# Lesson 2: Representing Ratios with Diagrams

## Goals

- Coordinate discrete diagrams and multiple written sentences describing the same ratios.
- Draw and label discrete diagrams to represent situations involving ratios.
- Practice reading and writing sentences describing ratios, e.g., "The ratio of these to those is $a : b$. The ratio of these to those is $a$ to $b$. For every $a$ of these, there are $b$ of those."

## Learning Targets

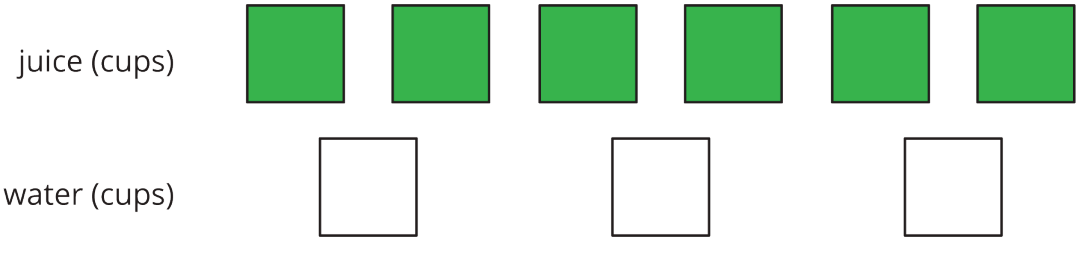
- I can draw a diagram that represents a ratio and explain what the diagram means.
- I include labels when I draw a diagram representing a ratio, so that the meaning of the diagram is clear.

## Lesson Narrative

Students used physical objects to learn about ratios in the previous lesson. Here they use diagrams to represent situations involving ratios and continue to develop ratio language. The use of diagrams to represent ratios involves some care so that students can make strategic choices about the tools they use to solve problems. Both the visual and verbal descriptions of ratios demand careful interpretation and use of language (MP6).

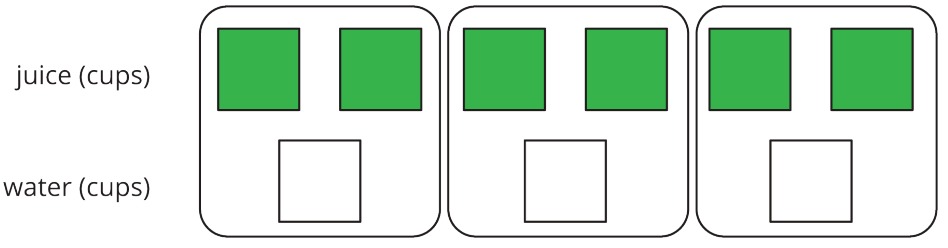
Students should see diagrams as a useful and efficient ways to represent ratios. There is not really a right or wrong way to draw a diagram; what is important is that it represents the mathematics and makes sense to the student, and the student can explain how the diagram is being used. However, a goal of this lesson is to help students draw useful diagrams efficiently.

For example, here is a diagram to show 6 cups of juice and 3 cups of soda water in a recipe.




When students are asked to draw diagrams, they often include unnecessary details such as making each cup look like an actual cup, which makes the diagrams inefficient to use for solving problems. Examples of very simple diagrams help guide students toward more abstract representations while still relying on visual or spatial cues to support reasoning.

Diagrams can also help students see associations between quantities in different ways. For example, we can see there are 2 cups of juice for 1 cup of soda water by grouping the items as shown below.




While students may say "for every 2 cups of juice there is 1 cup of soda," note that for now, we will not suggest writing the association as $2:1$. Equivalent ratios will be carefully developed in upcoming lessons. Diagrams like the one above are referred to as "discrete diagrams" in these materials, but students do not need to know this term. In student-facing materials they are simply called "diagrams."

The discrete diagrams in this lesson are meant to reflect the parallel structure of double number lines that students will learn later in the unit. But for now, students do not need to draw them this way as long as they can explain their diagrams and interpret discrete diagrams like the ones shown in the lesson.

## Alignments

### Building On

- 5.NF.B.3: Interpret a fraction as division of the numerator by the denominator ($a/b = a \div b$). Solve word problems involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by using visual fraction models or equations to represent the problem. For example, interpret $3/4$ as the result of dividing $3$ by $4$, noting that $3/4$ multiplied by $4$ equals $3$, and that when $3$ wholes are shared equally among $4$ people each person has a share of size $3/4$. If $9$ people want to share a $50$-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?

### Addressing

- 6.RP.A.1: Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. For example, "The ratio of wings to beaks in the bird house at the zoo was $2:1$, because for every $2$ wings there was $1$ beak." "For every vote candidate A received, candidate C received nearly three votes."

