

# Lesson 2: Using Diagrams to Represent Addition and Subtraction

## Goals

- Compare and contrast (orally and in writing) vertical calculations and base-ten diagrams that represent adding and subtracting decimals.
- Explain (in words and through other representations) that adding and subtracting decimals requires combining digits that represent like base-ten units.
- Interpret and create diagrams that represent 10 like base-ten units being composed into 1 unit of higher place value, e.g., 10 tenths as 1 one, and comprehend the word “bundle” to refer to this concept.

## Learning Targets

- I can use diagrams to represent and reason about addition and subtraction of decimals.
- I can use place value to explain addition and subtraction of decimals.
- I can use vertical calculations to represent and reason about addition and subtraction of decimals.

## Lesson Narrative

This lesson is optional. Prior to grade 6, students have added and subtracted decimals to the hundredths using a variety of methods, all of which focus on understanding place value. This lesson reinforces their understanding of place-value relationships in preparation for computing sums and differences of any decimals algorithmically.

In this lesson, students use two methods—base-ten diagrams and vertical calculations—to find the sum and differences of decimals. Central to both methods is an understanding about the meaning of each digit in the numbers and how the different digits are related. Students recall that we only add the values of two digits if they represent the same base-ten units. They also recall that when the value of a base-ten unit is 10 or more we can express it with a different unit that is 10 times higher in value. For example, 10 tens can be expressed as 1 hundred, and 12 hundredths can be expressed as 1 tenth and 2 hundredths. This idea is made explicit both in the diagrams and in vertical calculations.

## Alignments

### Building On

- 5.NBT.A.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and  $\frac{1}{10}$  of what it represents in the place to its left.

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

### Addressing

- 6.NS.B.3: Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.

### Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

### Required Materials

Graph paper

blackline master

Pre-printed slips, cut from copies of the

### Required Preparation

Students draw base-ten diagrams in this lesson. If drawing them is a challenge, consider giving students access to:

- Commercially produced base-ten blocks, if available.
- Print and cut up the Squares and Rectangles blackline master. Prepare one copy for every student. These tools will be useful throughout the unit, so consider printing on card stock and organizing them for easy reuse.
- Digital applet of base-ten representations <https://www.geogebra.org/m/FXEZD466>.

Some students might find graph paper helpful for aligning the digits for vertical calculations. Consider having graph paper accessible for these activities: Finding Sums in Different Ways, Representing Subtraction, and Why or Why Not?.

### Student Learning Goals

Let's represent addition and subtraction of decimals.

## 2.1 Changing Values

### Warm Up: 5 minutes

The purpose of this warm-up is for students to review place value when working with decimals. There are many ways students might find the numbers represented by the large rectangle and large square. However, the focus as students work should be on understanding that each place represents a unit that is 10 times larger than the unit immediately to its right. This understanding can be represented by diagrams or by multiplication expressions (MP8).

## Building On

- 5.NBT.A.1

## Instructional Routines

- Think Pair Share

## Launch

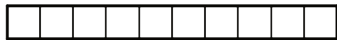
Arrange students in groups of 2. Tell students to look for patterns as they work. Give students 1–2 minute of quiet think time, followed by a brief partner discussion. Tell the partners to share their responses, come to an agreement on each answer, and discuss any patterns they noticed. Select students with correct responses to share during the whole-class discussion.

## Anticipated Misconceptions

Some students may continually use skip counting (by 10, by 0.1, etc.) to find the value of the rectangle and the square, rather than making connections to place value. To help these students see a pattern connected to their skip counting, ask them to also write the multiplication expression that relates the value of each of the smaller units to that of the larger unit they compose.

### Student Task Statement

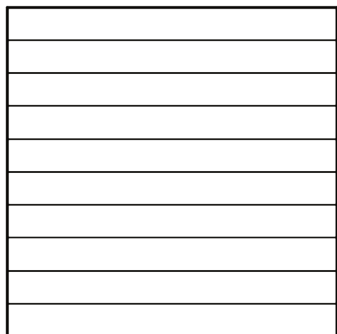
1. Here is a rectangle.



What number does the rectangle represent if each small square represents:

- a. 1
- b. 0.1
- c. 0.01
- d. 0.001

2. Here is a square.



What number does the square represent if each small rectangle represents:

- a. 10
- b. 0.1
- c. 0.00001

### Student Response

1. a. 10

- b. 1
  - c. 0.1
  - d. 0.01
2. a. 100
- b. 1
  - c. 0.0001

### Activity Synthesis

Ask selected students to share their values for the diagrams, how they found them, and what patterns they noticed. Record and display these values for all to see. Ask other students for the multiplication equation that represents each response to highlight that each decimal place value is 10 times the value of the unit to its right. For example, the recording for the first question should look like this:

$$10 \cdot 1 = 10$$

$$10 \cdot 0.1 = 1$$

$$10 \cdot 0.01 = 0.1$$

$$10 \cdot 0.001 = 0.01$$

If time permits, discuss some of these questions:

- “When you change what each small square represents, how did the change affect the value of the large rectangle?”
- “What might be some other numbers that the small square or long rectangle could represent?”
- “Why could these representations be called ‘base-ten diagrams’?”

## 2.2 Squares and Rectangles

**Optional: 15 minutes (there is a digital version of this activity)**

In this activity, students reinforce their understanding of the meaning of place value, i.e., that the value in ones place is 10 times the value of the place to its right (and  $\frac{1}{10}$  the value of the place to its left). They draw base-ten diagrams to represent addition of decimals; they connect the process of regrouping numbers in the addition algorithm to the “bundling” of pieces in the diagram. For example, they see that 10 medium squares (0.01) can be composed or “bundled” to make 1 medium rectangle (0.1). Likewise, when the numbers in the ones place add up to be at least 10, we can group them and add 1 to the tens place.

