Lesson 8: Finding Unknown Side Lengths

Goals

- Calculate unknown side lengths of a right triangle by using the Pythagorean Theorem, and explain (orally) the solution method.
- Label the "legs" and "hypotenuse" on a diagram of a right triangle.

Learning Targets

- If I know the lengths of two sides, I can find the length of the third side in a right triangle.
- When I have a right triangle, I can identify which side is the hypotenuse and which sides are the legs.

Lesson Narrative

The purpose of this lesson is to use the Pythagorean Theorem to find unknown side lengths of a right triangle.

Alignments

Addressing

- 8.G.B.7: Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.

Building Towards

- 8.G.B: Understand and apply the Pythagorean Theorem.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Think Pair Share
- Which One Doesn't Belong?

> **Student Learning Goals**
>
> Let's find missing side lengths of right triangles.

8.1 Which One Doesn't Belong: Equations

Warm Up: 5 minutes

The purpose of this warm-up is to prime students for solving equations that arise while using the Pythagorean Theorem.

Building Towards

- 8.G.B

Instructional Routines

- Which One Doesn’t Belong?

Launch

Arrange students in groups of 2–4. Display the equations for all to see. Ask students to indicate when they have noticed one that does not belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reason why a particular equation does not belong and together find at least one reason each question doesn’t belong.

> ### Student Task Statement
>
> Which one doesn’t belong?
>
> $3^2 + b^2 = 5^2$
>
> $b^2 = 5^2 - 3^2$
>
> $3^2 + 5^2 = b^2$
>
> $3^2 + 4^2 = 5^2$

Student Response

Answers vary. Sample responses:

$3^2 + b^2 = 5^2$: the only one where b is not isolated

$b^2 = 5^2 - 3^2$: the only one with one term on the left and two terms on the right, the only one with subtraction

$3^2 + 5^2 = b^2$: the only one that is not based on a 3-4-5 Pythagorean triple

$3^2 + 4^2 = 5^2$: the only one with all numbers

Activity Synthesis

Ask each group to share one reason why a particular equation does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students’ explanations and ensure the reasons given make sense.

8.2 Which One Is the Hypotenuse?

5 minutes

This activity helps students identify the hypotenuse in right triangles in different orientations.

Building Towards

- 8.G.B

Instructional Routines

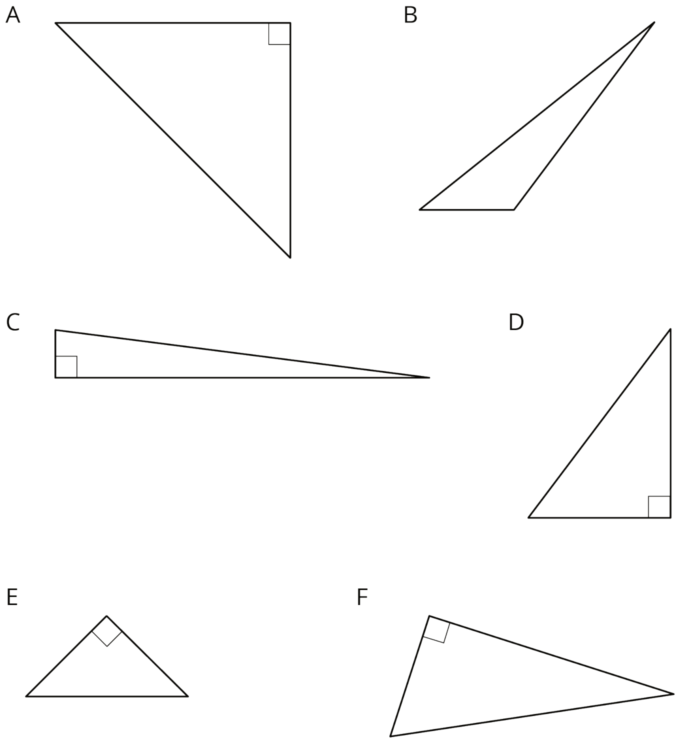
- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet work time and then have them compare with a partner. Follow with a whole-class discussion.

Student Task Statement

Label all the hypotenuses with c.



Student Response

All triangles except Triangle B should have a c on the side opposite the right angle. Triangle B is not a right triangle, therefore it does not have a hypotenuse.

Activity Synthesis

Ask students which triangles are right triangles, and then ask them which side is the hypotenuse for each one. Ask, "In a right triangle, does it matter which is a and which is b?" (No.)

