Lesson 15: Efficiently Solving Inequalities

Goals

- Compare and contrast (orally) solutions to equations and solutions to inequalities.
- Draw and label a graph on the number line that represents all the solutions to an inequality.
- Generalize (orally) that you can solve an inequality of the form $px + q > r$ or $px + q < r$ by solving the equation $px + q = r$ and then testing a value to determine the direction of the inequality in the solution.

Learning Targets

- I can graph the solutions to an inequality on a number line.
- I can solve inequalities by solving a related equation and then checking which values are solutions to the original inequality.

Lesson Narrative

In this lesson, students see more examples of inequalities. This time, many inequalities involve negative coefficients. This reinforces the point that solving an inequality is not as simple as solving the corresponding equation. After students find the boundary point, they must do some extra work to figure out the direction of inequality. This might involve reasoning about the context, substituting in values on either side of the boundary point, and reasoning about number lines. All of these techniques exemplify MP1: making the problem more concrete and visual and asking, “Does this make sense?”

It is important to understand that the goal is not to have students learn and practice an algorithm for solving inequalities like “whenever you multiply or divide by a negative, flip the inequality.” Rather, we want students to understand that solving a related equation tells you the lower or upper bound of an inequality. To know whether values greater than or less than the boundary number make the inequality true, it's best to test some values that are above and below the boundary number. This way of reasoning about inequalities will serve students well long into their future studies, whereas students are very likely to forget a procedure memorized for a special case.

Alignments

Addressing

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Building Towards

- 7.EE.B.4.b: Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. $

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

> **Student Learning Goals**
>
> Let's solve more complicated inequalities.

15.1 Lots of Negatives

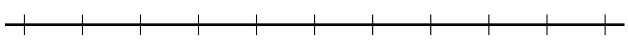
Warm Up: 5 minutes
This warm-up primes students for inequalities that include variables with negative coefficients without context. Students first predict a solution set, and then are given some values to test so that the solution emerges. Do *not* formalize a procedure for "flipping the inequality" when multiplying by a negative. Look for students who predict solution sets that are incorrect because of the sign.

Building Towards

- 7.EE.B.4.b

Launch

Give students 3 minutes of quiet work time followed by a whole-class discussion.

> **Student Task Statement**
>
> Here is an inequality: $-x \geq -4$.
>
> 1. Predict what you think the solutions on the number line will look like.
>
> 2. Select **all** the values that are solutions to $-x \geq -4$:
>   a. 3
>   b. -3
>   c. 4
>   d. -4
>   e. 4.001
>   f. -4.001
>
> 3. Graph the solutions to the inequality on the number line:
>
> 

Student Response

1. Answers vary.
2. a, b, c, d, f
3. A filled-in circle at 4 and all points to its left are graphed. The same graph that one would draw for $x \leq 4$.

Activity Synthesis

The purpose of the discussion is to highlight how negatives in the inequality sometimes make it hard to predict what the solutions will be. (It is important to reason carefully by first determining the value for which both sides are equal and then testing points to determine which on side of that value the solutions lie.) Select students to share how their predictions differed from their final solutions. Consider asking how the solutions to $-x \geq -4$ are different from the solutions to $x \geq 4$. (The solutions go in the opposite direction on the number line.)

15.2 Inequalities with Tables

15 minutes

The purpose of this activity is for students to get a visual feel for the relationship between an unsimplified inequality ($x - 3 > -2$) and its solution ($x > 1$). The tables suggest a way for students to see why the values of x that satisfy $x - 3 > -2$ should also satisfy $x > 1$. Graphing solutions on the number line reinforces this connection. The second and third questions, taken together, demonstrate how a negative coefficient can make the solutions to an inequality go “the other way.” The work in this activity suggests a procedure for solving inequalities: solve the corresponding equation, then test a number on either side. But the purpose of this activity is not to teach students a procedure, but rather to provide underlying knowledge and experience.

Building Towards

- 7.EE.B.4.b

Launch

A potentially challenging aspect of this task is that students must consider the two rows of a table at different times and relate the values in the table to solutions of an inequality. Consider displaying a table like this for all to see, and then asking some questions about it:

x	-4	-3	-2	-1	0	1	2	3	4
$x + 2$	-2	-1	0	1	2	3	4	5	6

After students have had a chance to look at the table, ask them some familiarizing questions:

- How are the numbers in the top row and bottom row related?

- Think about the equation $x + 2 = -2$. What value of x makes this true? Where do you see that in the table?
- Think about the inequality $x + 2 > 3$. What values of x make this true? Where do you see that in the table?