As alternatives to drawing diagrams, consider having students use physical base-ten blocks (if available), a paper version of the base-ten figures (from the blackline master), or this digital applet <https://ggbm.at/FXEZD466>.

### Building On

- 5.NBT.A.1
- 5.NBT.B.7

### Launch

Give students 1 minute of quiet time to study the diagrams and notice how they are structured. Then, have students identify the value of various units by pointing to them. For example, point to the medium square, and ask what it represents and how it relates to the large rectangle. Once students are familiar with the various pieces and their values, give students 10 minutes of quiet work time. Provide access to physical pattern blocks, cut-up paper copies of the base-ten figures, or the digital applet for representing base-ten numbers, if needed.



Classes using the digital activities have an interactive applet with virtual blocks. Note that the applet can only represent ones, tenths, and hundredths; this is so the pieces are large enough to manipulate. To use the bundling and unbundling features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool

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### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Activate or supply background knowledge. Some students may benefit from continued access to physical base-ten blocks (if available), a paper version of the base-ten figures (from the blackline master), or the digital applet. Encourage students to begin with physical representations before drawing a diagram.

*Supports accessibility for: Conceptual processing*

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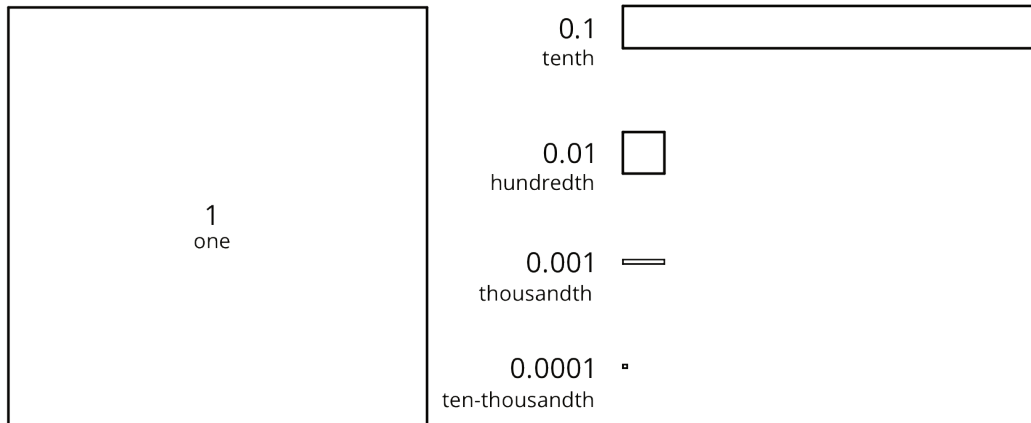
### Anticipated Misconceptions

Students will need a solid understanding of place value to be successful with the activity. If they are unclear about the terms tenths, hundredths, thousandths, and ten thousandths, review place value before proceeding with the activity. Consider having students practice by counting by a certain base-ten unit (6 and 8 tenths, 6 and 9 tenths, 7, 7 and 1 tenth. . .), completing place value charts, reading decimals aloud, or by matching a decimal that is read aloud to its written numerical representation.

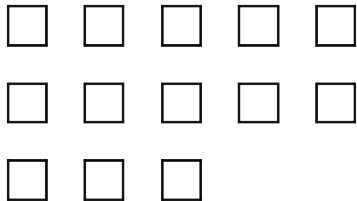
## Student Task Statement

You may be familiar with base-ten blocks that represent ones, tens, and hundreds. Here are some diagrams that we will use to represent base-ten units.

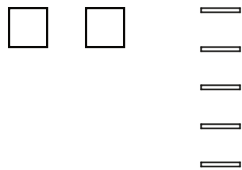
- A large square represents 1 one.
- A medium rectangle represents 1 tenth.
- A medium square represents 1 hundredth.
- A small rectangle represents 1 thousandth.
- A small square represents 1 ten-thousandth.



1. Here is the diagram that Priya drew to represent 0.13. Draw a different diagram that represents 0.13. Explain why both diagrams represent the same number.



2. Here is the diagram that Han drew to represent 0.025. Draw a different diagram that represents 0.025. Explain why both diagrams represent the same number.



3. For each number, draw or describe two different diagrams that represent it.

a. 0.1

b. 0.02

c. 0.004

4. Use diagrams of base-ten units to represent each sum. Think about how you could use as few units as possible to represent each number.

a.  $0.03 + 0.05$

b.  $0.006 + 0.007$

c.  $0.4 + 0.7$

### Student Response

Answers vary. Sample responses:

1. 1 tenth and 3 hundredths or 13 hundredths both represent 0.13. The tenth can be replaced with 10 of the hundredths to match what Priya drew.

2. 25 thousandths represent 0.025. 20 of the thousandths and can be replaced with the 2 hundredths to match what Han drew.

