Lesson 4: Color Mixtures

Goals

- Comprehend and respond (orally and in writing) to questions asking whether two ratios are equivalent, in the context of color mixtures.
- Draw and label a discrete diagram with circled groups to represent multiple batches of a color mixture.
- Explain equivalent ratios (orally and in writing) in terms of the amounts of each color in a mixture being multiplied by the same number to create another mixture that is the same shade.

Learning Targets

- I can explain the meaning of equivalent ratios using a color mixture as an example.
- I can use a diagram to represent a single batch, a double batch, and a triple batch of a color mixture.
- I know what it means to double or triple a color mixture.

Lesson Narrative

This is the second of two lessons that help students make sense of equivalent ratios through physical experiences. In this lesson, students mix different numbers of batches of a recipe for green water by combining blue and yellow water (created ahead of time with food coloring) to see if they produce the same shade of green. They also change the ratio of blue and yellow water to see if it changes the result. The activities here reinforce the idea that scaling a recipe up (or down) requires scaling the amount of each ingredient by the same factor (MP7). Students continue to use discrete diagrams as a tool to represent a situation.

For students who do not see color, the lesson can be adapted by having students make batches of dough with flour and water. 1 cup of flour to 5 tablespoons of water makes a very stiff dough, and 1 cup of flour to 6 tablespoons of water makes a soft (but not sticky) dough. In this case, doubling a recipe yields dough with the same tactile properties, just as doubling a colored-water recipe yields a mixture with the same color. The invariant property is stiffness rather than color. The principle that equivalent ratios yield products that are identical in some important way applies to both types of experiments.

Alignments

Building On

- 4.NBT.B.5: Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

Addressing

- 6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, “The ratio of wings to beaks in the bird house at the zoo was $2:1$, because for every 2 wings there was 1 beak.” “For every vote candidate A received, candidate C received nearly three votes.”

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Beakers
Food coloring
Graduated cylinders
Markers
Paper cups

Required Preparation

Mix blue water and yellow water; each group of 2 students will need 1 cup of each. To make colored water, add 1 teaspoon of food coloring to 1 cup of water. It would be best to give each mixture to students in a beaker or another container with a pour spout. If possible, conduct this lesson in a room with a sink.

Note that a digital version of this activity is available at this link: https://ggbm.at/Hcz2rDHc. It is embedded in the digital version of the student materials, but if classrooms using the print version of materials have access to enough student devices, it could be used in place of mixing actual colored water.

Student Learning Goals

Let’s see what color-mixing has to do with ratios.

4.1 Number Talk: Adjusting a Factor

Warm Up: 10 minutes
This number talk encourages students to use the structure of base ten numbers and the properties of operations to find the product of two whole numbers (MP7).

While many strategies may emerge, the focus of this string of problems is for students to see how adjusting a factor impacts the product, and how this insight can be used to reason about other problems. Four problems are given, however, it may not be possible to share every possible strategy. Consider gathering only two or three different strategies per problem. Each problem was chosen to elicit a slightly different reasoning, so as students explain their strategies, ask how the factors impacted how they approached the problem.

Building On

- 4.NBT.B.5

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find the value of each product mentally.

$6 \cdot 15$

$12 \cdot 15$

$6 \cdot 45$

$13 \cdot 45$

Student Response

- 90. Possible strategy: $(6 \cdot 10) + (6 \cdot 5) = 90$
- 180. Possible strategy: Since the 6 from the first question doubled to 12, and the 15 stayed the same, the product doubles to 180. This is because there are twice as many groups of 15 than in the first question.
- 270. Possible strategy: Since the 6 is the same as the in the first question, and the 15 tripled to 45, the product triples to 270. This is because the number of groups stayed the same, but the amount in each group got three times as large.

- 585. Possible strategy: Since the 45 is the same as the previous question, we can double the 6 and the product to get 540. We need one more group of 45, and $540 + 45 = 585$.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted the strategy choice. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone solve the problem the same way but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

4.2 Turning Green

35 minutes (there is a digital version of this activity)
In this activity, students mix different numbers of batches of a color recipe to obtain a certain shade of green. They observe how multiple batches of the same recipe produce the same shade of green as a single batch, which suggests that the ratios of blue to yellow for the two situations are equivalent. They also come up with a ratio that is not equivalent to produce a mixture that is a different shade of green.

