.1

## Instructional Routines

- MLR8: Discussion Supports

## Launch

Display the expression $7\frac{3}{4} + 3\frac{5}{6} - 1\frac{3}{4}$ and ask students to evaluate. After they have had a chance to think about the expression, read through the task statement together before setting students to work.

### Access for Students with Disabilities

*Representation: Access for Perception.* Read the dialogue between Lin and Kiran aloud. Students who both listen to and read the information will benefit from extra processing time. Consider having pairs of students role play the scenario together and repeat it as necessary in order to comprehend the situation.
*Supports accessibility for: Language*

### Access for English Language Learners

*Conversing, Speaking: MLR8 Discussion Supports.* Provide sentence frames to help students produce explanations about equivalent expressions. For example, "I agree/disagree that ____ is equivalent to $7\frac{3}{4}+3\frac{5}{6}-1\frac{3}{4}$ because . . . ." This will help students use the language of justification for comparing equivalent expressions related to the communicative property of addition.
*Design Principle(s): Optimize output (for justification)*

## Student Task Statement

Lin and Kiran are trying to calculate $7\frac{3}{4}+3\frac{5}{6}-1\frac{3}{4}$. Here is their conversation:

Lin: "I plan to first add $7\frac{3}{4}$ and $3\frac{5}{6}$, so I will have to start by finding equivalent fractions with a common denominator."

Kiran: "It would be a lot easier if we could start by working with the $1\frac{3}{4}$ and $7\frac{3}{4}$. Can we rewrite it like $7\frac{3}{4}+1\frac{3}{4}-3\frac{5}{6}$?"

Lin: "You can't switch the order of numbers in a subtraction problem like you can with addition; $2-3$ is not equal to $3-2$."

Kiran: "That's true, but do you remember what we learned about rewriting subtraction expressions using addition? $2-3$ is equal to $2+(\text{-}3)$."

1. Write an expression that is equivalent to $7\frac{3}{4}+3\frac{5}{6}-1\frac{3}{4}$ that uses addition instead of subtraction.
2. If you wrote the **terms** of your new expression in a different order, would it still be equivalent? Explain your reasoning.

## Student Response

1. $7\frac{3}{4}+3\frac{5}{6}+\left(\text{-}1\frac{3}{4}\right)$
2. Answers vary. Sample response: It works as long as the subtraction or negative sign is moved along with the number that follows. What doesn't work is moving the numbers but leaving the subtraction sign in the same place.

## Activity Synthesis

Ensure everyone agrees that $7\frac{3}{4}+3\frac{5}{6}-1\frac{3}{4}$ is equivalent to $7\frac{3}{4}+3\frac{5}{6}+\left(\text{-}1\frac{3}{4}\right)$ is equivalent to $7\frac{3}{4}+\text{-}1\frac{3}{4}+3\frac{5}{6}$. Use the language "commutative property of addition."

# 18.3 Organizing Work

**15 minutes**

Students learn that we can still organize our work with the distributive property in a familiar way, even with negative numbers where thinking in terms of area breaks down.

### Building On

- 7.NS.A.1.c

### Addressing

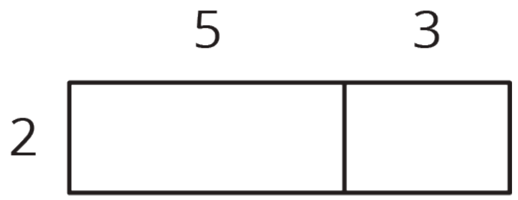
- 7.EE.A.1

### Instructional Routines

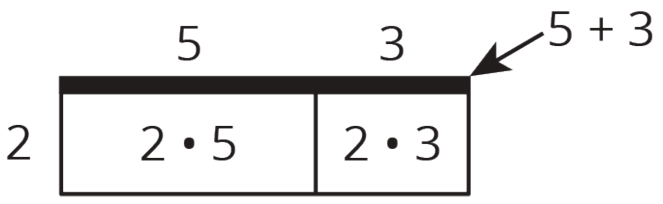
- MLR7: Compare and Connect

### Launch

Display the image and ask students to write an expression for the area of the big rectangle in at least 3 different ways.




Collect responses. If students simply say "16," ask them to explain how they calculated 16 and record these processes for all to see. Remind students that thinking about area gives us a way to understand the distributive property. This diagram can be used to show that $2 \cdot 5 + 2 \cdot 3 = 2(5 + 3)$. Be sure that students see you write the partial products in the diagram, and that they see every piece of the associated identity $2 \cdot 5 + 2 \cdot 3 = 2(5 + 3)$.



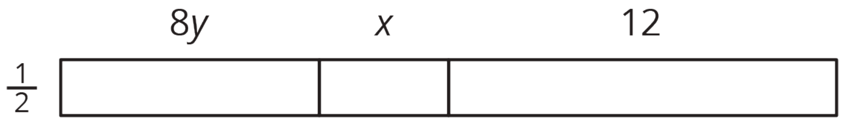

Tell students that when we are working with negative numbers, thinking about area doesn't work so well, but the distributive property still holds when there are negative numbers. The expressions involved still have the same structure, and we can still organize our work the same way.