Give 5–10 minutes of quiet work time to complete the tables and questions followed by a whole-class discussion. Depending on the needs of your class, you might instruct students to pause after each question for discussion before continuing with the next question.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity. Consider pausing after the first question for a brief class discussion before moving on.
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Some students may answer $x > 2$ for the first question, since that is the place where the value of $x - 3$ first surpasses the number -2. Remind these students that there are values between 1 and 2. Ask them whether 1.1 is a solution, for example.

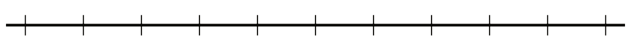
Some students may graph only whole-number solutions. Ask these students to think about whether values in between whole numbers are also solutions.

Student Task Statement

1. Let's investigate the inequality $x - 3 > -2$.

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7		-5				-1		1

   a. Complete the table.
   b. For which values of x is it true that $x - 3 = -2$?
   c. For which values of x is it true that $x - 3 > -2$?
   d. Graph the solutions to $x - 3 > -2$ on the number line:



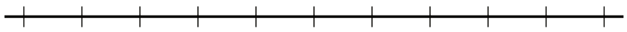
2. Here is an inequality: $2x < 6$.

a. Predict which values of x will make the inequality $2x < 6$ true.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$2x$									

c. Graph the solutions to $2x < 6$ on the number line:



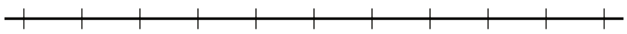
3. Here is an inequality: $-2x < 6$.

a. Predict which values of x will make the inequality $-2x < 6$ true.

b. Complete the table. Does it match your prediction?

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$									

c. Graph the solutions to $-2x < 6$ on the number line:



d. How are the solutions to $2x < 6$ different from the solutions to $-2x < 6$?

Student Response

1. For $x - 3 > -2$
   a. Table:

x	-4	-3	-2	-1	0	1	2	3	4
$x - 3$	-7	-6	-5	-4	-3	-2	-1	0	1

   b. $x = 1$

   c. $x > 1$

   d. The graph should have an open circle at $x = 1$ with all values greater than 1 shaded.

2. For $2x < 6$
   a. Answers vary. Any value less than 3 will work.

b. Table:

x	-4	-3	-2	-1	0	1	2	3	4
$2x$	-8	-6	-4	-2	0	2	4	6	8

c. The graph should have an open circle at $x = 3$ with all values less than 3 shaded.

3. For $-2x < 6$
   a. Answers vary. Sample response: Based on the solution to $2x < 6$, I predict that for $-2x < 6$, the solutions will be values less than -3.

   b. The table may or may not match the prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$-2x$	8	6	4	2	0	-2	-4	-6	-8

   c. The graph should have an open circle at $x = -3$, with all values greater than -3 shaded.

   d. The solution to $2x < 6$ is all values less than 3, but the solution to $-2x < 6$ is all values greater than -3.

Activity Synthesis

The main take-away is that solving the associated equation to an inequality gives the value that is the boundary between solutions and non-solutions. In this activity, students have a table to check on which side of the boundary are solutions and which side are not solutions. In order to transition to the next activity, ask students whether they need to complete an *entire* table to test on which side of the boundary the solutions are. The goal is to get students to understand that they only need to test one number. If that number is a solution, then all points on the same side of the boundary are solutions. If the point is not a solution, then the solutions are all the points on the other side of the boundary. The next activity will give students an opportunity to apply this insight and start to articulate such a procedure.

Resist the temptation to summarize the last two problems into a procedure like “whenever you multiply or divide by a negative, flip the inequality.”

15.3 Which Side are the Solutions?

15 minutes

In the previous activity, students saw that solving the equation associated with an inequality gives a boundary point that separates values that make the inequality true from values that make the inequality false. This activity builds on that understanding to solidify a process for solving inequalities: first solve the associated equation to find the boundary point, then test a value to determine on which side of that boundary the solutions lie. The first two questions offer more scaffolding, and the last two questions simply give an inequality and ask students to solve and graph.

Building Towards

- 7.EE.B.4.b

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

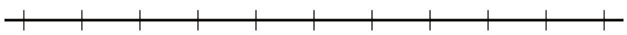
Arrange students in groups of 2. Give 5–10 minutes of quiet work time, time to share their responses and reasoning with a partner, and follow with a whole-class discussion.