As students make the mixtures, ensure that they measure accurately so they will get accurate outcomes. As students work, note the different diagrams students use to represent their recipes. Select a few examples that could be highlighted in discussion later.

Addressing

- 6.RP.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2–4. (Smaller groups are better, but group size might depend on available equipment.) Each group needs a beaker of blue water and one of yellow water, one graduated cylinder, a permanent marker, a craft stick, and 3 opaque white cups (either styrofoam, white paper, or with a white plastic interior).

For classes using the print-only version: Show students the blue and yellow water. Demonstrate how to pour from the beakers to the graduated cylinder to measure and mix 5 ml of blue water with 15 ml of yellow water. Demonstrate how to get an accurate reading on the graduated cylinder by working on a level surface and by reading the measurement at eye level. Tell students they will experiment with different mixtures of green water and observe the resulting shades.

For classes using the digital version: Display the dynamic color mixing cylinders for all to see. Tell students, "The computer mixes colors when you add colored water to each cylinder. You can add increments of 1 or 5. You can't remove water (once it's mixed, it's mixed), but you can start over. The computer mixes yellow and blue." Ask students a few familiarization questions before they start working on the activity:

- What happens when you mix yellow and blue? (A shade of green is formed.)
- What happens if you add more blue than yellow? (Darker green, blue green, etc.)

If necessary, demonstrate how it works by adding some yellows and blues to both the left and the right cylinder. Show how the "Reset" button lets you start over.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

If any students come up with an incorrect recipe, consider letting this play out. A different shade of green shows that the ratio of blue to yellow in their mixture is not equivalent to the ratio of blue to yellow in the other mixtures. Rescuing the incorrect mixture to display during discussion may lead to meaningful conversations about what equivalent ratios mean.

Student Task Statement

Your teacher mixed milliliters of blue water and milliliters of yellow water in the ratio $5 : 15$.

1. Doubling the original recipe:

a. Draw a diagram to represent the amount of each color that you will combine to double your teacher's recipe.

b. Use a marker to label an empty cup with the ratio of blue water to yellow water in this double batch.

c. Predict whether these amounts of blue and yellow will make the same shade of green as your teacher's mixture. Next, check your prediction by measuring those amounts and mixing them in the cup.

d. Is the ratio in your mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning.

2. Tripling the original recipe:

   a. Draw a diagram to represent triple your teacher's recipe.

   b. Label an empty cup with the ratio of blue water to yellow water.

   c. Predict whether these amounts will make the same shade of green. Next, check your prediction by mixing those amounts.

   d. Is the ratio in your new mixture equivalent to the ratio in your teacher's mixture? Explain your reasoning.

3. Next, invent your own recipe for a *bluer* shade of green water.

   a. Draw a diagram to represent the amount of each color you will combine.

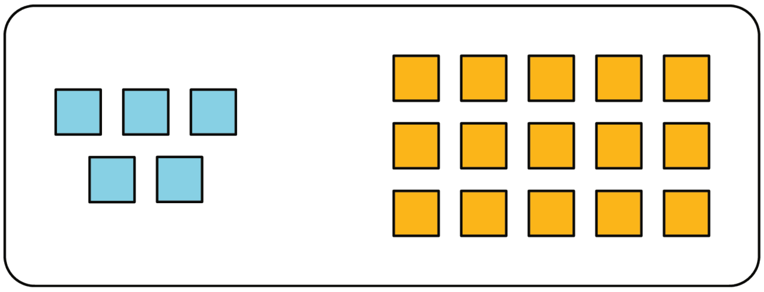
   b. Label the final empty cup with the ratio of blue water to yellow water in this recipe.

