Lesson 6: Methods for Multiplying Decimals

Goals

- Interpret different methods for computing the product of decimals, and evaluate (orally) their usefulness.
- Justify (orally, in writing, and through other representations) where to place the decimal point in the product of two decimals with multiple non-zero digits.

Learning Targets

- I can use area diagrams to represent and reason about multiplication of decimals.
- I know and can explain more than one way to multiply decimals using fractions and place value.

Lesson Narrative

In this lesson, students continue to develop methods for computing products of decimals, including using area diagrams. They multiply decimals by expressing them as fractions, or by interpreting each decimal as a product of a whole number and a power of 10 and $\frac{1}{10}$. To multiply $(0.25) \cdot (1.6)$, for example, students may first multiply 0.25 by 100 and 1.6 by 10 to have whole numbers 25 and 16, multiply the whole numbers to get 400, and then multiply 400 by $\frac{1}{1,000}$ to invert the initial multiplication by 1,000. They may also think of 0.25 and 1.6 as $\frac{25}{100}$ and $\frac{16}{10}$, multiply the fractions, and then express the fractional product as a decimal.

In earlier grades, students used the area of rectangles to represent and find products of whole numbers and fractions. Here they do the same to represent and find products of decimals. They see that a rectangle that represents $4 \cdot 2$, for instance, can also be used to reason about $(0.4) \cdot (0.2)$, $(0.004) \cdot (0.002)$, or $40 \cdot 20$ because they all share a common structure. In this lesson, students extend their understanding of multiplication of fractions and multiplication using area diagrams by using previous methods to multiply any pair of decimals.

Alignments

Building On

- 5.NBT.B.7: Add, subtract, multiply, and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

Addressing

- 6.NS.B: Compute fluently with multi-digit numbers and find common factors and multiples.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR8: Discussion Supports
- Think Pair Share

> **Student Learning Goals**
>
> Let’s look at some ways we can represent multiplication of decimals.

6.1 Equivalent Expressions

Warm Up: 5 minutes
The purpose of this warm-up is to help students recall that in an expression involving multiplication, the product is not affected by the order of the factors.

Building On

- 5.NBT.B.7

Launch

Set a timer for 15 seconds. Ask students to write as many expressions as they can think of that are equal to 0.6 without using addition or subtraction.

The purpose of the timer is to keep the number of unique expressions manageable, since you'll list them all during the activity synthesis. Adjust the duration of the timer if students need more or less time.

> **Student Task Statement**
>
> Write as many expressions as you can think of that are equal to 0.6. Do not use addition or subtraction.

Student Response

Answers vary. Sample responses:

- $6 \cdot 0.1$
- $0.1 \cdot 6$
- $\frac{6}{10}$
- $\frac{1}{10} \cdot 2 \cdot 3$
- $3 \cdot 0.2$

Activity Synthesis

Display for all to see all of the unique expressions that students create. Make sure everyone agrees that all of the expressions equal 0.6.

Make sure students see some expressions that illustrate the commutative property. For example, $0.1 \cdot 6$ and $6 \cdot 0.1$ are both equal to 0.6 because multiplication is commutative.

6.2 Using Properties of Numbers to Reason about Multiplication

20 minutes

This activity continues to develop the two methods for computing products of decimals introduced in the previous lesson. The first method uses the idea that multiplying a number by $\frac{1}{10}$ is the same as dividing the number by 10, multiplying by $\frac{1}{100}$ is the same as dividing by 100, and so on. The second method is to convert decimals to fractions, compute the product, then convert the product to a decimal. Students make sense of both methods and use one to solve a problem. As they continue to work through examples, students begin to notice a relationship between the location of decimal points in the factors and the product.

As they reason about the placement of the decimal and the relationship between decimals and fractions, students use the structure of the base-ten system (MP7). To reason correctly about the products of decimals, they also need to pay close attention to the digits and their place value.

