Lesson 3: Recipes

Goals

- Draw and label a discrete diagram with circled groups to represent multiple batches of a recipe.
- Explain equivalent ratios (orally and in writing) in terms of different sized batches of the same recipe having the same taste.
- Understand that doubling or tripling a recipe involves multiplying the amount of each ingredient by the same number, yielding something that tastes the same.

Learning Targets

- I can explain the meaning of equivalent ratios using a recipe as an example.
- I can use a diagram to represent a recipe, a double batch, and a triple batch of a recipe.
- I know what it means to double or triple a recipe.

Lesson Narrative

This is the first of two lessons that develop the idea of **equivalent ratios** through physical experiences. A key understanding is that if we scale a recipe up (or down) to make multiple batches (or a fraction of a batch), the result will still be “the same” in some meaningful way. Students see this idea in two contexts, taste and color:

- In this lesson, a mixture containing two batches of a recipe *tastes the same* as a mixture containing one batch. For example, 2 cups of water mixed thoroughly with 8 teaspoons of powdered drink mix tastes the same as 1 cup of water mixed with 4 teaspoons of powdered drink mix.
- In the next lesson, a mixture containing two batches of a recipe for colored water will produce the *same shade* of the color as a mixture containing one batch. For example, 10 ml of blue mixed with 30 ml of yellow produces the same shade of green as 5 ml of blue mixed with 15 ml of yellow.

The fact that two equivalent ratios yield the same taste or produce the same color is a physical manifestation of the equivalence of the ratios.

Students see that scaling a recipe up (or down) requires multiplying the amount of each ingredient by the same factor, e.g., doubling a recipe means doubling the amount of each ingredient (MP7). They also gain more experience using a discrete diagram as a tool to represent a situation.

Alignments

Addressing

- 6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was $2 : 1$, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Drink mix
A powder that is mixed with water to create a fruit-flavored or chocolate-flavored drink. Using a sugar-free drink mix is recommended, but *not* a mix that calls for adding a separate sweetener when mixing up the drink.

Empty containers
Markers
Paper cups
Teaspoon
Water

Required Preparation

Create two separate drink mixtures. Container A has one cup of water and one teaspoon of powdered drink mix. Container B has one cup of water and four teaspoons of powdered drink mix. You might have to stir the mixtures vigorously for a minute or more to ensure all the powder dissolves.

Get 6 small paper cups. Do not mark the cups. Put a small amount of mixture A in three of the cups and a small amount of mixture B in the other three cups. (Keep track of which is which, as you will give each of three volunteers one of each cup.) Discard the rest of the mixtures for now. (You will do a dramatic performance creating each mixture during class.)

During class, you will need three empty mixing containers with at least a 2-cup capacity each. One marked A, one marked B, and one marked C. You will also need a supply of water, a supply of drink mix, a measuring cup, and a teaspoon.

> **Student Learning Goals**
> Let’s explore how ratios affect the way a recipe tastes.

3.1 Flower Pattern

Warm Up: 5 minutes
The purpose of this warm-up is to quickly remind students of different ways to write ratios. They also have an opportunity to multiply the number of each type of shape by 2 to make two copies of the flower, which previews the process introduced in this lesson for making a double batch of a recipe.

Addressing

- 6.RP.A.1

Instructional Routines

- Think Pair Share

Launch

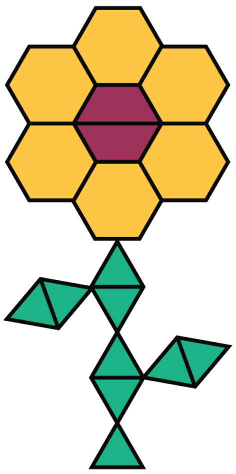
Arrange students in groups of 2. Ensure students understand there are 6 hexagons, 2 trapezoids, and 9 triangles in the picture, and that their job is to write ratios about the numbers of shapes. Give 2 minutes of quiet work time and then invite students to share their sentences with their partner, followed by whole-class discussion.