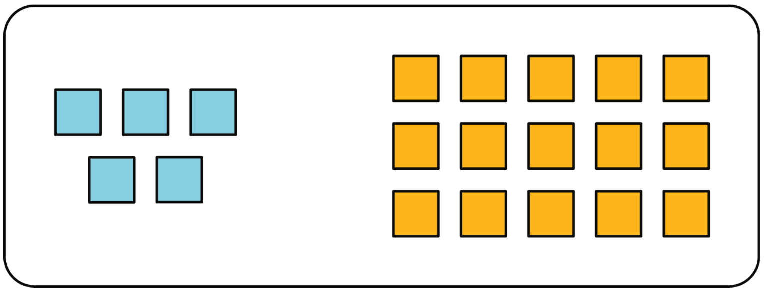
   c. Test your recipe by mixing a batch in the cup. Does the mixture yield a bluer shade of green?

   d. Is the ratio you used in this recipe equivalent to the ratio in your teacher's mixture? Explain your reasoning.

Student Response

1. Doubling the recipe.
   a. Here is one example of a diagram. Students may arrange the groups differently or use different symbols to represent 1 ml of water.





b. A cup is labeled $10:30$ or “10 to 30.”

c. If the recipe is correct, the shade of green is identical to the teacher’s.

d. $10:30$ is equivalent to $5:15$ because it is 2 batches of the same recipe. It creates an identical shade of green.

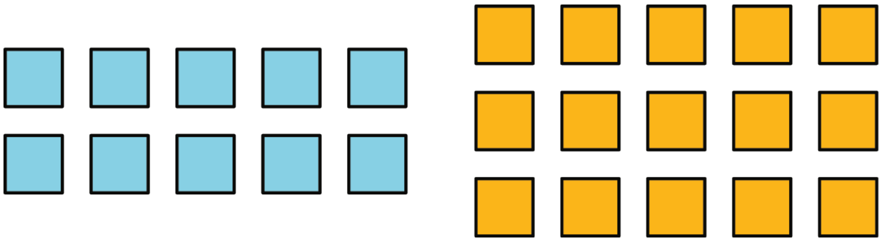
2. Tripling the recipe.
   a. Like the previous diagram, except showing 3 batches.

   b. A cup is labeled $15:45$ or “15 to 45.”

   c. If the recipe is correct, the shade of green is identical to the teacher’s.

   d. $15:45$ is equivalent to $5:15$ because it is 3 batches of the same recipe. It creates an identical shade of green.

3. A bluer shade of green.
   a. Answers vary. You might use more blue for the same amount of yellow, or less yellow for the same amount of blue. Sample response:



   b. Answers vary. Sample responses: $10:15$ (more blue for the same amount of yellow) or $5:10$ (less yellow for the same amount of blue).

c. If a correct ratio is used, the mixture should be a bluer shade of green than the other mixtures.

d. No, it was not the same shade of green. The first and second parts were not, respectively, obtained by multiplying 5 and 15 by the same number.

> **Are You Ready for More?**
>
> Someone has made a shade of green by using 17 ml of blue and 13 ml of yellow. They are sure it cannot be turned into the original shade of green by adding more blue or yellow. Either explain how more can be added to create the original green shade, or explain why this is impossible.

Student Response

You could add 3 ml of blue to get 20 ml of blue, and 47 ml of yellow to get 60 ml of yellow. The blue to yellow ratio of $20 : 60$ will make the same shade of green as $5 : 15$. It's a quadruple batch.

Activity Synthesis

After each group has completed the task, have the students rotate through each group's workspace to observe the mixtures and diagrams. As they circulate, pose some guiding questions. (For students using the digital version, these questions refer to the mixtures on their computers.)

- Are each group's results for the first two mixtures the same shade of green?
- Are the ratios representing the double batch, the triple batch, and your new mixture all equivalent to each other? How do you know?
- What are some different ways groups drew diagrams to represent the ratios?

Highlight the idea that a ratio is equivalent to another if the two ratios describe different numbers of batches of the same recipe.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Assign one member from each group to stay behind to answer questions from students visiting from other groups. Provide visitors with question prompts such as, "These look like the same shade, how can we be sure the ratios are equivalent?", "If you want a smaller amount with the same shade, what can you do?" or "What is the ratio for your new mixture?"
Design Principle(s): Cultivate conversation

4.3 Perfect Purple Water

Optional: 10 minutes

Students revisit color mixing—this time by producing purple-colored water—to further understand equivalent ratios. They recall that doubling, tripling, or halving a recipe for colored water yields the same resulting color, and that equivalent ratios can represent different numbers of batches of the same recipe.