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR2: Collect and Display
- MLR8: Discussion Supports
- Number Talk
- Take Turns

### Required Materials

**Colored pencils**
**Copies of blackline master**
**Pre-printed slips, cut from copies of the blackline master**

**Tools for creating a visual display**
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

### Required Preparation

For the Card Sort: Spaghetti Sauce activity, make 1 copy of the blackline master for each group of 2 students, plus a few extras. The blackline master shows the correct matches. Keep the extra copies whole to serve as answer keys. Cut up the rest of the slips for students to use, and throw away the cut slips that say "The above diagram also matches this sentence." It may be helpful to copy each group's slips on a different color of paper, so that misplaced slips can quickly be put back.

### Student Learning Goals

Let's use diagrams to represent ratios.

# 2.1 Number Talk: Dividing by 4 and Multiplying by $\frac{1}{4}$

**Warm Up: 10 minutes**
This number talk helps students recall that dividing by a number is the same as multiplying by its reciprocal. Four problems are given, however, they do not all require the same amount of time. Consider spending 6 minutes on the first three questions and 4 on the fourth question.

In grade 4, students multiplied a fraction by a whole number, using their understanding of multiplication as groups of a number as the basis for their reasoning. In grade 5, students multiply fractions by whole numbers, reasoning in terms of taking a part of a part, either by using division or partitioning a whole. In both grade levels, the context of the problem played a significant role in how students reasoned and notated the problem and solution. Two important ideas that follow from this work and that will be relevant to future work should be emphasized during discussions:

- Dividing by a number is the same as multiplying by its reciprocal.
- We can multiply numbers in any order if it makes it easier to find the answer.

### Building On

- 5.NF.B.3

### Instructional Routines

- MLR8: Discussion Supports
- Number Talk

### Launch

Display one problem at a time. Give students 1 minute of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Allow students to share their answers with a partner and note any discrepancies. Pause after the third question and ask, "What do you notice about the first three questions? Do you notice the same thing if we divide 5 by 4? Why?"

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* To support working memory, provide students with sticky notes or mini whiteboards.
*Supports accessibility for: Memory; Organization*

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### Student Task Statement

Find the value of each expression mentally.

$24 \div 4$

$\frac{1}{4} \cdot 24$

$24 \cdot \frac{1}{4}$

$5 \div 4$

### Student Response

- $24 \div 4 = 6$; Possible strategies: Divide 24 into 4 equal groups or know that $4 \cdot 6 = 24$.
- $\frac{1}{4} \cdot 24 = 6$; Possible strategies: Divide 24 into 4 equal groups or know that $4 \cdot 6 = 24$.
- $24 \cdot \frac{1}{4} = 6$; Possible strategies: Divide 24 into 4 equal groups or know that $4 \cdot 6 = 24$ or Commutative Property from the second question.
- $5 \div 4 = \frac{5}{4}$ or equivalent; Possible strategies: Distributive Property $(4 + 1) \div 4 = (4 \div 4) + (1 \div 4)$ or know that $5 \cdot \frac{1}{4} = \frac{5}{4}$.

### Activity Synthesis

Ask students to share what they noticed about the first three problems. Record student explanations that connect dividing by a number with multiplying by its reciprocal. Revisit the meaning of "reciprocal" when the term comes up (or bring it up if it's not mentioned by students). Help students recall that the product of a number and its reciprocal is 1.

Discuss how students could use their observations on the first three questions to divide 5 by 4, and then any two whole numbers.

**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.*: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
*Design Principle(s): Optimize output (for explanation)*

# 2.2 A Collection of Snap Cubes

**Optional: 10 minutes**
Here students read ratio information from a picture and represent it as a diagram. The activity serves two purposes: to reinforce ratio language introduced in the previous lesson, and to better understand the meaning of the term “diagram.”

As students work, check that they use ratios in their sentences and draw appropriate diagrams. Examples of sentences with ratios (from the previous lesson) should be posted in the room. Draw students’ attention to these existing examples as needed.

Look for students who write ratios involving the same values, e.g., “the ratio of blue to white is 1 to 1,” or make note if no one does so. If all examples of ratios students have come across so far involve pairs or sets with different numbers for their values, students may mistakenly conclude that quantities that have the same values cannot be expressed as a ratio.

For non-sighted or color-blind students, this activity can be adapted by giving them blocks of different shapes.