Anticipated Misconceptions

Students might get off track by attending to the area each shape covers. Clarify that this task is only concerned with the number of each shape and not the area covered.

Student Task Statement

This flower is made up of yellow hexagons, red trapezoids, and green triangles.



1. Write sentences to describe the ratios of the shapes that make up this pattern.
2. How many of each shape would be in two copies of this flower pattern?

Student Response

1. Answers vary. Sample responses:
    - For every 2 hexagons there are 3 triangles.
    - There are 3 hexagons for every trapezoid.
    - The ratio of trapezoids to triangles is 2 to 9.
    - The ratio of hexagons to trapezoids to triangles is $6 : 2 : 9$.
2. There would be 12 yellow hexagons, 4 red trapezoids and 18 green triangles.

Activity Synthesis

Invite a student to share a sentence that describe the ratios of shapes in the picture. Ask if any students described the same relationship a different way. For example, three ways to describe the same ratio are: The ratio of hexagons to trapezoids is $6:2$. The ratio of trapezoids to hexagons is 2 to 6. There are 3 hexagons for every trapezoid.

Ask a student to describe why two copies of the picture would have 12 hexagons, 4 trapezoids, and 18 triangles. If no student brings it up, be sure to point out that each number in one copy of the picture can be multiplied by 2 to find the number of each shape in two copies.

3.2 Powdered Drink Mix

15 minutes

In this activity, three student volunteers participate in a taste test of two drink mixtures. Mixture A is made with 1 cup of water and 1 teaspoon of drink mix. Mixture B is made with 1 cup of water and 4 teaspoons of drink mix. The taste testers match diagrams with each mixture and explain their reasoning.

After the taste test, in front of students, recreate mixture A (1 cup water with 1 teaspoon of drink mix) and mixture B (1 cup water with 4 teaspoons of drink mix). Ask students to describe how the diagrams correspond with these mixtures. Then, conduct a demonstration in which 3 teaspoons of drink mix are added to Mixture A and a new diagram is drawn. Once Mixture A and Mixture B both contain one cup of water and 4 teaspoons of drink mix, both mixtures are combined in a third container labeled Mixture C.

As part of their work on the task, students reason that this combined mixture tastes the same as each individual batch. Students then conclude that a mixture containing two batches tastes the same as the mixture that contains just one batch, because mixing two things together that taste the same will produce a mixture that tastes the same. They should also note that each ingredient was doubled in the mixture.

Addressing

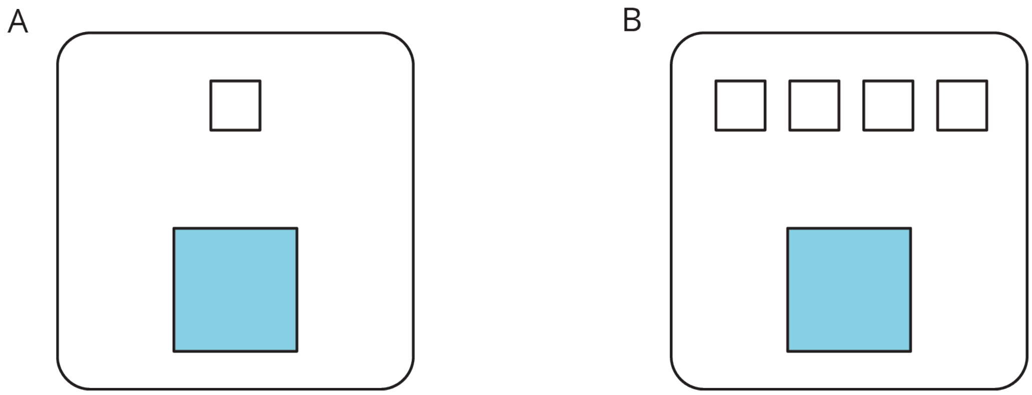
- 6.RP.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Display the diagram for all to see:



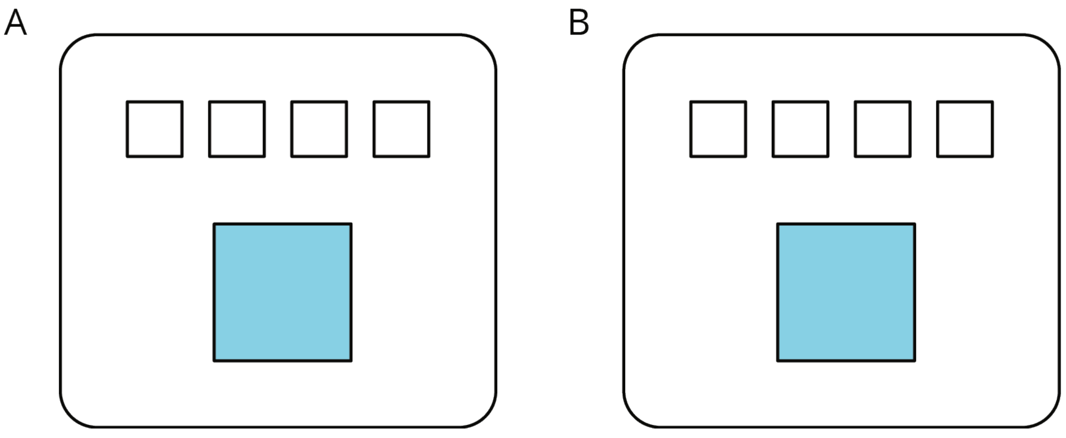
Taste test: Recruit three volunteers for a taste test. Give each volunteer two unmarked cups—one each of a small amount of Mixture A and Mixture B. Explain that their job is to take a tiny sip of each sample, match the diagrams to the samples, and explain their matches.

Demonstration: Conduct a dramatic demonstration of mixing powdered drink mix and water. Start with two empty containers labeled A and B. To Container A, add 1 cup of water and 1 teaspoon of drink mix. To Container B, add 1 cup of water and 4 teaspoons of drink mix. Mix them both thoroughly. The first diagram should still be displayed.

Discuss:

- Which mixture has a stronger flavor? (B has more drink mix in the same quantity of water).
- How can we make Mixture A taste like Mixture B? (Put 3 more teaspoons of drink mix into Container A.)

Add 3 more teaspoons of drink mix to Container A. Display a new diagram to represent the situation:



Discuss:

- Describe the ratio of ingredients that is now in Container A. (The ratio of cups of water to teaspoons of drink mix is $1:4$.)

- Describe the ratio of ingredients that is in Container B. (The ratio of cups of water to teaspoons of drink mix is also 1 to 4.)
- How do you think they compare in taste? (They taste the same. If desired, you can have volunteers verify that they taste the same, but this might not be necessary.)

Pour the contents of both A and B into a larger container labeled C and mix them thoroughly.

Discuss:

- How would the taste of Mixture C compare to the taste of Mixture A and Mixture B? (The new mixture would taste the same as each component mixture.)

Following this demonstration, students individually interpret the drink mixture diagrams. The work in the task will reiterate what happened in the demonstration.

Access for English Language Learners

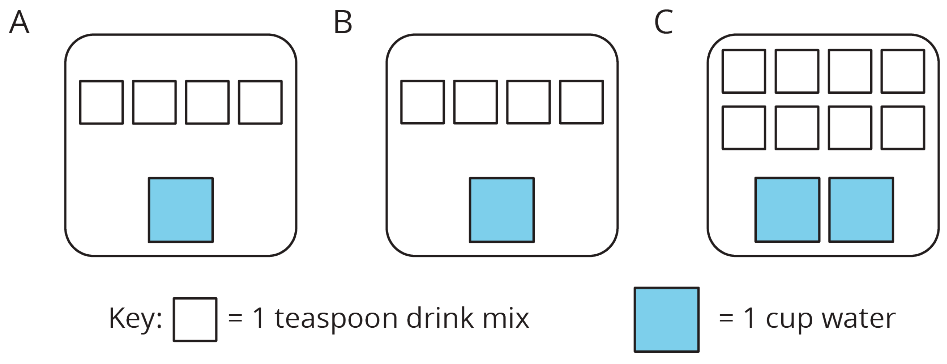
Conversing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Ask "What if I add a half cup of water to C?" or "What if I add a teaspoon of drink mix to B?" To help students justify their reasoning, display a sentence frame such as: Mixtures B and C will taste _____ because . . . ."
Design Principle(s): Support sense-making