As students work, monitor for students who use different representations to answer both questions, as well as students who come up with different ratios for the second question.

Addressing

- 6.RP.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

Arrange students in groups of 2. Remind students of the previous "Turning Green" activity. Ask students to discuss the following questions with a partner. Then, discuss responses together as a whole class:

- Why did the different mixtures of blue and yellow water result in the same shade of green? (If mixed correctly, the amount of the ingredients were all doubled or all tripled. The ratio of blue water to yellow water was equivalent within each recipe.)
- How were you able to get a darker shade of green? (We changed the ratio of ingredients, so there was more blue for the same amount of yellow.)

Explain to students that the task involves producing purple-colored water, but they won't actually be mixing colored water. Ask students to use the ideas just discussed from the previous activity to predict the outcomes of mixing blue and red water.

Ensure students understand the abbreviation for milliliters is ml.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes, blocks or printed representations.
Supports accessibility for: Conceptual processing

Anticipated Misconceptions

At a quick glance, students may think that since Andre is mixing a multiple of 8 with a multiple of 3, it will also result in Perfect Purple Water. If this happens, ask them to take a closer look at how the values are related or draw a diagram showing batches.

Student Task Statement

The recipe for Perfect Purple Water says, "Mix 8 ml of blue water with 3 ml of red water."

Jada mixes 24 ml of blue water with 9 ml of red water. Andre mixes 16 ml of blue water with 9 ml of red water.

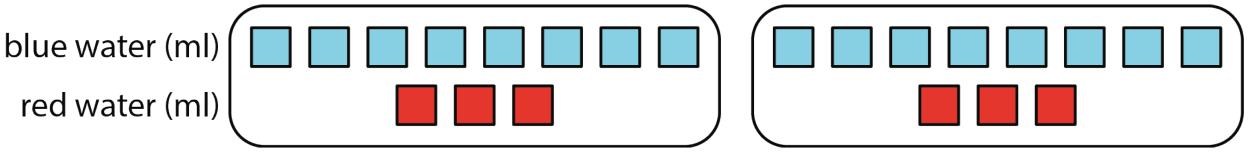
1. Which person will get a color mixture that is the same shade as Perfect Purple Water? Explain or show your reasoning.

2. Find another combination of blue water and red water that will also result in the same shade as Perfect Purple Water. Explain or show your reasoning.

Student Response

1. Jada's mixture will result in the same shade of purple, because both ingredients were tripled. $8 \cdot 3 = 24$ and $3 \cdot 3 = 9$. Andre's mixture will not result in the same shade of purple, because the amount of red water is doubled, but the amount of blue water was tripled.

2. Answers vary. One possible answer is $16 : 6$ (each ingredient is doubled or multiplied by 2. $8 \cdot 2 = 16$, there are 16 ml blue. $3 \cdot 2 = 6$, there are 6 ml red.)



Activity Synthesis

Select students to share their answers to the questions.

- For the first question, emphasize that not only did Jada triple each amount of red and blue, but this means that amount of each color is being *multiplied by the same value*, in this case, 3.

- For the second question, list all the different ratios students brought up for all to see. Discuss how each ratio differed from that for the original mixture. Point out that as long as both terms are multiplied by the same quantity, the resulting ratio will be *equivalent* and will yield the same shade of purple.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their combination of blue water and red water that will make the same shade as Perfect Purple Water, present a a flawed response. For example, “Mixing 18 ml of blue water with 13 ml of red water would result in the same shade of purple because I added 10 ml of each color.” Ask students to identify the error, critique the reasoning, and write a correct explanation. Invite students to share their critiques and corrected combinations and explanations with the class. Listen for and amplify the language students use to justify the ratios are equivalent. This may include such language as multiply by the same quantity or represent the same shade. This helps students evaluate, and improve upon, the written mathematical arguments of others, as they clarify their understanding of equivalent ratios.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Lesson Synthesis

The important take-aways from this lesson are:

- To create more batches of a color recipe that will come out to be the same shade of the color, multiply each ingredient by the same number.
- We can think of equivalent ratios as representing different numbers of batches of the same recipe.