Building On

- 5.NBT.B.7

Addressing

- 6.NS.B

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Arrange students in groups of 2. Give students 5–6 minutes to complete the first set of questions. Ask each student to study one of the two methods and then explain that method to their partner. After making sense of both methods together, each partner applies one method to solve a new problem. After the first question, have students pause for a brief discussion. Invite a student from each camp to share their reasoning. If not already mentioned in students' explanations, ask:

- Why might have Elena multiplied by 0.23 by 100 and 1.5 by 10? (Elena might have multiplied the factors by 100 and 10 to get them into whole numbers, which are easier to multiply.) What might be her reason for dividing 345 by 1,000? (Because she multiplied the original factors by

$(100 \cdot 10)$ or 1,000, so the product 345 is 1,000 times the original product and must therefore be divided by 1,000).

- How is Noah's method different than Elena's? (Noah converted each decimal into fractions and multiplied the fractions.)

Then give students another 7–8 minutes of quiet time to work on the second set of questions.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as "_____'s method makes more sense to me because . . . ."
Supports accessibility for: Language; Organization

Student Task Statement

Elena and Noah used different methods to compute $(0.23) \cdot (1.5)$. Both calculations were correct.

Elena's Method	Noah's Method
$(0.23) \cdot 100 = 23$	$0.23 = \frac{23}{100}$
$(1.5) \cdot 10 = 15$	$1.5 = \frac{15}{10}$
$23 \cdot 15 = 345$	$\frac{23}{100} \cdot \frac{15}{10} = \frac{345}{1,000}$
$345 \div 1,000 = 0.345$	$\frac{345}{1,000} = 0.345$

1. Analyze the two methods, then discuss these questions with your partner.
    - Which method makes more sense to you? Why?
    - What might Elena do to compute $(0.16) \cdot (0.03)$? What might Noah do to compute $(0.16) \cdot (0.03)$? Will the two methods result in the same value?
2. Compute each product using the equation $21 \cdot 47 = 987$ and what you know about fractions, decimals, and place value. Explain or show your reasoning.
    a. $(2.1) \cdot (4.7)$
    b. $21 \cdot (0.047)$
    c. $(0.021) \cdot (4.7)$

Student Response

1. Answers vary. Sample responses:
    - Elena might multiply 0.16 by 100 to get 16 and 0.03 by 100 to get 3, multiply 16 and 3 to get 48, and then divide 48 by 10,000 (because $100 \cdot 100 = 10{,}000$). Noah might write 0.16 as $\frac{16}{100}$ and 0.03 as $\frac{3}{100}$, and then multiply $\frac{16}{100} \cdot \frac{3}{100}$ to get $\frac{48}{10{,}000}$. Both methods would result in 0.0048.

2.
    a. 9.87. Multiply each of the original factors by 10 (making a product of 100), and then divide 987 by 100. $987 \div 100 = 9.87$.

    b. 0.987. Multiply the second original factor by 1,000, then divide 987 by 1,000. $987 \div 1{,}000 = 0.987$.

    c. 0.0987. Multiply the original factors by 1,000 and 10 (making a product of $10{,}000$), so then divide 987 by $10{,}000$. $987 \div 10{,}000 = 0.0987$.

Activity Synthesis

Invite a couple of students to summarize the two different methods used in this activity. Poll the class to see which method they used in the second question. Ask if they preferred one method over the other and, if so, which one and why.

It is important students understand that the two methods presented here are mathematically equivalent. In Noah's method, the product of the numerators of his fractions (23 and 15) is the same as the product of Elena's whole numbers. Both of them then move the decimal point three places to the left because they need to divide by 1,000. For Noah, 1,000 is the denominator of his fraction. For Elena, the division by 1,000 reverts the initial multiplication by 1,000 she had performed so she could have whole-number factors.