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. As students share, press for details in students' reasoning by asking how they know the side they selected is the hypotenuse. Listen for and amplify the language students use to describe the important features of the hypotenuse (e.g., longest side of a right triangle, side opposite the right angle). Then ask students to explain why Triangle B does not have a hypotenuse. This will support rich and inclusive discussion about strategies for identifying the hypotenuse of a right triangle.
Design Principle(s): Support sense-making

8.3 Find the Missing Side Lengths

20 minutes
The purpose of this activity is to give students practice finding missing side lengths in a right triangle using the Pythagorean Theorem.

Addressing

- 8.G.B.7

Instructional Routines

- MLR3: Clarify, Critique, Correct
- Think Pair Share

Launch

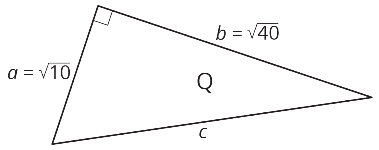
Arrange students in groups of 2. Give students 10 minutes of quiet work time and then have them compare with a partner. If partners disagree about any of their answers, ask them to explain their reasoning to one another until they reach agreement. Follow with a whole-class discussion.

Access for Students with Disabilities

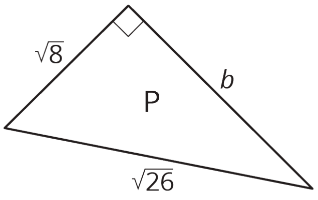
Representation: Internalize Comprehension. Activate or supply background knowledge by displaying the Pythagorean Theorem with a labeled diagram. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Student Task Statement

1. Find c.



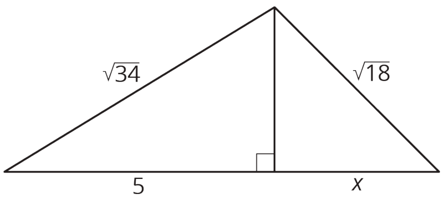
2. Find b.



3. A right triangle has sides of length 2.4 cm and 6.5 cm. What is the length of the hypotenuse?

4. A right triangle has a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$. What is the length of the other side?

5. Find the value of x in the figure.

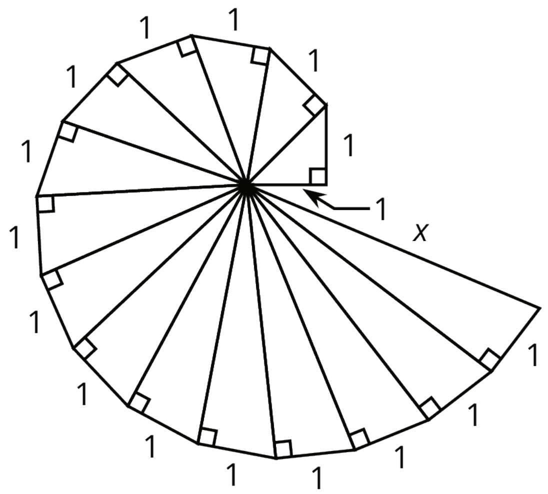


Student Response

1. $\sqrt{50}$
2. $\sqrt{18}$
3. $\sqrt{48.01}$
4. $\sqrt{\frac{7}{144}}$
5. $x = 3$

Are You Ready for More?

The spiral in the figure is made by starting with a right triangle with both legs measuring one unit each. Then a second right triangle is built with one leg measuring one unit, and the other leg being the hypotenuse of the first triangle. A third right triangle is built on the second triangle’s hypotenuse, again with the other leg measuring one unit, and so on.



Find the length, x, of the hypotenuse of the last triangle constructed in the figure.

Student Response

We can repeatedly apply the Pythagorean Theorem. The first hypotenuse equals $\sqrt{2}$, since $(\sqrt{2})^2 = 1^2 + 1^2$. The second right triangle has legs 1 and $\sqrt{2}$, so has a hypotenuse of $\sqrt{3}$, since $(\sqrt{3})^2 = (\sqrt{2})^2 + 1^2$. This pattern continues, with the next hypotenuses having length $\sqrt{4}$, the $\sqrt{5}$, etc. By counting until the end, we find that the 15th and last hypotenuse has a length x equal to $\sqrt{16}$, so $x = 4$.

Activity Synthesis

Ask students to share how they found the missing side lengths. If students drew triangles for the two questions that did not have an image, display a few of these for all to see, noting any differences between them. For example, students may have drawn triangles with different orientations or labeled different sides as a and b.