Remind students of the work done and observations made in this lesson. Some questions to guide the discussion might include:

- How did you decide that 10 ml blue and 30 ml yellow would make 2 batches of 5 ml blue and 15 ml yellow? (Multiply each part by 2.)
- How did you decide that 15 ml blue and 45 ml yellow would make 3 batches? (Multiply each part by 3.)
- How did we know that $5 : 15$, $10 : 30$, and $15 : 45$ were equivalent? (They created the same shade of green. Also, $10 : 30$ has both parts of the original recipe multiplied by 2, and $15 : 45$ has both parts of the original recipe multiplied by 3.)

4.4 Orange Water

Cool Down: 5 minutes
Addressing

- 6.RP.A.1

Student Task Statement

A recipe for orange water says, "Mix 3 teaspoons yellow water with 1 teaspoon red water." For this recipe, we might say: "The ratio of teaspoons of yellow water to teaspoons of red water is $3:1$."

1. Write a ratio for 2 batches of this recipe.

2. Write a ratio for 4 batches of this recipe.

3. Explain why we can say that any two of these three ratios are equivalent.

Student Response

1. The ratio of teaspoons of yellow to teaspoons of red is $6:2$ (or any sentence that accurately states this ratio). Note: a statement like "The ratio of yellow to red is $6:2$" describes the association between the colors but does not indicate the amount of stuff in the mixture.

2. The ratio of teaspoons of yellow to teaspoons of red is $12:4$ (or any sentence that accurately states this ratio).

3. These are equivalent ratios because they describe different numbers of batches of the same recipe. To make 2 batches, multiply the amount of each color by 2. To make 4 batches, multiply the amount of each color by 4. As long as you multiply the amounts for both colors by the same number, you will get a ratio that is equivalent to the ratio in the recipe.

Student Lesson Summary

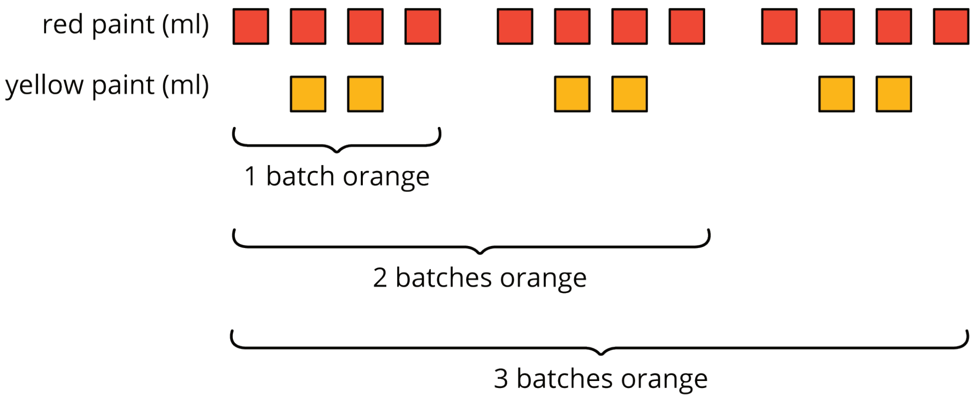
When mixing colors, doubling or tripling the amount of each color will create the same shade of the mixed color. In fact, you can always multiply the amount of *each* color by *the same number* to create a different amount of the same mixed color.

For example, a batch of dark orange paint uses 4 ml of red paint and 2 ml of yellow paint.

- To make two batches of dark orange paint, we can mix 8 ml of red paint with 4 ml of yellow paint.

- To make three batches of dark orange paint, we can mix 12 ml of red paint with 6 ml of yellow paint.

Here is a diagram that represents 1, 2, and 3 batches of this recipe.



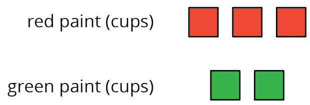
We say that the ratios $4 : 2$, $8 : 4$, and $12 : 6$ are *equivalent* because they describe the same color mixture in different numbers of batches, and they make the same shade of orange.