Access for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. Use this routine to help students improve their writing, by providing them with multiple opportunities to clarify their explanations through conversation. Give students time to meet with 2–3 partners, to share and get feedback on their responses. Display questions for students to ask their partners such as, "Can you say more about why you think Elena do that?" and "Why do you think Noah would do that?" Give students 1–2 minutes to revise their writing based on the feedback they received. *Design Principle(s): Optimize output (for explanation); Maximize meta-awareness*

6.3 Using Area Diagrams to Reason about Multiplication

Optional: 15 minutes

Students have used area diagrams to reason about multiplication of whole numbers and fractions in previous grades. This task prepares them to use area diagrams to find products of decimals in upcoming lessons. In addition to using the structure of base-ten numbers in their reasoning, students also use the structure of the area diagram to help them find products of decimals (MP7). This activity illustrates how students' previous understandings of multiplication using area diagrams can be applied to the multiplication of any pair of decimals.

As students work, look and listen for different ways students might reason about the area of each unit square and of the large rectangle given particular side lengths. Identify a few students with correct reasoning so they can share later.

Building On

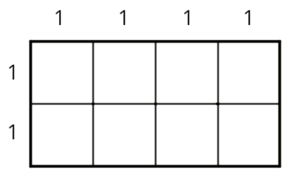
- 5.NBT.B.7

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Display this image for all to see.



Ask students to consider a rectangle composed of 8 squares. If the side length of each square is 1 unit, as shown:

- What is the area of each square? (The area of each square is 1 square unit, because $1 \cdot 1 = 1$.)
- What is the area of the rectangle? (The area of the rectangle is 8 square units, because it is made up of 8 squares and $8 \cdot 1 = 8$.)
- How can we express the area of the rectangle in terms of its length and width? (The area of the rectangle can also be expressed as $4 \cdot 2$.)

After this discussion, give students 5 minutes of quiet work time and 2–3 minutes to discuss their responses with a partner. Follow with a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, present one question at a time and monitor students to ensure they are making progress throughout the activity.
Supports accessibility for: Organization; Attention

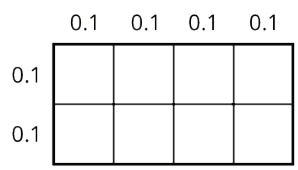
Anticipated Misconceptions

Some students might think that $(0.1) \cdot (0.1) = 0.1$ (just like $1 \cdot 1 = 1$). If this happens, have them write 0.1 in fraction form so they see that $\frac{1}{10} \cdot \frac{1}{10} = \frac{1}{100}$ or 0.01.

Student Task Statement

1. In the diagram, the side length of each square is 0.1 unit.

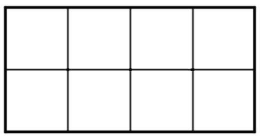
    a. Explain why the area of each square is *not* 0.1 square unit.

    b. How can you use the area of each square to find the area of the rectangle? Explain or show your reasoning.

    c. Explain how the diagram shows that the equation $(0.4) \cdot (0.2) = 0.08$ is true.

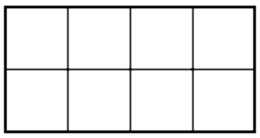
2. Label the squares with their side lengths so the area of this rectangle represents $40 \cdot 20$.

    a. What is the area of each square?

    b. Use the squares to help you find $40 \cdot 20$. Explain or show your reasoning.

3. Label the squares with their side lengths so the area of this rectangle represents $(0.04) \cdot (0.02)$.

    Next, use the diagram to help you find $(0.04) \cdot (0.02)$. Explain or show your reasoning.

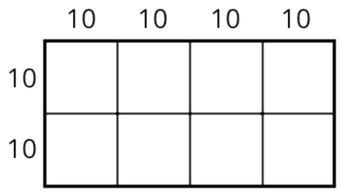
Student Response

1. a. The area is not 0.1 square unit because $(0.1) \cdot (0.1)$ is 1 tenth times 1 tenth, which is 1 hundredth or 0.01 square units.

   b. There are 8 squares in the rectangle, so the area of the rectangle is $8 \cdot (0.01) = 0.08$ square units.

   c. The rectangle has side lengths of 0.4 and 0.2. The area of a rectangle is its length times its width, so the area of this rectangle is $(0.4) \cdot (0.2)$. We found out earlier that the area of the rectangle is 0.08 square units, so $(0.4) \cdot (0.2) = 0.08$.