Access for Students with Disabilities

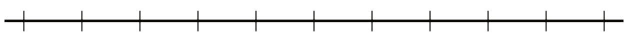
Representation: Internalize Comprehension. Activate or supply background knowledge. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

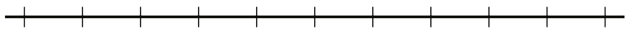
1. Let's investigate $-4x + 5 \geq 25$.
    a. Solve $-4x + 5 = 25$.
    b. Is $-4x + 5 \geq 25$ true when x is 0? What about when x is 7? What about when x is -7?
    c. Graph the solutions to $-4x + 5 \geq 25$ on the number line.



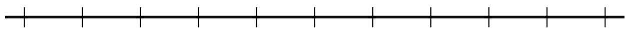
2. Let's investigate $\frac{4}{3}x + 3 < \frac{23}{3}$.
    a. Solve $\frac{4}{3}x + 3 = \frac{23}{3}$.
    b. Is $\frac{4}{3}x + 3 < \frac{23}{3}$ true when x is 0?
    c. Graph the solutions to $\frac{4}{3}x + 3 < \frac{23}{3}$ on the number line.



3. Solve the inequality $3(x + 4) > 17.4$ and graph the solutions on the number line.



4. Solve the inequality $-3\left(x - \frac{4}{3}\right) \leq 6$ and graph the solutions on the number line.



Student Response

1. For $-4x \geq 25$:
   a. -5

   b. No, no, yes

   c. A closed circle at -5 and all values to the left shaded.

2. For $\frac{4}{3}x + 3 < \frac{23}{3}$:
   a. $\frac{7}{2}$ (or equivalent)

   b. Yes

   c. An open circle at $\frac{7}{2}$ and all values to the left shaded.

3. The solution is $x > 1.8$. An open circle at 1.8 and all values to the right shaded.

4. The solution is $x \geq -\frac{2}{3}$. A closed circle at $-\frac{2}{3}$ and all values to the right shaded.

Are You Ready for More?

Write at least three different inequalities whose solution is $x > -10$. Find one with x on the left side that uses a $<$.

Student Response

Answers vary. Possible responses: $2x > -20$, $x + 50 > 40$. Responses that involve $x <$: $-5x < 50$, $\frac{x}{-6} < 60$.

Activity Synthesis

For each question, ask one student to demonstrate and explain their process for solving the inequality. For each, highlight the moment when they find the boundary value (the solution to the related equation) and then when they test one or more numbers on either side to decide which side has values that make the inequality true.

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. During the whole-class discussion, ask students to discuss with their partner, what is the same and what is different between the demonstrated processes for solving each inequality. To call students' attention to the different ways the boundary values were determined, ask students, "Which numbers can be used to check the direction of the solution?" This will help students produce language, including symbols, common to inequalities as they reason about procedures for solving inequalities. *Design Principle(s): Maximize meta-awareness; Cultivate conversation*

Lesson Synthesis

By the time students have finished this lesson, they should have a variety of methods for solving inequalities, all of which involve some kind of final decision about the direction of inequality. Students will have had some summative practice with this in the final activity and the cool-down.

Ask students to consider, "What if someone asked for your help with how to solve inequalities? What would you tell them? How would you describe to someone how to solve any inequality?" Ask them to either write this down or share their thoughts with their partner. Consider creating a persistent display showing the procedure using language the class develops, along with an example.

15.4 Testing for Solutions

Cool Down: 5 minutes

The purpose of this cool-down is to check whether students can determine the direction of inequality. The questions involve using algebra to find boundary points, then testing values of x. Since the boundary points are given, some students may skip directly to testing points.

Addressing

- 7.EE.B.4

Student Task Statement

For each inequality, decide whether the solution is represented by $x < 2.5$ or $x > 2.5$.

1. $-4x + 5 > -5$
2. $-25 > -5(x + 2.5)$

Student Response

1. $x < 2.5$
2. $x > 2.5$

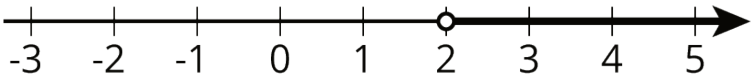
Student Lesson Summary

Here is an inequality: $3(10 - 2x) < 18$. The solution to this inequality is all the values you could use in place of x to make the inequality true.

In order to solve this, we can first solve the related equation $3(10 - 2x) = 18$ to get the solution $x = 2$. That means 2 is the boundary between values of x that make the inequality true and values that make the inequality false.