For the last question, ask students to say what they did first to try and solve for x. For example, while many students may have found the length of the unknown altitude first and then used that value to find x, others may have set up the equation $34 - 5^2 = 18 - x^2$.

Point out that when you know two sides of a right triangle, you can always find the third by using the Pythagorean identity $a^2 + b^2 = c^2$. Remind them that it is important to keep track of which side is the hypotenuse.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their method for the questions that did not have an image, present an incorrect solution based on a common error related to labeling the sides of a right triangle. For the right triangle with a side of length $\frac{1}{4}$ and a hypotenuse of length $\frac{1}{3}$, draw a right triangle with the legs labeled $\frac{1}{4}$ and $\frac{1}{3}$. Provide an incorrect explanation such as: "I know that $a = \frac{1}{4}$ and $b = \frac{1}{3}$, so when I use the Pythagorean Theorem, I get the equation $(\frac{1}{4})^2 + (\frac{1}{3})^2 = c^2$." Ask students to identify the error, critique the reasoning, and revise the original statement. As students discuss in partners, listen for students who clarify the meaning of the hypotenuse and identify c as the length of the hypotenuse in the Pythagorean Theorem. This routine will engage students in meta-awareness as they critique and correct a common error when labeling the sides of a right triangle.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

The purpose of this discussion is to check that students understand the Pythagorean Theorem and how it can be used to determine information about triangles. Ask students to draw a right triangle and label 2 of the 3 sides. Tell them to swap triangles with another student, solve for the missing length, then swap back to check the other person's work. Select a few groups to share their triangles and, if possible, display them for all to see while sharing how they solved for the unknown length.

8.4 Could be the Hypotenuse, Could be a Leg

Cool Down: 5 minutes

Addressing

- 8.G.B.7

Student Task Statement

A right triangle has sides of length 3, 4, and x.

1. Find x if it is the hypotenuse.

2. Find x if it is one of the legs.

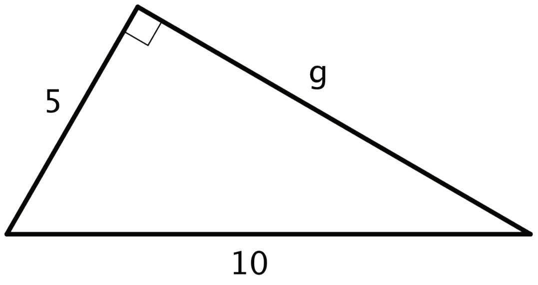
Student Response

1. $x = \sqrt{25}$ or $x = 5$

2. $x = \sqrt{7}$

Student Lesson Summary

There are many examples where the lengths of two legs of a right triangle are known and can be used to find the length of the hypotenuse with the Pythagorean Theorem. The Pythagorean Theorem can also be used if the length of the hypotenuse and one leg is known, and we want to find the length of the other leg. Here is a right triangle, where one leg has a length of 5 units, the hypotenuse has a length of 10 units, and the length of the other leg is represented by g.

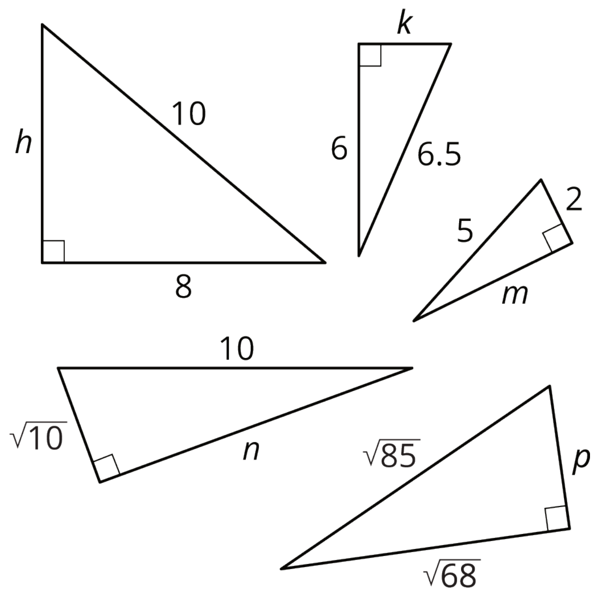


Start with $a^2 + b^2 = c^2$, make substitutions, and solve for the unknown value. Remember that c represents the hypotenuse: the side opposite the right angle. For this triangle, the hypotenuse is 10.