Anticipated Misconceptions

Students may not initially realize that Mixtures C and B taste the same. You could ask them to imagine ordering a smoothie from a takeout window. Would a small size smoothie taste the same as a size that is double that amount? If you double the amount of each ingredient, the mixture tastes the same.

Student Task Statement

Here are diagrams representing three mixtures of powdered drink mix and water:



1. How would the taste of Mixture A compare to the taste of Mixture B?

2. Use the diagrams to complete each statement:

   a. Mixture B uses ______ cups of water and ______ teaspoons of drink mix. The ratio of cups of water to teaspoons of drink mix in Mixture B is ________.

   b. Mixture C uses ______ cups of water and ______ teaspoons of drink mix. The ratio of cups of water to teaspoons of drink mix in Mixture C is ________.

3. How would the taste of Mixture B compare to the taste of Mixture C?

Student Response

1. Mixtures A and B will taste the same because the have the same amount of water and drink mix.
2. Mixture B uses 1 cup of water and 4 teaspoons of drink mix. The ratio of cups of water to teaspoons of drink mix in Mixture B is $1 : 4$.
3. Mixture C uses 2 cups of water and 8 teaspoons of drink mix. The ratio of cups of water to teaspoons of drink mix in mixture C is $2 : 8$.
4. Mixtures B and C will taste the same. This is because Mixture C was made by doubling Mixture B or by mixing A and B together, which taste the same, so the mixture would still taste the same.

Are You Ready for More?

Sports drinks use sodium (better known as salt) to help people replenish electrolytes. Here are the nutrition labels of two sports drinks.

A

Nutrition Facts

Serving Size 8 fl oz (240 mL)
Serving Per Container 4

Amount Per Serving
Calories 50

		% Daily Value*
Total Fat	0 g	0%
Sodium	110 mg	5%
Potassium	30 mg	1%
Total Carbohydrate	14 g	5%
Sugars	14 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

B

Nutrition Facts

Serving Size 12 fl oz (355 mL)
Serving Per Container about 2.5

Amount Per Serving
Calories 80

		% Daily Value*
Total Fat	0 g	0%
Sodium	150 mg	6%
Potassium	35 mg	1%
Total Carbohydrate	21 g	7%
Sugars	20 g	
Protein	0 g	

% Daily Value are based on a 2,000 calorie diet.

1. Which of these drinks is saltier? Explain how you know.
2. If you wanted to make sure a sports drink was less salty than both of the ones given, what ratio of sodium to water would you use?

Student Response

1. Drink A. Sample reasoning: Drink A has 110 mg of sodium in an 8 ounce serving. Drink B has 150 mg of sodium in a 12 ounce serving. If we had 24 ounces of each drink, drink A would have 330 mg of sodium and drink B would have 300 mg of sodium. Therefore, drink A is saltier.
2. To be less salty than both drinks, the new drink would have to be less salty than drink B. So, for a 12-ounce serving, you would have to use less than 150 mg of sodium. For example, the ratio of ounces of drink to milligrams of sodium could be 12 to 100.

Activity Synthesis

Mixing 1 cup of water with 4 teaspoons of powdered drink mix makes a mixture that tastes exactly the same as mixing 2 cups of water with 8 teaspoons of powdered drink mix. We say that $1:4$ and $2:8$ are **equivalent ratios**. Ask students to discuss what they think "equivalent" means. Some ways they might respond are:

- Mixtures that taste the same use equivalent ratios.
- A double batch of a recipe—doubling each ingredient—is an equivalent ratio to a single batch.