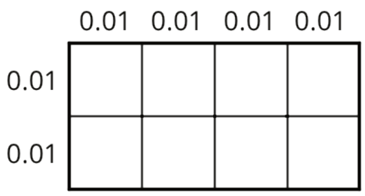
2. The side length of each square is 10 units.



   a. The area of each square is $(10 \cdot 10)$ or 100 square units.

   b. $40 \cdot 20 = 800$. Sample reasoning: There are 8 squares in the rectangle, so the area of the rectangle is $8 \cdot 100$ or 800 square units. Multiplying the side lengths of the rectangle, which are 40 units and 20 units, gives an area of 800 square units.

3. The side length of each square is 0.01 unit. $(0.04) \cdot (0.02) = 0.0008$. Sample reasoning: The area of each square is $(0.01) \cdot (0.01)$ or 0.0001 square units. There are 8 squares in the rectangles, so the area of the rectangle is $8 \cdot (0.0001)$ square units, which is 0.0008 square units.



Activity Synthesis

Select previously identified students to share their explanations and ask the class for agreement and disagreement.

Point out how the situation described in this task arises whenever we want to express an area given in one unit of measurement in terms of a different unit of measurement. Display for all to see a 2 by 4 rectangle with the side length of squares marked as 1 cm. After giving quiet think time, invite 1–2 students to explain their thinking for the following kinds of questions:

- If the sides of each small square are 1 cm by 1 cm, what is its area in cm^2? (1). What is the area of the rectangle in cm^2? (8)

- If the sides of each small square are 1 cm by 1 cm, what is the area of each small square in mm^2? (100). What is the area of the full rectangle in mm^2? (800)
- If the dimensions of each small square are 1 cm by 1 cm, what is the area of each small square in m^2? (There are 100 cm in a meter.)
  ($\frac{1}{100} \cdot \frac{1}{100} = \frac{1}{10,000}$).
- What is the area of the full rectangle in m^2? ($\frac{8}{10,000}$, or 0.0008)

Also point out that area diagrams are similar to the pieces in a base-ten diagram in that they can represent different values. Just as the same collection of base-ten figures can represent 103 or 0.103 (or many other numbers), so can the area of a rectangle composed of small-sized squares represent many different products. Students will continue to use area diagrams to multiply in the next lesson.

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. After each student shares, provide the class with the following sentence frames to help them respond: "I agree because . . .” or "I disagree because . . . .” If necessary, revoice student ideas to demonstrate mathematical language, and invite students to chorally repeat phrases that include relevant vocabulary in context.
Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, we saw additional ways to find the product of decimals: by converting the decimals to fractions and multiplying the fractions, and by using the area of a rectangle to represent multiplication.

- How can changing decimals to fractions help to find decimal products? (Writing the decimals as fractions allows us to use multiplication and division of whole numbers. It also tells us the size of the decimals relative to powers of $\frac{1}{10}$. Both kinds of information allow us to find the products.)
- How can an area diagram represent decimal products? (If the side lengths of a rectangle represent two factors, then the area of the rectangle represents the product of those factors. We can specify the unit of length to match that of the decimals, find the area of one unit square, and use the area of each unit square to find the area of the rectangle.)

6.4 Finding Products of Decimals

Cool Down: 5 minutes

Addressing

- 6.NS.B

Student Task Statement

1. Use the equation $135 \cdot 42 = 5{,}670$ and what you know about fractions, decimals, and place value to explain how to place the decimal point when you compute $(1.35) \cdot (4.2)$.

2. Which of the following is the correct value of $(0.22) \cdot (0.4)$? Show your reasoning.

   a. 8.8

   b. 0.88

   c. 0.088

   d. 0.0088

Student Response

1. $(1.35) \cdot (4.2) = 5.67$, because $135 \cdot 42 \cdot (0.01) \cdot (0.1) = (5{,}670) \cdot (0.001) = 5.67$.