3. a. 1 tenth or 10 hundredths

b. 2 hundredths or 20 thousandths

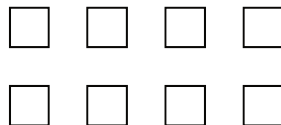
c. 4 thousandths or 40 ten-thousandths

4. a. 8 hundredths, 0.08

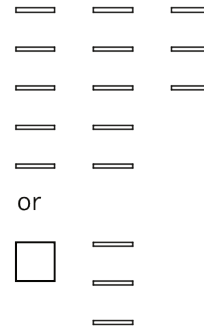
b. 13 thousandths or 1 hundredth and 3 thousandths, 0.013

c. 11 tenths or 1 one and 1 tenth, 1.1

a.



b.



c.

		or	

### Activity Synthesis

Select a few students to share their responses to the questions, or display the correct answers for all to see. Discuss the following questions:

- “Is it always helpful or important to use as few base-ten figures as possible when representing addition of two numbers?” (Yes: fewer pieces mean less counting and smaller likelihood of making a counting mistake. No: once the pieces are bundled, it is harder to see the two numbers that make up the sum.)
- “When can you bundle a base-ten figure?” (When there are at least 10 of a unit.)
- “What would a figure that represents 10 look like?” (An extra-large rectangle, composed of 10 big squares.) “What about 100?” (A giant square made from 10 of the extra-large rectangles representing 10.)
- “Why might using base-ten diagrams for addition be cumbersome with larger multi-digit numbers?” (We would have to draw a lot of squares and rectangles. When there are more units, the diagrams become more complex.)

## 2.3 Finding Sums in Different Ways

**Optional: 15 minutes (there is a digital version of this activity)**

In this activity, students use symbols and diagrams to find a sum that requires regrouping of base-ten units. Use this activity to give students more explicit instruction on how to bundle smaller units into a larger one and additional practice on using addition algorithm to add decimals.

Again, consider having physical base-ten blocks, a paper version of the base-ten figures, or this digital applet [ggbm.at/n9yaWPQj](https://ggbm.at/n9yaWPQj) available as alternatives to diagram drawing, or to more concretely illustrate the idea of bundling and unbundling.

### Building On

- 5.NBT.B.7



## Addressing

- 6.NS.B.3

## Instructional Routines

- MLR7: Compare and Connect

## Launch

Arrange students in groups of 2. Give groups 4–5 minutes to discuss and answer the first question, and then 7–8 minutes of quiet time to complete the remaining questions. Provide access to base-ten representations, if needed.



Classes using the digital activities have an interactive applet with virtual blocks. Note that the applet can only represent ones, tenths, and hundredths, so the pieces are large enough to manipulate. To use the bundling and unbundling features, the pieces must be aligned on the light blue grids. To bring a piece into the workspace, select one of the green tool icons and then click on the workspace. To move it, you must click on the Move tool

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### Access for Students with Disabilities

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.

*Supports accessibility for: Organization; Attention*

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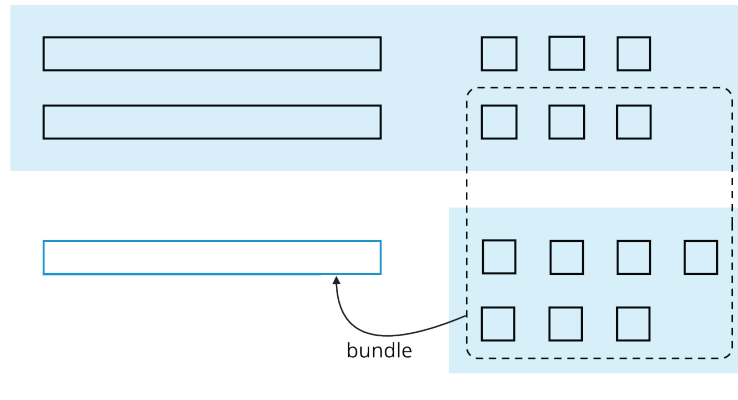
## Anticipated Misconceptions

If students have difficulty drawing the diagrams to represent bundling, it might be helpful for them to work with actual base-ten blocks, paper printouts of base-ten blocks, or digital base-ten blocks so that they can physically trade 10 hundredths for 1 tenth or 10 tenths for 1 whole.

### Student Task Statement

1. Here are two ways to calculate the value of  $0.26 + 0.07$ . In the diagram, each rectangle represents 0.1 and each square represents 0.01.

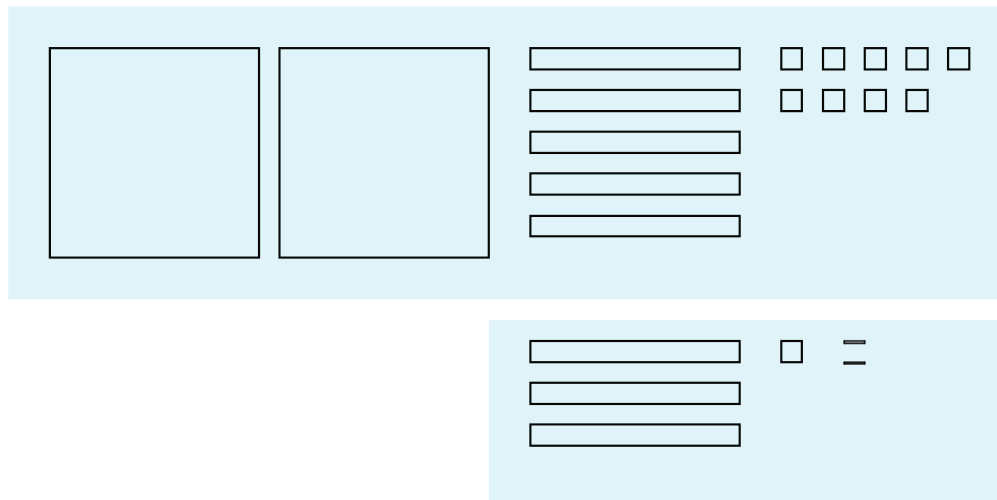
$$\begin{array}{r}
 \phantom{0.}1 \\
 0.26 \\
 + 0.07 \\
 \hline
 0.33
 \end{array}$$



Use what you know about base-ten units and addition to explain:

- Why ten squares can be “bundled” into a rectangle.
  - How this “bundling” is represented in the vertical calculation.
- Find the value of  $0.38 + 0.69$  by drawing a diagram. Can you find the sum without bundling? Would it be useful to bundle some pieces? Explain your reasoning.
  - Calculate  $0.38 + 0.69$ . Check your calculation against your diagram in the previous question.
  - Find each sum. The larger square represents 1.

a.

