# Lesson 5: Construction Techniques 3: Perpendicular Lines and Angle Bisectors

• Let's use tools to solve some construction challenges.

### 5.1: Two Circles

Points *A* and *B* are each at the centers of circles of radius *AB*.



- 1. Compare the distance EA to the distance EB. Be prepared to explain your reasoning.
- 2. Compare the distance FA to the distance FB. Be prepared to explain your reasoning.
- 3. Draw line EF and write a conjecture about its relationship with segment AB.



## 5.2: Make It Right

Here is a line  $\ell$  with a point labeled *C*. Use straightedge and compass moves to construct a line perpendicular to  $\ell$  that goes through *C*.



### 5.3: Bisect This

Here is an angle:



- 1. Estimate the location of a point D so that angle ABD is approximately congruent to angle CBD.
- 2. Use compass and straightedge moves to create a ray that divides angle CBA into 2 congruent angles. How close is the ray to going through your point D?



- 3. Take turns with your partner, drawing and bisecting other angles.
  - a. For each angle that you draw, explain to your partner how each straightedge and compass move helps you to bisect it.
  - b. For each angle that your partner draws, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.

#### Are you ready for more?

For thousands of years since the ancient Greeks started playing with straightedge and compass constructions, people strived to find a construction to trisect an arbitrary angle into three equal angles. Many claimed to have found such a construction, but there was always some flaw in their reasoning. Finally, in 1837, Pierre Wantzel used a new field of mathematics to prove it was impossible—which still did not stop some from claiming to have found a construction. If we allow other tools besides just a straightedge and compass, though, it is possible. For example, try this method of using origami (paper folding) to trisect an angle.



Video 'Trisecting an Angle with Origami' available here: https://player.vimeo.com/video/ 298418799.

#### Lesson 5 Summary

We can construct a line that is perpendicular to a given line. We can also bisect a given angle using only a straightedge and compass. The line that bisects an angle is called the **angle bisector**. Both constructions use 2 circles that go through each other's centers:



For the perpendicular line, start by finding 2 points on the line the same distance from the given point. Then create the 2 circles that go through each other's centers. Connect the intersection points of those circles to draw a perpendicular line.

For the angle bisector, start by finding 2 points on the rays the same distance from the vertex. Then create the 2 circles that go through each other's centers. Connect the intersection points of those circles to draw the angle bisector.

In fact, we can think of creating a perpendicular line as bisecting a 180 degree angle!