To solve the inequality, we can check numbers greater than 2 and less than 2 and see which ones make the inequality true.

Let’s check a number that is greater than 2: $x = 5$. Replacing x with 5 in the inequality, we get $3(10 - 2 \cdot 5) < 18$ or just $0 < 18$. This is true, so $x = 5$ is a solution. This means that all values greater than 2 make the inequality true. We can write the solutions as $x > 2$ and also represent the solutions on a number line:



Notice that 2 itself is not a solution because it's the value of x that makes $3(10 - 2x)$ *equal* to 18, and so it does not make $3(10 - 2x) < 18$ true.

For confirmation that we found the correct solution, we can also test a value that is less than 2. If we test $x = 0$, we get $3(10 - 2 \cdot 0) < 18$ or just $30 < 18$. This is false, so $x = 0$ and all values of x that are less than 2 are not solutions.

Lesson 15 Practice Problems

Problem 1

Statement

a. Consider the inequality $-1 \leq \frac{x}{2}$.

   i. Predict which values of x will make the inequality true.

   ii. Complete the table to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$									

b. Consider the inequality $1 \leq \frac{-x}{2}$.

   i. Predict which values of x will make it true.

   ii. Complete the table to check your prediction.

x	-4	-3	-2	-1	0	1	2	3	4
$-\frac{x}{2}$									

Solution

a. i. $x \geq -2$

ii.

x	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$	-2	-1.5 (or $\frac{-3}{2}$)	-1	-0.5 (or $\frac{-1}{2}$)	0	0.5 (or $\frac{1}{2}$)	1	1.5 (or $\frac{3}{2}$)	2

b. i. $x \leq -2$

ii.

x	-4	-3	-2	-1	0	1	2	3	4
$\frac{x}{2}$	2	1.5 (or $\frac{3}{2}$)	1	0.5 (or $\frac{1}{2}$)	0	-0.5 (or $\frac{-1}{2}$)	-1	-1.5 (or $\frac{-3}{2}$)	-2

Problem 2

Statement

Diego is solving the inequality $100 - 3x \geq -50$. He solves the equation $100 - 3x = -50$ and gets $x = 50$. What is the solution to the inequality?

A. $x < 50$

B. $x \leq 50$

C. $x > 50$

D. $x \geq 50$

Solution

B

Problem 3

Statement

Solve the inequality $-5(x - 1) > -40$, and graph the solution on a number line.

Solution

a. $x < 9$

b. A number line with an open circle at 9 and the arrow going to the left

Problem 4

Statement

Select **all** values of x that make the inequality $-x + 6 \geq 10$ true.

A. -3.9

B. 4

C. -4.01

D. -4

E. 4.01

F. 3.9

G. 0

H. -7

Solution

["C", "D", "H"]

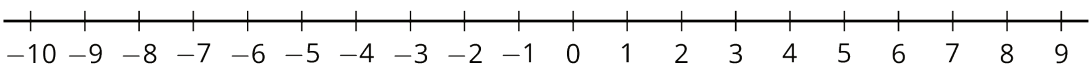
(From Unit 6, Lesson 13.)

Problem 5

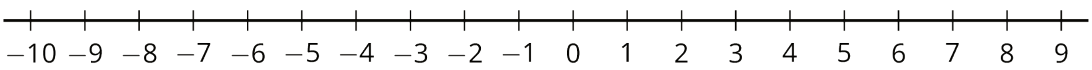
Statement

Draw the solution set for each of the following inequalities.

a. $x > 7$

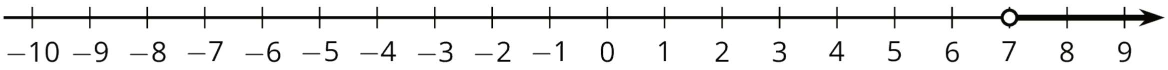


b. $x \geq -4.2$

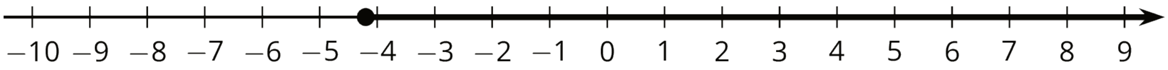


Solution

a.



b.



(From Unit 6, Lesson 13.)

Problem 6

Statement

The price of a pair of earrings is \$22 but Priya buys them on sale for \$13.20.

a. By how much was the price discounted?

b. What was the percentage of the discount?

Solution

a. \$8.80

b. 40%

(From Unit 4, Lesson 12.)