3.3 Batches of Cookies

15 minutes
Students continue to use diagrams to represent the ratio of ingredients in a recipe as well as mixtures that contain multiple batches. They come to understand that a change in the number of batches changes the quantities of the ingredients, but the end product tastes the same. They then use this observation to come up with a working definition for equivalent ratio.

Addressing

- 6.RP.A.1

Launch

Launch the task with a scenario and a question: "Let's say you are planning to make cookies using your favorite recipe, and you're going to 'double the recipe.' What does it mean to double a recipe?"

There are a few things you want to draw out in this conversation:

- If we double a recipe, we need to double the amount of *every ingredient*. If the recipe calls for 3 eggs, doubling it means using 6 eggs. If the recipe calls for $\frac{1}{3}$ teaspoon of baking soda, we use $\frac{2}{3}$ teaspoon of baking soda, etc.
- We expect to end up with twice as many cookies when we double the recipe as we would when making a single batch.
- However, we expect the cookies from 2 batches of a recipe to taste *exactly the same* as those from a single batch.

Tell students they will now think about making different numbers of batches of a cookie recipe.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "I noticed _____, so I . . .", "In my diagram, _____ represents . . .", and “To find an equivalent ratio, first I _____ because . . . .” This will help students produce mathematical language as they make sense of equivalent ratios using diagrams.
Supports accessibility for: Language; Social-emotional skills

Anticipated Misconceptions

For the fourth question, students may not multiply both the amount of flour and the amount of vanilla by the same number. If this happens, refer students to the previous questions in noting that the amount of each ingredient was changed in the same way.

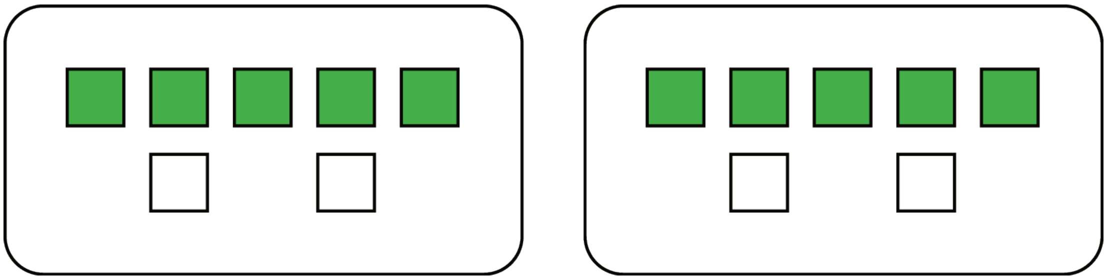
Student Task Statement

A recipe for one batch of cookies calls for 5 cups of flour and 2 teaspoons of vanilla.

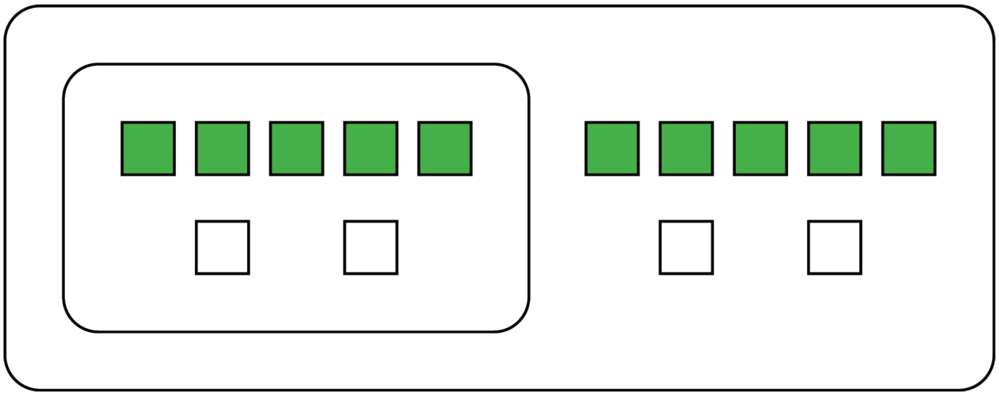
1. Draw a diagram that shows the amount of flour and vanilla needed for *two* batches of cookies.
2. How many batches can you make with 15 cups of flour and 6 teaspoons of vanilla? Show the additional batches by adding more ingredients to your diagram.
3. How much flour and vanilla would you need for 5 batches of cookies?
4. Whether the ratio of cups of flour to teaspoons of vanilla is $5:2$, $10:4$, or $15:6$, the recipes would make cookies that taste the same. We call these **equivalent ratios**.
    a. Find another ratio of cups of flour to teaspoons of vanilla that is equivalent to these ratios.
    b. How many batches can you make using this new ratio of ingredients?