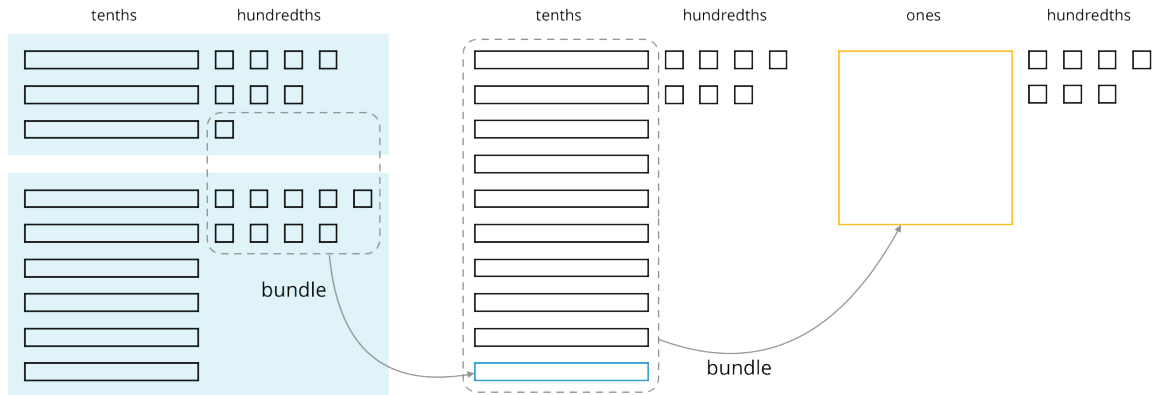


b.

$$\begin{array}{r}
 6.03 \\
 + 0.098 \\
 \hline
 \end{array}$$

## Student Response

- Ten squares can be bundled into a rectangle because the squares each represent  $\frac{1}{100}$ , and the rectangles represent  $\frac{1}{10}$ . There are ten hundredths in a tenth.
  - In the computation, the 7 hundredths from 0.07 are combined with 3 of the hundredths from 0.26 to make a tenth.
- $0.38 + 0.69 = 1.07$



$$\begin{array}{r}
 1 \ 1 \\
 0.38 \\
 + 0.69 \\
 \hline
 1.07
 \end{array}$$

- 
- 
- 
- 2.902
  - 6.128

### Are You Ready for More?

A distant, magical land uses jewels for their bartering system. The jewels are valued and ranked in order of their rarity. Each jewel is worth 3 times the jewel immediately below it in the ranking. The ranking is red, orange, yellow, green, blue, indigo, and violet. So a red jewel is worth 3 orange jewels, a green jewel is worth 3 blue jewels, and so on.

- If you had 500 violet jewels and wanted to trade so that you carried as few jewels as possible, which jewels would you have?
- Suppose you have 1 orange jewel, 2 yellow jewels, and 1 indigo jewel. If you're given 2 green jewels and 1 yellow jewel, what is the fewest number of jewels that could represent the value of the jewels you have?

## Student Response

- 2 orange, 1 blue, 1 indigo, 2 violet

2. 2 orange, 2 green, 1 indigo

### Activity Synthesis

Focus the whole-class debriefing on the idea of choosing appropriate tools to solve a problem, which is an important part of doing mathematics (MP5). Highlight how drawings can effectively help us understand what is happening when we add base-ten numbers before moving on to a more generalized method. Discuss:

- “In which place(s) did bundling happen when adding 0.38 and 0.69?” (In the hundredth and tenth places.) “Why?” (There is a total of 17 hundredths, and 10 hundredths can be bundled to make 1 tenth. This 1 tenth is added to the 3 tenths and 6 tenths, which makes 10 tenths. Ten tenths can be bundled into 1 one.)
- “How can the bundling process be represented in vertical calculations?” (We can show that the 8 hundredths and 9 hundredths make 1 tenth and 7 hundredths by recording 7 hundredths and writing a 1 above the 3 tenths in 0.38.)
- “Which method of calculating is more efficient?” (It depends on the complexity and size of the numbers. The drawings become hard when there are lots of digits or when the digits are large. The algorithm works well in all cases, but it is more abstract and requires that all bundling be recorded in the right places.)

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### Access for English Language Learners

*Listening, Speaking, Representing: MLR7 Compare and Connect.* Use this routine when students discuss different ways to calculate and represent decimal sums. Display three different representations for calculating the sum of 6.03 and 0.098 (using base-ten blocks, diagrams, and vertically). Ask students to identify where the bundling occurred and how it is shown in each representation. Draw students’ attention to how the action of bundling is represented. For instance, in the base-ten blocks the pieces are physically joined or traded, in the diagram the hundredths are circled, and in the vertical calculation the “1” is notated. Emphasize the mathematical language used to make sense of the different ways to represent bundling. These exchanges strengthen students’ mathematical language use and reasoning.