Lesson 4 Practice Problems

Problem 1

Statement

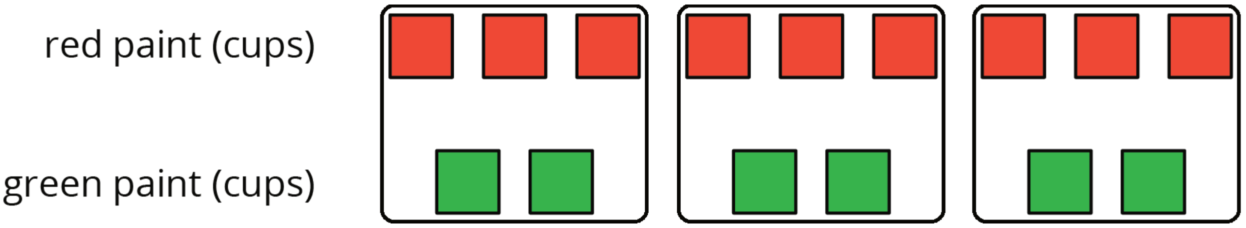
Here is a diagram showing a mixture of red paint and green paint needed for 1 batch of a particular shade of brown.



Add to the diagram so that it shows 3 batches of the same shade of brown paint.

Solution

Answers vary. Sample response:



Problem 2

Statement

Diego makes green paint by mixing 10 tablespoons of yellow paint and 2 tablespoons of blue paint. Which of these mixtures produce the same shade of green paint as Diego's mixture? Select **all** that apply.

A. For every 5 tablespoons of blue paint, mix in 1 tablespoon of yellow paint.

B. Mix tablespoons of blue paint and yellow paint in the ratio $1 : 5$.

C. Mix tablespoons of yellow paint and blue paint in the ratio 15 to 3.

D. Mix 11 tablespoons of yellow paint and 3 tablespoons of blue paint.

E. For every tablespoon of blue paint, mix in 5 tablespoons of yellow paint.

Solution

["B", "C", "E"]

Problem 3

Statement

To make 1 batch of sky blue paint, Clare mixes 2 cups of blue paint with 1 gallon of white paint.

a. Explain how Clare can make 2 batches of sky blue paint.

b. Explain how to make a mixture that is a darker shade of blue than the sky blue.

c. Explain how to make a mixture that is a lighter shade of blue than the sky blue.

Solution

a. Mix 4 cups of blue paint and 2 gallons of white paint.

b. Answers vary. Sample response: 3 cups of blue paint and 1 gallon of white paint. Mixing the same amount of white paint with *more* blue paint will make a darker shade of blue.

c. Answers vary. Sample response: 2 cups of blue paint and 2 gallons of white paint. Mixing the same amount of blue paint with *more* white paint will make a lighter shade of blue.

Problem 4

Statement

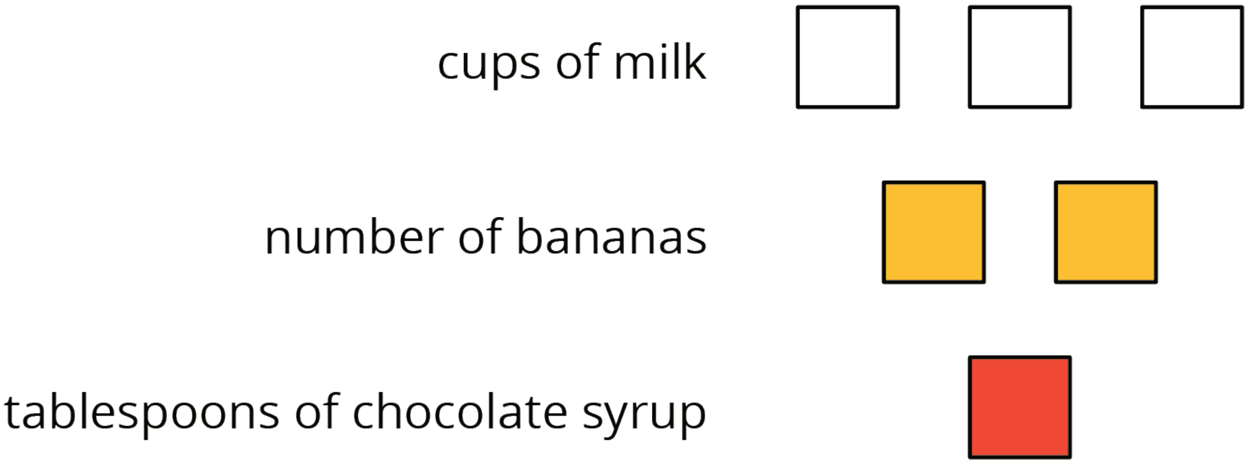
A smoothie recipe calls for 3 cups of milk, 2 frozen bananas and 1 tablespoon of chocolate syrup.