2. C: 0.088. Sample explanations:
   - 0.22 is $\frac{22}{100}$ and 0.4 is $\frac{4}{10}$. The product is 0.088 because $\frac{22}{100} \cdot \frac{4}{10} = \frac{88}{1{,}000}$, and $\frac{88}{1{,}000}$ is 0.088.
   - 0.22 is $22 \cdot \frac{1}{100}$ and 0.4 is $4 \cdot \frac{1}{10}$, so $(0.22) \cdot (0.4) = 22 \cdot 4 \cdot \frac{1}{100} \cdot \frac{1}{10}$. Multiplying the whole numbers and the decimals gives: $88 \cdot \frac{1}{1{,}000}$, which equals 0.088.

Student Lesson Summary

Here are three other ways to calculate a product of two decimals such as $(0.04) \cdot (0.07)$.

- First, we can multiply each decimal by the same power of 10 to obtain whole-number factors.

  $$(0.04) \cdot 100 = 4$$

  $$(0.07) \cdot 100 = 7$$

  $$4 \cdot 7 = 28$$

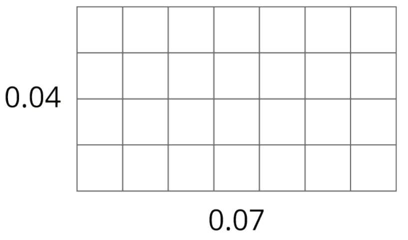
  Because we multiplied both 0.04 and 0.07 by 100 to get 4 and 7, the product 28 is $(100 \cdot 100)$ times the original product, so we need to divide 28 by 10,000.

  $$28 \div 10{,}000 = 0.0028$$

- Second, we can write each decimal as a fraction, $0.04 = \frac{4}{100}$ and $0.07 = \frac{7}{100}$, and multiply them.

  $$\frac{4}{100} \cdot \frac{7}{100} = \frac{28}{10{,}000} = 0.0028$$

- Third, we can use an area model. The product $(0.04) \cdot (0.07)$ can be thought of as the area of a rectangle with side lengths of 0.04 unit and 0.07 unit.



In this diagram, each small square is 0.01 unit by 0.01 unit. The area of each square, in square units, is therefore $\left(\frac{1}{100} \cdot \frac{1}{100}\right)$, which is $\frac{1}{10,000}$.

Because the rectangle is composed of 28 small squares, the area of the rectangle, in square units, must be:

$$28 \cdot \frac{1}{10,000} = \frac{28}{10,000} = 0.0028$$

All three calculations show that $(0.04) \cdot (0.07) = 0.0028$.

Lesson 6 Practice Problems

Problem 1

Statement

Find each product. Show your reasoning.

a. $(1.2) \cdot (0.11)$

b. $(0.34) \cdot (0.02)$

c. $120 \cdot (0.002)$

Solution

a. 0.132. Sample reasoning: 1.2 is a tenth of 12 and 0.11 is a hundredth of 11, so the product of 1.2 and 0.11 is a thousandth of $12 \cdot 11$ or $\frac{1}{1,000} \cdot 132$, which is 0.132.

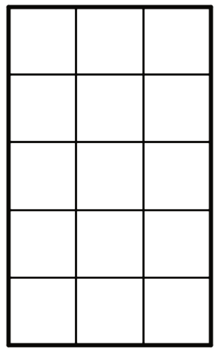
b. 0.0068. Sample reasoning: $\frac{34}{100} \cdot \frac{2}{100} = \frac{68}{1,000}$ or 0.0068.

c. 0.24. Sample reasoning: 0.002 is 2 thousandths or $2 \cdot \frac{1}{1,000}$, so the product of 120 and 0.002 is $120 \cdot 2 \cdot \frac{1}{1,000}$, which equals $\frac{240}{1,000}$ or 0.24.