*Design Principle(s): Support sense-making; Maximize meta-awareness*

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## 2.4 Representing Subtraction

**Optional: 15 minutes**

In this activity, students use base-ten diagrams and vertical calculations to perform subtraction. As with addition of decimals, students need to pay close attention to place value when calculating differences. They identify the need to pair the digits of like base-ten units when subtracting decimals and why it is helpful to line up the decimal points.

There is no decomposition or “unbundling” of a base-ten unit into smaller units in this activity, that will come up in the next activity.

As in previous activities, consider having students use physical base-ten blocks (if available), a paper version of the base-ten diagrams (from the blackline master), or this digital applet <https://ggbm.at/n9yaWPQj>, as alternatives to drawing diagrams.

### Building On

- 5.NBT.B.7

### Addressing

- 6.NS.B.3

### Instructional Routines

- Think Pair Share

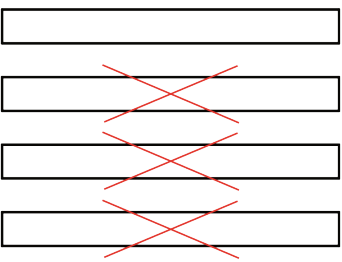
### Launch

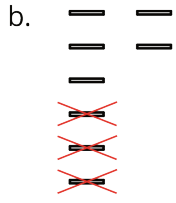
Arrange students in groups of 2. Review that the term “difference” means the result of a subtraction. Tell students that in this lesson they will use base-ten diagrams to determine differences and, in the diagrams, use X’s to indicate what is being taken away.

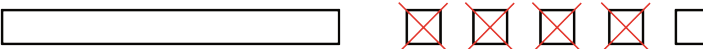
Give students 10 minutes of quiet work time. Ask them to pause after the third question, share their diagrams and calculations with a partner, and resolve any differences before finishing the activity.

### Student Task Statement

- Here are diagrams that represent differences. Removed pieces are marked with Xs. The larger rectangle represents 1 tenth. For each diagram, write a numerical subtraction expression and determine the value of the expression.

a. 

b. 

c. 

- Express each subtraction in words.

a.  $0.05 - 0.02$

b.  $0.024 - 0.003$

c.  $1.26 - 0.14$

3. Find each difference by drawing a diagram and by calculating with numbers. Make sure the answers from both methods match. If not, check your diagram and your numerical calculation.

a.  $0.05 - 0.02$

b.  $0.024 - 0.003$

c.  $1.26 - 0.14$

### Student Response

1. a.  $0.4 - 0.3 = 0.1$

b.  $0.008 - 0.003 = 0.005$

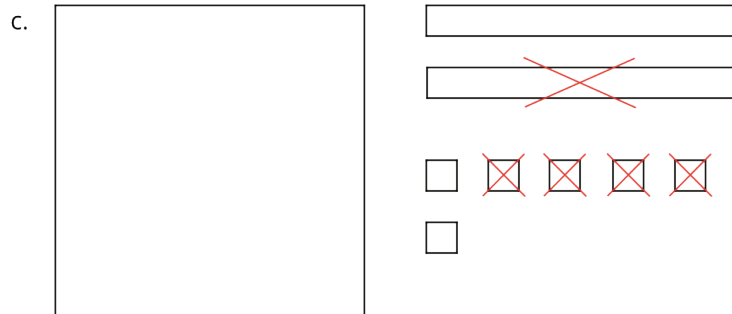
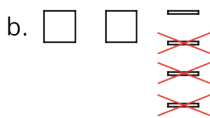
c.  $0.15 - 0.04 = 0.11$

2. Answers vary. Possible responses:

a. five hundredths minus two hundredths

b. the difference between twenty four thousandths and three thousandths

c. subtract fourteen hundredths from one and twenty six hundredths



3.

a.

$$\begin{array}{r} 0.05 \\ - 0.02 \\ \hline 0.03 \end{array}$$

b.

$$\begin{array}{r} 0.024 \\ - 0.003 \\ \hline 0.021 \end{array}$$

c.

$$\begin{array}{r} 1.26 \\ - 0.14 \\ \hline 1.12 \end{array}$$

### Activity Synthesis

The goal of the whole-class discussion is to make sure students understand that when we perform subtraction without diagrams, it is essential to pay close attention to place value in the numbers. Select a few students to share their responses and reasonings for the last two questions. Highlight how the different sizes of the base-ten units in the diagram informs how we subtract one decimal from another. Then, discuss:

- How are addition and subtraction of decimal numbers similar? (It is important to attend to place value and to add or subtract numbers that represent the same base-ten units.)
- Did anyone find different results when using diagrams versus when calculating vertically? If so, where did the error happen and what might have caused it?
- Why is it helpful to line up the decimal points when calculating differences of decimals? (Aligning the points helps us align digits with the same place value.)
- Which is more efficient, using base-ten blocks or calculating the difference? (For some numbers, such as  $1.26 - 0.14$ , both methods are efficient. If the numbers contain more decimal places or larger digits, the diagrams would take a lot of time to draw.)

## Lesson Synthesis

One main idea in this lesson is that addition of decimals beyond hundredths works the same way as addition of whole numbers and decimals up to hundredths: all of them rely on combining the values of like base-ten units. Another main idea is bundling: we can group 10 of any base-ten unit into 1 of a base-ten unit that is 10 times as large. The methods we used for adding in the lesson reflect both ideas.