a. Create a diagram to represent the quantities of each ingredient in the recipe.

b. Write 3 different sentences that use a ratio to describe the recipe.

Solution

a. Answers vary. Sample response:



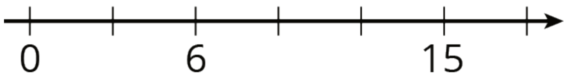
2. Answers vary. Sample response: The ratio of cups of milk to number of bananas is $3 : 2$, the ratio of bananas to tablespoons of chocolate syrup is 2 to 1, for every tablespoon of chocolate syrup, there are 3 cups of milk.

(From Unit 2, Lesson 2.)

Problem 5

Statement

Write the missing number under each tick mark on the number line.



Solution

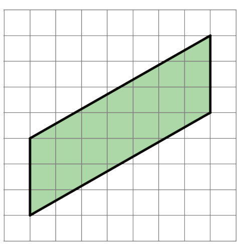
0, 3, 6, 9, 12, 15, 18 (intervals of 3)

(From Unit 2, Lesson 1.)

Problem 6

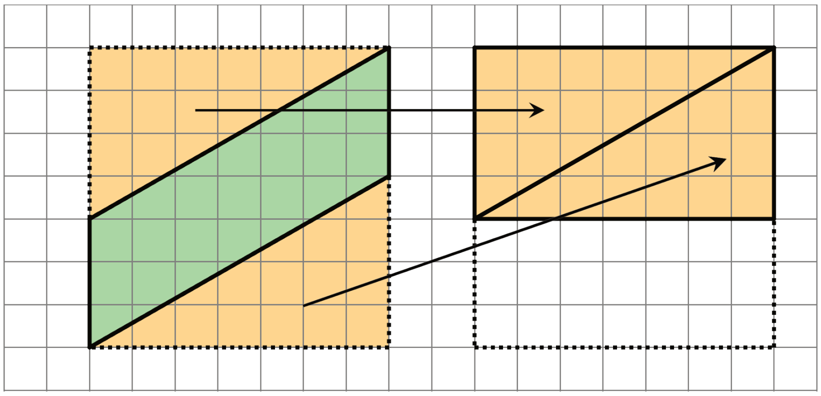
Statement

Find the area of the parallelogram. Show your reasoning.



Solution

21 square units. Reasoning varies. Sample reasoning: Draw a square around the parallelogram; its area is 49 square units, because $7 \cdot 7 = 49$. Rearrange the triangles above and below the parallelogram to form a rectangle; the area of this rectangle is 28 square units, because $4 \cdot 7 = 28$. Subtracting the area of the triangles from the area of the square, we have 21 square units. $49 - 28 = 21$.



(From Unit 1, Lesson 4.)

Problem 7

Statement

Complete each equation with a number that makes it true.

a. $11 \cdot \frac{1}{4} =$ ________

b. $7 \cdot \frac{1}{4} =$ ________

c. $13 \cdot \frac{1}{27} =$ ________

a. $13 \cdot \frac{1}{99} =$ ________

b. $x \cdot \frac{1}{y} =$ ________
(As long as y does not equal 0.)

Solution

a. $\frac{11}{4}$ (or equivalent)

b. $\frac{7}{4}$ (or equivalent)

c. $\frac{13}{27}$ (or equivalent)

d. $\frac{13}{99}$ (or equivalent)

e. $\frac{x}{y}$ (or equivalent)

(From Unit 2, Lesson 1.)