Student Response

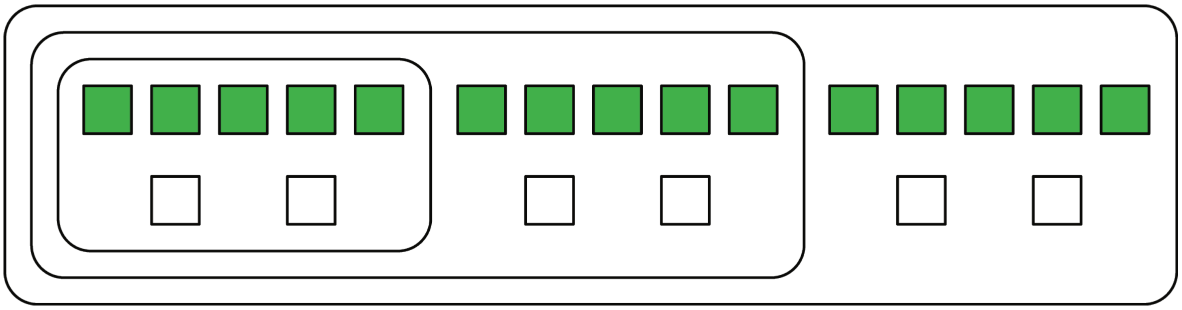
1. Diagrams may look different, but should clearly show two groups of 5 and 2. Here are two ways that students might circle the batches.
   If they see each batch individually, they might draw something like this:



If they think of a “double batch” as a single thing, they might circle it like this:



2. You can make 3 batches. Sample diagram:



3. You would need 25 cups of flour and 10 teaspoons of vanilla. (The diagram may be expanded to reflect this.)
4. Answers vary. Sample responses:
    a. $20 : 8$
    b. 4 batches

Activity Synthesis

Invite a few students to share their responses and diagrams with the class. A key point to emphasize during discussion is that when we double (or triple) a recipe, we also have to double (or triple) each ingredient. Record a working (but not final) definition for equivalent ratio that can be displayed for at least the next several lessons. Here is an example: “Cups of flour and teaspoons of vanilla in the ratio $5 : 2$, $10 : 4$, or $15 : 6$ are equivalent ratios because they describe different numbers of batches of the same recipe.” Include a diagram in this display.

Lesson Synthesis

The four main ideas you want to draw out to conclude the lesson are:

- To double a recipe, you need to double the amount of each ingredient.
- To scale a recipe generally, you need to multiply each ingredient by the same number.
- Scaling a recipe results in a substance that tastes the same as the original recipe.
- We say that a ratio that represents a recipe is *equivalent to* a ratio that represents multiple batches of the same recipe.

Discuss:

- When doubling a recipe, how does the amount of each individual ingredient change? (Each ingredient is doubled. We call the new ratio of ingredients an **equivalent ratio**.)
- When tripling a recipe, how does the amount of each individual ingredient change? (Each ingredient is tripled. We call the new ratio of ingredients an equivalent ratio.)
- How do different numbers of batches of the same recipe taste? (They taste exactly the same.)