- How do the pieces representing ones, tenths, hundredths, etc. of a base-ten diagram help us add two decimals? (We can combine the pieces that represent the same unit and see the value for each decimal place.)
- When might we want to bundle some of the base-ten pieces? (When we have at least 10 of the same unit.) Why? (It would make it simpler to show or tell the sum.)
- How is adding with vertical calculations similar to and different from using base-ten diagrams? (We still combine numbers based on their place values but without drawing figures to represent each number.)
- When using vertical calculations, how do we make sure that we add like base-ten units? (We line up digits that represent the same place value or line up the decimal point.)
- Which method of calculating is more efficient? (It depends on the size of the numbers, but vertical calculations tend to be quicker. Drawing becomes hard when the numbers have lots of digits, e.g.,  $2.315641$ , or when the digits are large, e.g.,  $9.999$ .)

## 2.5 Why or Why Not?

Cool Down: 5 minutes

### Addressing

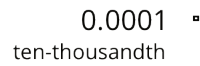
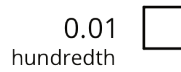
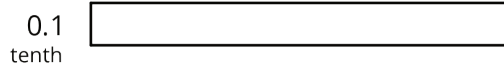
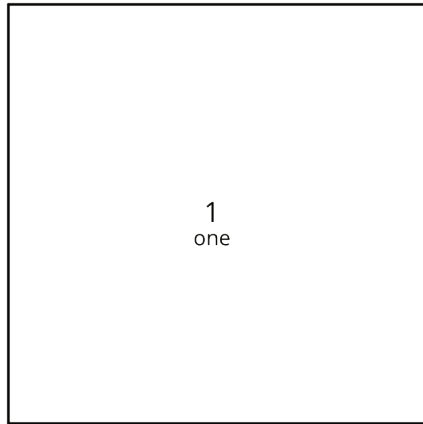
- 6.NS.B.3

### Student Task Statement

Is this equation true?

$$0.025 + 0.17 = 0.042$$

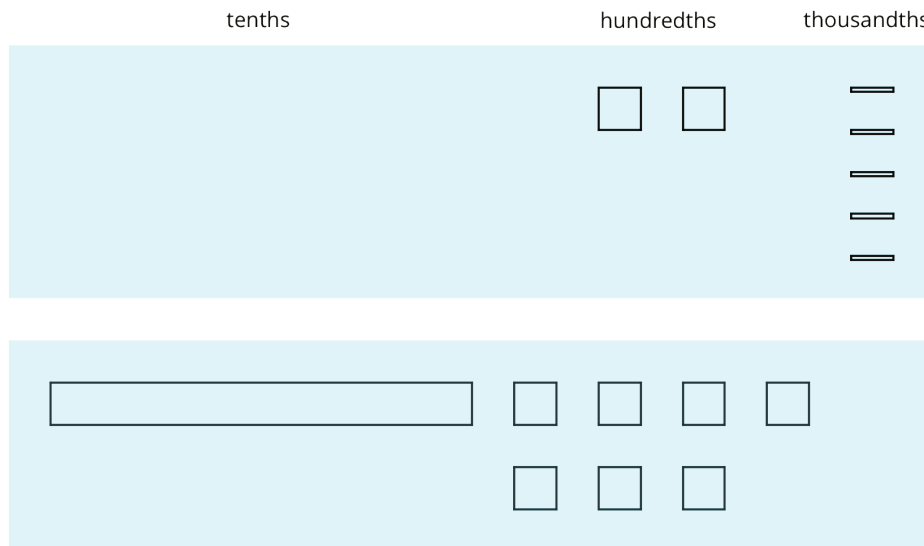
Use a diagram or numerical calculation to explain or show your reasoning. Here are diagrams that you could use to represent base-ten units.



### Student Response

The equation is not true. Sample reasoning:

- First, 0.17 is larger than 0.042, so 0.042 cannot be the sum of 0.17 and another decimal.
- The diagram should show 1 medium rectangle (1 tenth), 9 medium squares (9 hundredths), and 5 small rectangles (5 thousandths).



- $0.025 + 0.17 = 0.02 + 0.005 + 0.1 + 0.07 = 0.125 + 0.07 = 0.195$ .
- Calculation with numbers should show the decimal points lining up and a sum of 0.195.

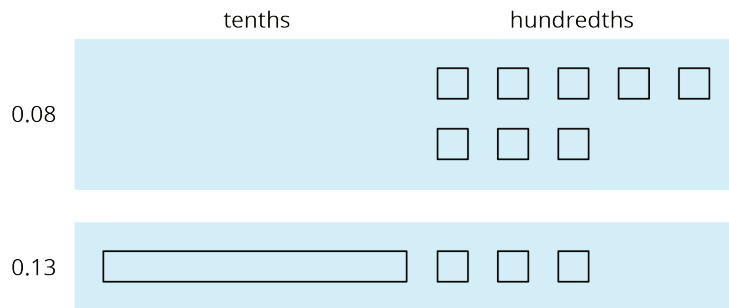


$$\begin{array}{r}
 0.025 \\
 + 0.170 \\
 \hline
 0.195
 \end{array}$$

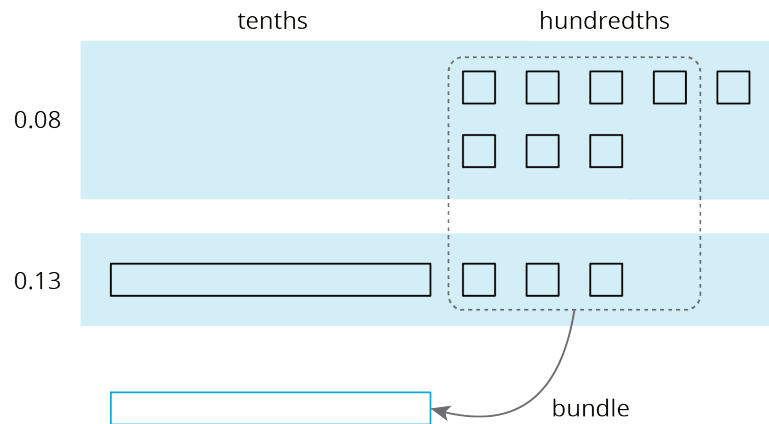
## Student Lesson Summary

Base-ten diagrams represent collections of base-ten units—tens, ones, tenths, hundredths, etc. We can use them to help us understand sums of decimals.