### Addressing

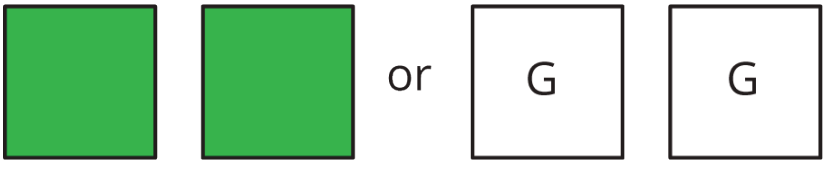
- 6.RP.A.1

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Orient students to the picture (if you have real cubes, use them). Review the meaning of "diagram." For example, to represent two green snap cubes, you might draw two green squares on the board, or two squares labeled "G" if colors are not available.




Arrange students in groups of 2. Provide access to colored pencils.

#### Access for Students with Disabilities

*Representation: Develop Language and Symbols.* Use virtual or concrete manipulatives to connect symbols to concrete objects or values. Provide students with snap cubes or blocks or different shapes.
*Supports accessibility for: Conceptual processing*

### Anticipated Misconceptions

Students might not draw discrete diagrams at first. They might be inclined to draw more detailed drawings. Emphasize that a diagram represents the number and type of objects, and does not need to represent details about the shapes of the snap cubes.

### Student Task Statement

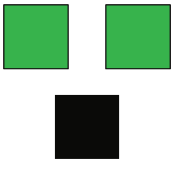
Here is a collection of snap cubes.

1. Choose two of the colors in the image, and draw a diagram showing the number of snap cubes for these two colors.
2. Trade papers with a partner. On their paper, write a sentence to describe a ratio shown in their diagram. Your partner will do the same for your diagram.
3. Return your partner's paper. Read the sentence written on your paper. If you disagree, explain your thinking.

### Student Response

Answers vary. Sample response:

- The ratio of green cubes to black cubes is $2:1$.
- The ratio of black cubes to green cubes is 1 to 2.
- For every two green cubes, there is one black cube.



### Activity Synthesis

Invite one or two pairs of students to share their sentences. Press for details as they explain, asking them to clarify, elaborate, or give examples. Revoice student ideas to demonstrate mathematical language. Discuss whether or not students were able to interpret one another's drawings accurately. If not, what may have led to confusion?

If no one wrote ratios in which all numbers are the same (e.g., 1 to 1, or $3:3$), ask if the following sentence is acceptable and why or why not: "The ratio of green cubes to blue cubes is 2 to 2." If

students suspect that ratios are only used to associate quantities with different values, clarify that this is not the case.

**Access for English Language Learners**

*Representing: MLR8 (Discussion Supports).* Clarify mathematical use of the term "to" (as in 1 to 1 or 2 to 2) highlighting its use to compare the quantities. Listen for any misunderstanding about the use of the word "to".
*Design Principle: Maximize meta-awareness*

# 2.3 Blue Paint and Art Paste

**10 minutes**
In this activity, students continue to draw connections between a diagram and the ratios it represents. Students work in pairs to discuss different ways to use ratio language to describe discrete diagrams. They first identify statements that would correctly describe a given diagram. Then, they create both a diagram and corresponding statements to represent a new situation involving ratio.

As students work, monitor for different ways in which students draw and discuss diagrams of the paste recipe. Identify a few pairs who draw different diagrams and use ratio language differently to share later. A few things to anticipate:

- Some students may draw very literal drawings of cups and pints. Encourage them to use simpler representations.
- Students may choose to draw letters (X's) or other symbols or marks instead of squares and rectangles.
- Students may use equivalent ratios to describe a situation, even though these have not been explicitly taught (e.g., they may say the ratio of cups of flour to pints of water is $4:1$ instead of $8:2$). Though this is correct, be careful here. We have previously regrouped objects and might say, for example, that with a ratio $8:2$, "for every 4 cups of flour there is 1 cup of water," but we have not asserted that this ratio can be written as $4:1$ yet. The idea of equivalent ratios is sophisticated and will be developed over the next several lessons.
- Correct descriptions may include fractions (e.g., for every tablespoon of blue paint, there is $\frac{1}{3}$ cup of white paint). Although students are not expected to work with fractions in this lesson, responses involving fractions are fine.

### Addressing

- 6.RP.A.1

### Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- Group Presentations
- MLR2: Collect and Display

### Launch

Arrange students in groups of 2. Provide them with the tools needed for creating a large visual display for the second part of the task. Ensure students understand they are supposed to select more than one statement for the first question. Consider having students take turns reading each statement and deciding whether they think it describes the situation or not.

