## Lesson 17: Changing the Vertex

* Let’s write new quadratic equations in vertex form to produce certain graphs.

### 17.1: Graphs of Two Functions



Here are graphs representing the functions $f$ and $g$, given by $f(x)=x(x+6)$ and $g(x)=x(x+6)+4$.

1. Which graph represents each function? Explain how you know.
2. Where does the graph of $f$ meet the $x$-axis? Explain how you know.

### 17.2: Shifting the Graph

1. How would you change the equation $y=x^{2}$ so that the vertex of the graph of the new equation is located at the following coordinates and the graph opens as described?
	1. $(0,11)$, opens up
	2. $(7,11)$, opens up
	3. $(7,-3)$, opens down
2. Use graphing technology to verify your predictions. Adjust your equations if necessary.
3. Kiran graphed the equation $y=x^{2}+1$ and noticed that the vertex is at $(0,1)$. He changed the equation to $y=(x−3)^{2}+1$ and saw that the graph shifted 3 units to the right and the vertex is now at $(3,1)$.
* Next, he graphed the equation $y=x^{2}+2x+1$, observed that the vertex is at $(-1,0)$. Kiran thought, “If I change the squared term $x^{2}$ to $(x−5)^{2}$, the graph will move 5 units to the right and the vertex will be at $(4,0)$.”
* Do you agree with Kiran? Explain or show your reasoning.

### 17.3: A Peanut Jumping over a Wall

Mai is learning to create computer animation by programming. In one part of her animation, she uses a quadratic function to model the path of the main character, an animated peanut, jumping over a wall.



Mai uses the equation $y=-0.1(x−h)^{2}+k$ to represent the path of the jump. $y$ represents the height of the peanut as a function of the horizontal distance it travels, $x$.

On the screen, the base of the wall is located at $(22,0)$, with the top of the wall at $(22,4.5)$. The dashed curve in the picture shows the graph of 1 equation Mai tried, where the peanut fails to make it over the wall.



1. What are the values of $h$ and $k$ in this equation?
2. Starting with Mai’s equation, choose values for $h$ and $k$ that will guarantee the peanut stays on the screen but also makes it over the wall. Be prepared to explain your reasoning.

### 17.4: Smiley Face

Do you see 2 “eyes” and a smiling “mouth” on the graph? The 3 arcs on the graph all represent quadratic functions that were initially defined by $y=x^{2}$, but whose equations were later modified.

1. Write equations to represent each curve in the smiley face.
2. What domain is used for each function to create this graph?



### Lesson 17 Summary

The graphs of $y=x^{2}$, $y=x^{2}+12$ and $y=(x+3)^{2}$ all have the same shape but their locations are different. The graph that represents $y=x^{2}$ has its vertex at $(0,0)$.



Notice that adding 12 to $x^{2}$ raises the graph by 12 units, so the vertex of that graph is at $(0,12)$. Replacing $x^{2}$ with $(x+3)^{2}$ shifts the graph 3 units to the left, so the vertex is now at $(-3,0)$.

We can also shift a graph both horizontally and vertically.

The graph that represents $y=(x+3)^{2}+12$ will look like that for $y=x^{2}$ but it will be shifted 12 units up and 3 units to the left. Its vertex is at $(-3,12)$.



The graph representing the equation $y=-(x+3)^{2}+12$ has the same vertex at $(-3,12)$, but because the squared term $(x+3)^{2}$ is multiplied by a negative number, the graph is flipped over horizontally, so that it opens downward.





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