3.4 A Smaller Batch of Bird Food

Cool Down: 5 minutes

Addressing

- 6.RP.A.1

Launch

If necessary, explain that some people like to observe birds. These people might put bird food in a bird feeder outside their homes to attract birds, so they can watch them through a window.

> ### Student Task Statement
>
> Usually when Elena makes bird food, she mixes 9 cups of seeds with 6 tablespoons of maple syrup. However, today she is short on ingredients. Think of a recipe that would yield a smaller batch of bird food but still taste the same. Explain or show your reasoning.

Student Response

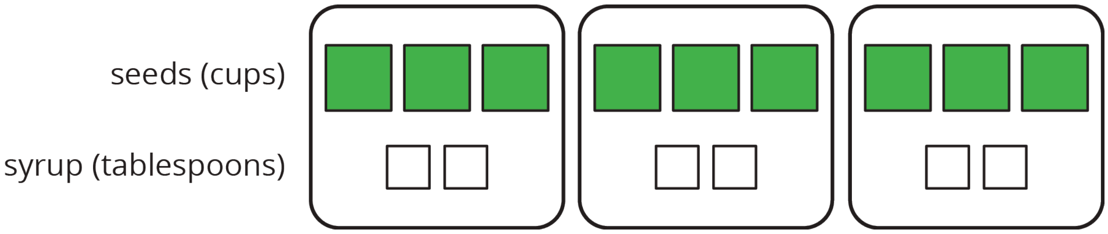
Two likely valid answers are:

- 3 cups of seeds and 2 tablespoons of syrup
- 6 cups of seeds and 4 tablespoons of syrup

Explanations and diagrams vary. Here are some possibilities:

$3 : 2$ represents the cups of seeds to the tablespoons of syrup.

$3 : 2$ is equivalent to $9 : 6$.

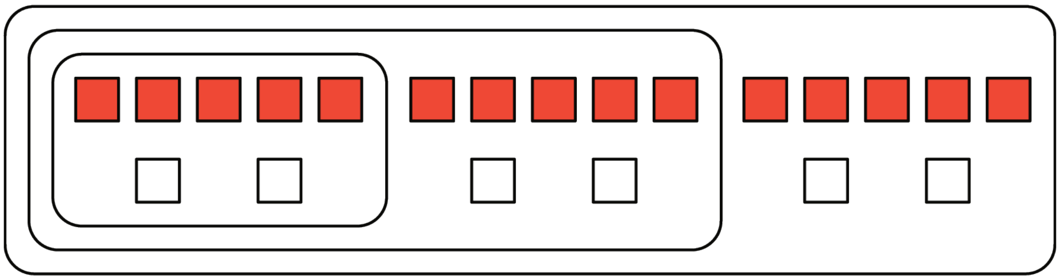


Student Lesson Summary

A recipe for fizzy juice says, “Mix 5 cups of cranberry juice with 2 cups of soda water.”

To double this recipe, we would use 10 cups of cranberry juice with 4 cups of soda water. To triple this recipe, we would use 15 cups of cranberry juice with 6 cups of soda water.

This diagram shows a single batch of the recipe, a double batch, and a triple batch:



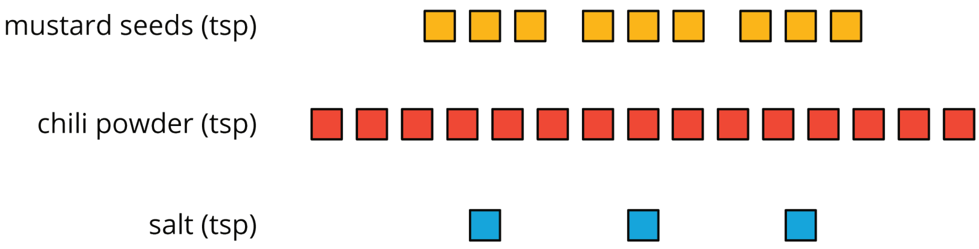
We say that the ratios $5 : 2$, $10 : 4$, and $15 : 6$ are **equivalent**. Even though the amounts of each ingredient within a single, double, or triple batch are not the same, they would make fizzy juice that tastes the same.