## Access for Students with Disabilities

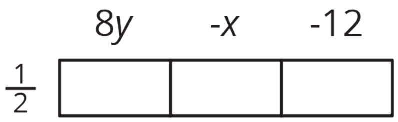
*Representation: Internalize Comprehension.* Differentiate the degree of difficulty or complexity by beginning with an example with more accessible values, such as the one given. Highlight connections between representations by recording the calculations in the boxes. In addition, consider creating a display showing a general example using only variables and keeping it as a reference throughout the remainder of the unit.
*Supports accessibility for: Conceptual processing*

### Student Task Statement

1. Write two expressions for the area of the big rectangle.




2. Use the distributive property to write an expression that is equivalent to $\frac{1}{2}(8y + \text{-}x + \text{-}12)$. The boxes can help you organize your work.




3. Use the distributive property to write an expression that is equivalent to $\frac{1}{2}(8y - x - 12)$.

### Student Response

1. $\frac{1}{2}(8y + x + 12)$ and $4y + \frac{1}{2}x + 6$
2. $4y + \text{-}\frac{1}{2}x + \text{-}6$
3. $4y - \frac{1}{2}x - 6$

### Are You Ready for More?

Here is a calendar for April 2017.

| April 2017 | | | | | | |
|---|---|---|---|---|---|---|
| Sunday | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| | | | | | | 1 |
| 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 9 | 10 | 11 | 12 | 13 | 14 | 15 |
| 16 | 17 | 18 | 19 | 20 | 21 | 22 |
| 23 | 24 | 25 | 26 | 27 | 28 | 29 |
| 30 | | | | | | |

Let's choose a date: the 10th. Look at the numbers above, below, and to either side of the 10th: 3, 17, 9, 11.

1. Average these four numbers. What do you notice?
2. Choose a different date that is in a location where it has a date above, below, and to either side. Average these four numbers. What do you notice?
3. Explain why the same thing will happen for any date in a location where it has a date above, below, and to either side.

### Student Response

1. The average of the four numbers is 10, because $(3 + 17 + 9 + 11) \div 4 = 40 \div 4 = 10$. The average of the four surrounding numbers equals the original date chosen.
2. Answers vary. Sample response: Let's choose 21. The four numbers are 14, 28, 20, 22. The average of these is 21, because $(14 + 28 + 20 + 22) \div 4 = 84 \div 4 = 21$. The average of the four surrounding numbers equals the original date chosen.
3. Answers vary. Sample response using algebra. If the original date chosen is represented by $x$, then the date above is $x - 7$ because it must be 7 days prior. The date below is $x + 7$ because it must be 7 days after. The date to the left is $x - 1$ and the date to the right is $x + 1$. The sum of these four dates is $x - 7 + x + 7 + x - 1 + x + 1$ which equals $4x$. To find the average, I would divide this by 4, giving the original date chosen, $x$.

### Activity Synthesis

Solicit responses to the second question and demonstrate thinking about one product at a time:

| | $8y$ | $-x$ | $-12$ |
|---|---|---|---|
| $\frac{1}{2}$ | $4y$ | $-\frac{1}{2}x$ | $-6$ |

Then ask students to share how they approached the last question. Highlight responses where students noticed that $\frac{1}{2}(8y - x - 12)$ can be rewritten like $\frac{1}{2}(8y + -x + -12)$ (because of what they talked about in the warm-up). So the two questions have the same answer.

### Access for English Language Learners

*Speaking, Representing: MLR7 Compare and Connect.* Use this routine when students present their expressions. Ask students "What is the same and what is different?" about the approaches and representations involving subtraction with the distributive property. Help students connect how the expressions that have a subtraction operation are equivalent to expressions that add its additive inverse. These exchanges strengthen students' mathematical language use and reasoning with the distributive property and the subtraction operation.
*Design Principle(s): Maximize meta-awareness*

## Lesson Synthesis

Display two expressions like $x + 2 - 3x - 10$ and $x + 3x - 2 - 10$. Ask students to think about why these expressions are *not* equivalent and explain to a partner. Two explanations should be highlighted:

- Subtraction isn't commutative. $2 - 3x$ and $3x - 2$ are not equivalent; you can't just switch terms around a subtraction sign.
- Since $-3x$ is the same as $+\text{-}3x$, the negative sign needs to stay with the $3x$ when terms are rearranged.

Ask students how they could fix the second expression to make it equivalent to the first. Ensure that everyone agrees and understands why $x + \text{-}3x + 2 + \text{-}10$ and $x - 3x + 2 - 10$ are equivalent to the first expression.