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#### Access for English Language Learners

*Speaking, Writing: MLR2 Collect and Display.* Circulate and listen to student talk during partner or group work, and display publicly common or important words and phrases (e.g., for every, the ratio of, for each) students are using. Refer back to this list, and ask students to clarify their meaning, explain how they are useful, and to reflect on which words and phrases help to communicate ideas more precisely. This will provide access to important language for students to use as they are needed.
*Design Principle(s): Support sense-making*

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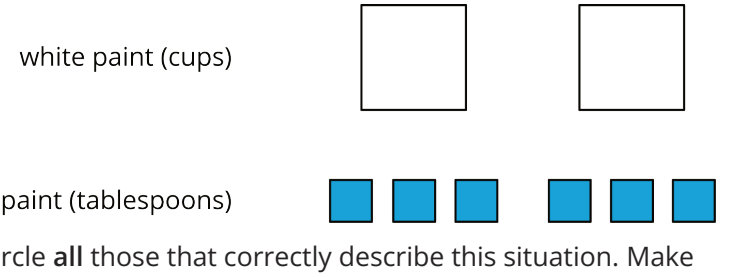
### Anticipated Misconceptions

Some students may think all of the statements about the paint mixture are accurate descriptions. If so, suggest that there are two false statements. Have students discuss the statements again in determining which two are false.

### Student Task Statement

Elena mixed 2 cups of white paint with 6 tablespoons of blue paint.

Here is a diagram that represents this situation.




1. Discuss each statement, and circle **all** those that correctly describe this situation. Make sure that both you and your partner agree with each circled answer.

    a. The ratio of cups of white paint to tablespoons of blue paint is $2 : 6$.

    b. For every cup of white paint, there are 2 tablespoons of blue paint.

c. There is 1 cup of white paint for every 3 tablespoons of blue paint.

d. There are 3 tablespoons of blue paint for every cup of white paint.

e. For each tablespoon of blue paint, there are 3 cups of white paint.

f. For every 6 tablespoons of blue paint, there are 2 cups of white paint.

g. The ratio of tablespoons of blue paint to cups of white paint is 6 to 2.

2. Jada mixed 8 cups of flour with 2 pints of water to make paste for an art project.

   a. Draw a diagram that represents the situation.

   b. Write at least two sentences describing the ratio of flour and water.

### Student Response

1. The following statements describe the paint mixture:
   A. The ratio of cups of white paint to tablespoons of blue paint is $2 : 6$.
   C. There is 1 cup of white paint for every 3 tablespoons of blue paint.
   D. There are 3 tablespoons of blue paint for every cup of white paint.
   F. For every 6 tablespoons of blue paint, there are 2 cups of white paint.
   G. The ratio of tablespoons of blue paint to cups of white paint is 6 to 2.

   The following statements do *not* describe the paint mixture:
   B. For every cup of white paint there are 2 tablespoons of blue paint.
   E. For each tablespoon of blue paint there are 3 cups of white paint.

2. Answers vary. Sample responses:
   - The ratio of cups of flour to pints of water is $8 : 2$.
   - The ratio of pints of water to cups of flour is 2 to 8.
   - For each pint of water, there are 4 cups of flour.
   - For every 8 cups of flour, there are 2 pints of water.
   - For every 4 cups of flour, there is 1 pint of water.
   - There are 2 pints of water for every 8 cups of flour.

### Activity Synthesis

Select students to share their paste diagrams and sentences with the class. Sequence the diagrams from most elaborate to most simple. Connect the many ways in which the paste can be represented and described. Compare more detailed pictures with a discrete diagram; point out how the discrete diagram is a more efficient way of showing the paste recipe.

# 2.4 Card Sort: Spaghetti Sauce

**15 minutes**
Writing and using ratio language requires attention to detail. This task further develops students' ability to describe ratio situations precisely by attending carefully to the quantities, their units, and their order in the ratio.

Students work in pairs to match ratios of sauce ingredients to discrete diagrams and to explain reasoning (MP3).

### Addressing

- 6.RP.A.1

### Instructional Routines

- MLR8: Discussion Supports
- Take Turns

### Launch

Arrange students in groups of 2. Place two copies of uncut blackline masters in envelopes to serve as answer keys.

Demonstrate how to set up and play the matching game. Choose a student to be your partner. Discuss what all the symbols mean. Mix up the cards and place them face-up. Point out that the cards contain either diagrams or sentences. Select one of each style of card and then explain to your partner why you think the cards do or do not match. Demonstrate productive ways to agree or disagree, e.g., by explaining your mathematical thinking, asking clarifying questions, etc.

Give each group cut-up cards for matching. Tell students to check their matches after they complete the activity using the answer keys.