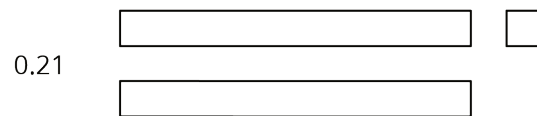
Suppose we are finding  $0.08 + 0.13$ . Here is a diagram where a square represents 0.01 and a rectangle (made up of ten squares) represents 0.1.



To find the sum, we can “bundle” (or compose) 10 hundredths as 1 tenth.



We now have 2 tenths and 1 hundredth, so  $0.08 + 0.13 = 0.21$ .

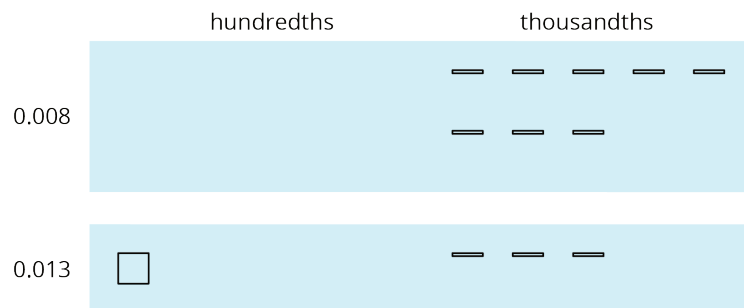


We can also use vertical calculation to find  $0.08 + 0.13$ .

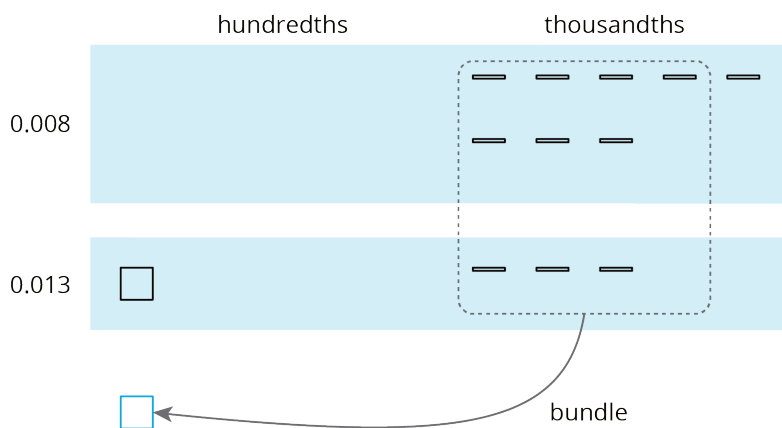
$$\begin{array}{r}
 \phantom{0.}1\phantom{3} \\
 0.1\phantom{3} \\
 + 0.0\phantom{8} \\
 \hline
 0.2\phantom{1}
 \end{array}$$

Notice how this representation also shows 10 hundredths are bundled (or composed) as 1 tenth.

This works for any decimal place. Suppose we are finding  $0.008 + 0.013$ . Here is a diagram where a small rectangle represents 0.001.



We can "bundle" (or compose) 10 thousandths as 1 hundredth.



The sum is 2 hundredths and 1 thousandth.

$$\begin{array}{r}
 \phantom{0.}02\phantom{1} \\
 0.02\phantom{1} \\
 \phantom{0.}02\phantom{1} \\
 \phantom{0.}02\phantom{1} \\
 \phantom{0.}02\phantom{1}
 \end{array}$$

Here is a vertical calculation of  $0.008 + 0.013$ .

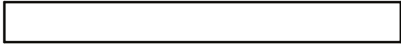
$$\begin{array}{r}
 \phantom{0.}0\phantom{1}\phantom{3} \\
 0.0\phantom{1}\phantom{3} \\
 + 0.0\phantom{0}\phantom{8} \\
 \hline
 0.0\phantom{2}\phantom{1}
 \end{array}$$


# Lesson 2 Practice Problems

## Problem 1


### Statement

Use the given key to answer the questions.

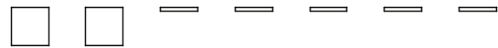
0.1  
tenth 

0.01  
hundredth 

0.001  
thousandth 

0.0001  
ten-thousandth 

a. What number does this diagram represent?

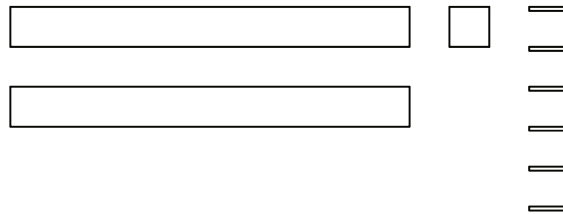


b. Draw a diagram that represents 0.216.

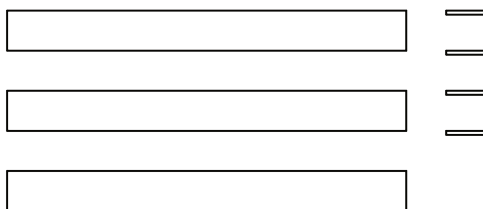
c. Draw a diagram that represents 0.304.

### Solution

a. 0.025



b.