# 18.4 Equivalent to $4 - x$

**Cool Down: 5 minutes**
**Addressing**

- 7.EE.A.1
- 7.NS.A.1.c

**Student Task Statement**

1. Select **all** the expressions that are equivalent to $4 - x$.

   a. $x - 4$

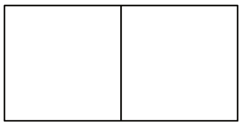
   b. $4 + \text{-}x$

   c. $\text{-}x + 4$

   d. $\text{-}4 + x$

e. $4 + x$

2. Use the distributive property to write an expression that is equivalent to $5(-2x - 3)$. If you get stuck, use the boxes to help organize your work.



**Student Response**

1. b, c
2. $-10x - 15$ or equivalent

## Student Lesson Summary

Working with subtraction and signed numbers can sometimes get tricky. We can apply what we know about the relationship between addition and subtraction—that subtracting a number gives the same result as adding its opposite—to our work with expressions. Then, we can make use of the properties of addition that allow us to add and group in any order. This can make calculations simpler. For example:

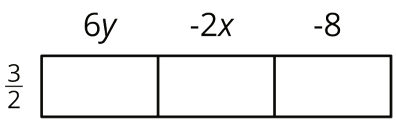
$$\frac{5}{8} - \frac{2}{3} - \frac{1}{8}$$

$$\frac{5}{8} + -\frac{2}{3} + -\frac{1}{8}$$

$$\frac{5}{8} + -\frac{1}{8} + -\frac{2}{3}$$

$$\frac{4}{8} + -\frac{2}{3}$$

We can also organize the work of multiplying signed numbers in expressions. The product $\frac{3}{2}(6y - 2x - 8)$ can be found by drawing a rectangle with the first factor, $\frac{3}{2}$, on one side, and the three terms inside the parentheses on the other side:



Multiply $\frac{3}{2}$ by each term across the top and perform the multiplications:

| | $6y$ | $-2x$ | $-8$ |
|---|---|---|---|
| $\frac{3}{2}$ | $\frac{3}{2} \cdot 6y$ | $\frac{3}{2} \cdot -2x$ | $\frac{3}{2} \cdot -8$ |

| | $6y$ | $-2x$ | $-8$ |
|---|---|---|---|
| $\frac{3}{2}$ | $9y$ | $-3x$ | $-12$ |

Reassemble the parts to get the expanded version of the original expression:

$$\frac{3}{2}(6y - 2x - 8) = 9y - 3x - 12$$

## Glossary

- term

# Lesson 18 Practice Problems

## Problem 1

### Statement

For each expression, write an equivalent expression that uses only addition.

a. $20 - 9 + 8 - 7$

b. $4x - 7y - 5z + 6$

c. $-3x - 8y - 4 - \frac{8}{7}z$

### Solution

a. $20 + -9 + 8 + -7$

b. $4x + -7y + -5z + 6$

c. $-3x + -8y + -4 + -\frac{8}{7}z$

## Problem 2

### Statement

Use the distributive property to write an expression that is equivalent to each expression. If you get stuck, consider drawing boxes to help organize your work.

a. $9(4x - 3y - \frac{2}{3})$

b. $-2(-6x + 3y - 1)$

c. $\frac{1}{5}(20y - 4x - 13)$

d. $8(-x - \frac{1}{2})$

e. $-8(-x - \frac{3}{4}y + \frac{7}{2})$

## Solution

a. $36x - 27y - 6$

b. $12x - 6y + 2$

c. $4y - \frac{4}{5}x - \frac{13}{5}$

d. $-8x - 4$

e. $8x + 6y - 28$

# Problem 3

## Statement

Kiran wrote the expression $x - 10$ for this number puzzle: "Pick a number, add -2, and multiply by 5."

Lin thinks Kiran made a mistake.

a. How can she convince Kiran he made a mistake?

b. What would be a correct expression for this number puzzle?

## Solution

a. Answers vary. Sample response: for $x = 1$ the number puzzle should result in $-5$. But Kiran's expression gives $1 - 10 = -9$.

b. $(x - 2) \cdot 5$ (or $5x - 10$)

# Problem 4

## Statement

The output from a coal power plant is shown in the table:

| energy in megawatts | number of days |
|---|---|
| 1,200 | 2.4 |
| 1,800 | 3.6 |
| 4,000 | 8 |
| 10,000 | 20 |

Similarly, the output from a solar power plant is shown in the table:

| energy in megawatts | number of days |
|---|---|
| 100 | 1 |
| 650 | 4 |
| 1,200 | 7 |
| 1,750 | 10 |

Based on the tables, is the energy output in proportion to the number of days for either plant? If so, write an equation showing the relationship. If not, explain your reasoning.

## Solution

The coal power plant could be a proportional relationship. Its equation would be $E = 500 \cdot d$ where $E$ is the energy output in megawatts and $d$ is the number of days. The solar power plant would not be a proportional relationship since the ratio between the number of days and the energy output is not constant.

(From Unit 2, Lesson 7.)