Problem 2

Statement

You can use a rectangle to represent $(0.3) \cdot (0.5)$.



a. What must the side length of each square represent for the rectangle to correctly represent $(0.3) \cdot (0.5)$?

b. What area is represented by each square?

c. What is $(0.3) \cdot (0.5)$? Show your reasoning.

Solution

a. 0.1

b. 0.01 square units

c. The area is 0.15 because there are 15 squares, and $15 \cdot (0.01) = 0.15$.

Problem 3

Statement

One gallon of gasoline in Buffalo, New York costs \$2.29. In Toronto, Canada, one liter of gasoline costs \$0.91. There are 3.8 liters in one gallon.

a. How much does one gallon of gas cost in Toronto? Round your answer to the nearest cent.

b. Is the cost of gas greater in Buffalo or in Toronto? How much greater?

Solution

a. \$. $(3.8) \cdot (0.91) = 3.458$, and this is closer to 3.46 than to 3.45.

b. The cost of one gallon of gas is \$ more in Toronto.

Problem 4

Statement

Calculate each sum or difference.

$10.3 + 3.7$ $\quad$ $20.99 - 4.97$ $\quad$ $15.99 + 23.51$ $\quad$ $1.893 - 0.353$

Solution

a. 14

b. 16.02

c. 39.5

d. 1.54

(From Unit 5, Lesson 2.)

Problem 5

Statement

Find the value of $\frac{49}{50} \div \frac{7}{6}$ using any method.

Solution

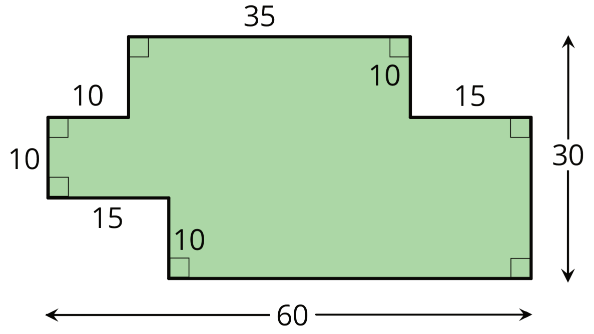
$\frac{21}{25}$ (or equivalent)

(From Unit 4, Lesson 11.)

Problem 6

Statement

Find the area of the shaded region. All angles are right angles. Show your reasoning.



Solution

1,400 square units. Reasoning varies. Sample reasoning: The region can be enclosed with a 60-by-30 rectangle, which has an area of 1,800 square units. Three of the corners of that rectangle have a rectangular region removed. The removed areas are 100 square units (upper left), 150 square units (lower left), and 150 square units (upper right). The area of the shaded region, in square units, is $1,800 - (100 + 150 + 150)$ or $1,800 - 400$, which is 1,400.

(From Unit 1, Lesson 1.)

Problem 7

Statement

a. Priya finds $(1.05) \cdot (2.8)$ by calculating $105 \cdot 28$, then moving the decimal point three places to the left. Why does Priya's method make sense?

b. Use Priya's method to calculate $(1.05) \cdot (2.8)$. You can use the fact that $105 \cdot 28 = 2{,}940$.

c. Use Priya's method to calculate $(0.0015) \cdot (0.024)$.

Solution

a. $1.05 = \frac{1}{100} \cdot 105$ and $2.8 = \frac{1}{10} \cdot 28$, so $(1.05) \cdot (2.8) = \frac{1}{1{,}000} \cdot (105 \cdot 28)$. This is the same as finding $105 \cdot 28$ and then moving the decimal point three places to the left.

b. Since $105 \cdot 28 = 2{,}940$, $(1.05) \cdot (2.8) = 2.940$ because the decimal point in 2,940 moved three places to the left.

c. $15 \cdot 24 = 360$. The decimal needs to be moved 7 places to the left because the decimal point of 0.0015 was moved four places to the right to get 15, and the decimal point of 0.024 was moved three places to the right to get 24. So the answer is 0.0000360.