Lesson 3 Practice Problems

Problem 1

Statement

A recipe for 1 batch of spice mix says, “Combine 3 teaspoons of mustard seeds, 5 teaspoons of chili powder, and 1 teaspoon of salt.” How many batches are represented by the diagram? Explain or show your reasoning.



Solution

The diagram represents 3 batches of spice mix. It shows 3 times the amount of each ingredient in the recipe: 9 teaspoons of mustard ($3 \cdot 3$), 15 teaspoons of chili powder ($3 \cdot 5$), and 3 teaspoons of salt ($3 \cdot 1$).

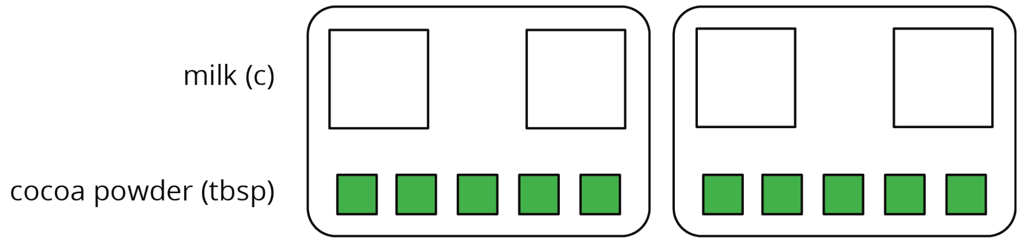
Problem 2

Statement

Priya makes chocolate milk by mixing 2 cups of milk and 5 tablespoons of cocoa powder. Draw a diagram that clearly represents two batches of her chocolate milk.

Solution

Answers vary. Sample response:



Problem 3

Statement

In a recipe for fizzy grape juice, the ratio of cups of sparkling water to cups of grape juice concentrate is 3 to 1.

a. Find two more ratios of cups of sparkling water to cups of juice concentrate that would make a mixture that tastes the same as this recipe.

b. Describe another mixture of sparkling water and grape juice that would taste different than this recipe.

Solution

Answers vary. Sample responses:

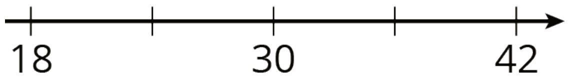
a. 6 to 2

b. 6 to 3

Problem 4

Statement

Write the missing number under each tick mark on the number line.



Solution

24, 36 (intervals of 6)

(From Unit 2, Lesson 1.)

Problem 5

Statement

At the kennel, there are 6 dogs for every 5 cats.

a. The ratio of dogs to cats is ______ to ______.

b. The ratio of cats to dogs is ______ to ______.

c. For every ______ dogs there are ______ cats.

d. The ratio of cats to dogs is ______ : ______.

Solution

a. 6 to 5

b. 5 to 6

c. 6, 5

d. $5 : 6$

(From Unit 2, Lesson 1.)

Problem 6

Statement

Elena has 80 unit cubes. What is the volume of the largest cube she can build with them?

Solution

64 cubic units (from a 4 by 4 by 4 cube)

(From Unit 1, Lesson 17.)

Problem 7

Statement

Fill in the blanks to make each equation true.

a. $3 \cdot \frac{1}{3} =$ ________

b. $10 \cdot \frac{1}{10} =$ ________

c. $19 \cdot \frac{1}{19} =$ ________

d. $a \cdot \frac{1}{a} =$ ________
(As long as a does not equal 0.)

a. $5 \cdot$ ________ $= 1$

b. $17 \cdot$ ________ $= 1$

c. $b \cdot$ ________ $= 1$

Solution

a. 1 (or equivalent)

b. 1 (or equivalent)

c. 1 (or equivalent)

a. 1 (or equivalent)

b. $\frac{1}{5}$ (or equivalent)

c. $\frac{1}{17}$ (or equivalent)

a. $\frac{1}{b}$ (or equivalent)

(From Unit 2, Lesson 1.)