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
5^2 + g^2 &= 10^2 \\
g^2 &= 10^2 - 5^2 \\
g^2 &= 100 - 25 \\
g^2 &= 75 \\
g &= \sqrt{75}
\end{aligned}
$$

Use estimation strategies to know that the length of the other leg is between 8 and 9 units, since 75 is between 64 and 81. A calculator with a square root function gives $\sqrt{75} \approx 8.66$.

Lesson 8 Practice Problems
Problem 1
Statement
Find the exact value of each variable that represents a side length in a right triangle.



Solution
$h = 6$ (because $100 - 64 = 36$ and $\sqrt{36} = 6$)

$k = 2.5$ (because $42.25 - 36 = 6.25$ and $\sqrt{6.25} = 2.5$)

$m = \sqrt{21}$ because $25 - 4 = 21$

$n = \sqrt{90}$ because $100 - 10 = 90$

$p = \sqrt{17}$ because $85 - 68 = 17$

Problem 2
Statement
A right triangle has side lengths of a, b, and c units. The longest side has a length of c units. Complete each equation to show three relations among a, b, and c.

- $c^2 =$
- $a^2 =$
- $b^2 =$

Solution

- $c^2 = a^2 + b^2$ or $c^2 = b^2 + a^2$
- $a^2 = c^2 - b^2$
- $b^2 = c^2 - a^2$

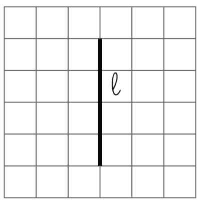
(From Unit 8, Lesson 7.)

Problem 3

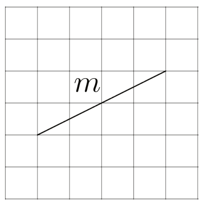
Statement

What is the exact length of each line segment? Explain or show your reasoning. (Each grid square represents 1 square unit.)

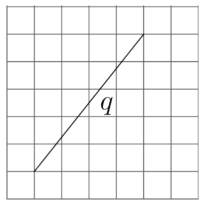
a.



b.



c.



Solution

a. 4 units. The segment is along the grid lines, so count the squares.

b. $\sqrt{20}$ because $4^2 + 2^2 = 20$

c. $\sqrt{41}$ because $4^2 + 5^2 = 41$

(From Unit 8, Lesson 7.)

Problem 4

Statement

In 2015, there were roughly 1×10^6 high school football players and 2×10^3 professional football players in the United States. About how many times more high school football players are there? Explain how you know.

Solution

There are approximately 500 times more high school football players. $\frac{1 \times 10^6}{2 \times 10^3} = 0.5 \times 10^3 = 5 \times 10^2$

(From Unit 7, Lesson 15.)

Problem 5

Statement

Evaluate:

a. $\left(\frac{1}{2}\right)^3$

b. $\left(\frac{1}{2}\right)^{-3}$

Solution

a. $\frac{1}{8}$

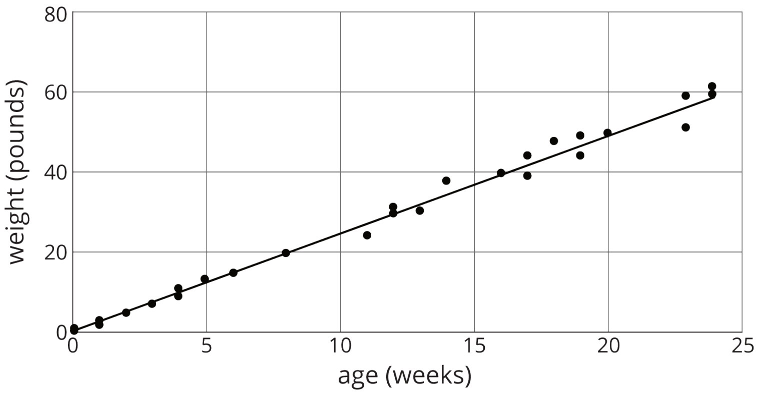
b. 8

(From Unit 7, Lesson 6.)

Problem 6

Statement

Here is a scatter plot of weight vs. age for different Dobermans. The model, represented by $y = 2.45x + 1.22$, is graphed with the scatter plot. Here, x represents age in weeks, and y represents weight in pounds.



a. What does the slope mean in this situation?

b. Based on this model, how heavy would you expect a newborn Doberman to be?

Solution

a. The slope means that a doberman can be expected to gain 2.45 pounds per week.

b. 1.22 pounds (the y-intercept of the function).

(From Unit 6, Lesson 6.)