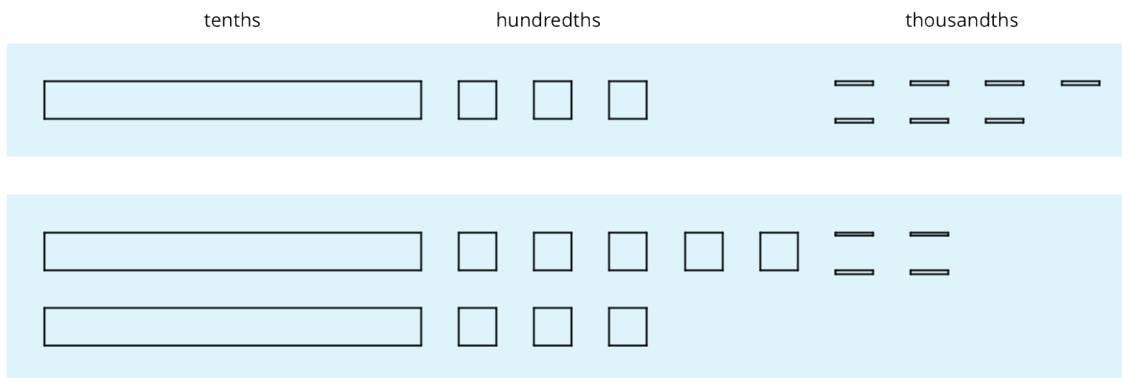


c.

## Problem 2

### Statement

Here are diagrams that represent 0.137 and 0.284.



a. Use the diagram to find the value of  $0.137 + 0.284$ . Explain your reasoning.

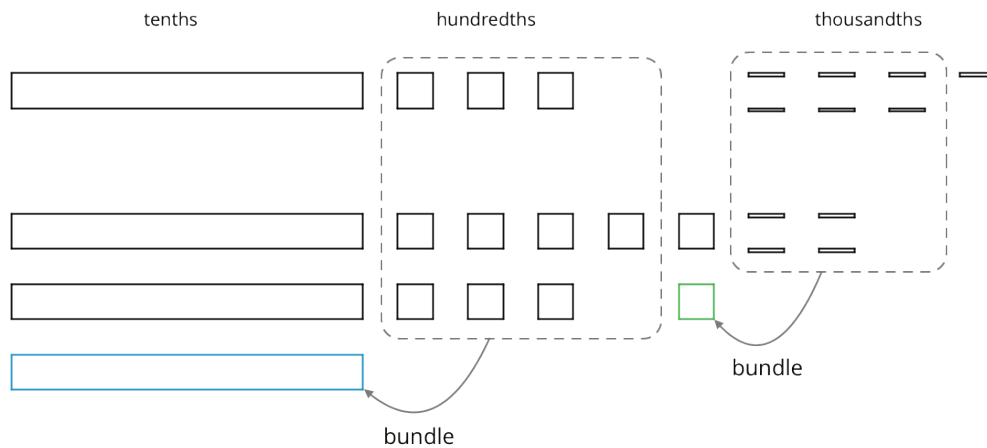
b. Calculate the sum vertically.

$$\begin{array}{r}
 0.137 \\
 + 0.284 \\
 \hline
 \begin{array}{|c|c|c|c|}
 \hline
 & . & & & \\
 \hline
 \end{array}
 \end{array}$$

c. How was your reasoning about  $0.137 + 0.284$  the same with the two methods? How was it different?

## Solution

a.



b.

$$\begin{array}{r}
 0.137 \\
 + 0.284 \\
 \hline
 0.421
 \end{array}$$

c. Responses vary. Sample response: Using the diagrams, 10 thousandths can be bundled to make 1 hundredth. Then 10 hundredths can be bundled to make 1 tenth. These values can then be combined. Without diagrams, 10 of the thousandths can be converted into 1 hundredth and 10 of the hundredths to 1 tenth. The methods are similar. The diagrams show the bundling, but the method without a diagram is faster.

### Problem 3

#### Statement

For the first two problems, circle the vertical calculation where digits of the same kind are lined up. Then, finish the calculation and find the sum. For the last two problems, find the sum using vertical calculation.

a.  $3.25 + 1$

$$\begin{array}{r} 3.25 \\ + 1.0 \\ \hline \end{array}$$

$$\begin{array}{r} 3.25 \\ + 1.0 \\ \hline \end{array}$$

$$\begin{array}{r} 3.25 \\ + 1 \\ \hline \end{array}$$

b.  $0.5 + 1.15$

$$\begin{array}{r} 0.5 \\ + 1.15 \\ \hline \end{array}$$

$$\begin{array}{r} 0.5 \\ + 1.15 \\ \hline \end{array}$$

$$\begin{array}{r} 0.50 \\ + 1.150 \\ \hline \end{array}$$

c.  $10.6 + 1.7$

d.  $123 + 0.2$

#### Solution

- The second arrangement is correct. The sum is 4.25.
- The first arrangement is correct. The sum is 1.65.
- 12.3
- 123.2

### Problem 4

#### Statement

Andre has been practicing his math facts. He can now complete 135 multiplication facts in 90 seconds.

- If Andre is answering questions at a constant rate, how many facts can he answer per second?
- Noah also works at a constant rate, and he can complete 75 facts in 1 minute. Who is working faster? Explain or show your reasoning.

## **Solution**

a. 1.5 facts per second ( $135 \div 9 = 1.5$ )

b. Andre is faster, because Noah can only answer 1.25 facts per second. ( $75 \div 60 = 1.25$ )

(From Unit 2, Lesson 9.)