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**Access for Students with Disabilities**

*Action and Expression: Develop Expression and Communication.* To help get students started, display sentence frames such as, "_____ and _____ are a match because . . . ."
*Supports accessibility for: Language; Organization*

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### Anticipated Misconceptions

If students disagree about a match, encourage them to figure out the correct answer through discussion and use of the answer key. Make sure that when students use the answer key, they discuss any errors rather than just make changes.

Students may think the shapes in the diagram need to be drawn in the same order the ingredients appear in the description. This is not the case. You could turn a diagram card upside down and it

would still represent the same situation. The diagram just shows ingredients that get mixed together in a pot. It *is* the case, however, that within the description, the order of the words in the sentence must correspond with the terms within the ratio.

### Student Task Statement

Your teacher will give you cards describing different recipes for spaghetti sauce. In the diagrams:

- a circle represents a cup of tomato sauce
- a square represents a tablespoon of oil
- a triangle represents a teaspoon of oregano

1. Take turns with your partner to match a sentence with a diagram.
    a. For each match that you find, explain to your partner how you know it's a match.
    b. For each match that your partner finds, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
2. After you and your partner have agreed on all of the matches, check your answers with the answer key. If there are any errors, discuss why and revise your matches.
3. There were two diagrams that each matched with two different sentences. Which were they?
    - Diagram _____ matched with both sentences _____ and _____.
    - Diagram _____ matched with both sentences _____ and _____.
4. Select one of the other diagrams and invent another sentence that could describe the ratio shown in the diagram.

### Student Response

1.
    - Diagram A matches with sentence 4.
    - Diagram B matches with sentences 2 and 8.
    - Diagram C matches with sentence 1.
    - Diagram D matches with sentence 5.
    - Diagram E matches with sentences 3 and 7.
    - Diagram F matches with sentence 6.
2. No answer necessary.
3. a. Diagram B matches with sentences 2 and 8.
    b. Diagram E matches with sentences 3 and 7.

4. Answers vary. Sample responses:
    - For diagram A, the ratio of cups of tomato sauce to tablespoons of oil is $3:1$.
    - For diagram D, the ratio of tablespoons of oil to cups of tomato sauce is 2 to 5.

> **Are You Ready for More?**
>
> Create a diagram that represents any of the ratios in a recipe of your choice. Is it possible to include more than 2 ingredients in your diagram?

### Student Response

Answers vary.

### Activity Synthesis

Once all groups have completed the matching, discuss the following:

- Which matches were tricky? Explain why.
- Did any pairs need to make adjustments in their matches? What might have caused an error? What adjustments were made?
- What if you were making this tasty sauce and got the ratios wrong? What would happen?

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**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports*. To demonstrate mathematical language and to help students with communicating their reasoning clearer, revoice students' ideas and press for details in explanations. Request that students challenge an idea, elaborate on an idea. For example, if a student says that they matched D with 5 ask, "What did you see in Diagram D that matched with the words of Sentence 5?"
*Design Principle(s): Optimize output (for explanation)*

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## Lesson Synthesis

This lesson used diagrams to represent ratios. These diagrams omit details that are not necessary for understanding and solving the problem at hand. Discuss:

- What are some good things to remember when you draw a diagram of a ratio? (You only need necessary information. You could include shapes, color-coded boxes, or initials to represent each object within the set. It is helpful to organize the types of items in rows, and to arrange smaller groups so they are easier to see.)
- How can a diagram help you make sense of a situation involving a ratio? (It is easier to write correct statements about them. Also, you can see how the objects can be grouped.)

# 2.5 Paws, Ears, and Tails

**Cool Down: 5 minutes**

## Addressing

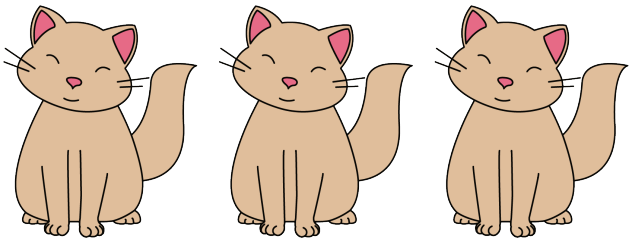
- 6.RP.A.1

### Anticipated Misconceptions

In the second question, students may not realize that the order of the words in the sentence must correspond with the terms within the ratio. Ears : paws : tails must correspond with $6 : 12 : 3$. In the fourth question, students may not write the sentence for every *one* ear. If this is the case, prompt them to draw a circle around each set of two paws and one ear to help them see this relationship.

### Student Task Statement

There are 3 cats in a room and no other creatures. Each cat has 2 ears, 4 paws, and 1 tail.



1. Draw a diagram that shows an association between numbers of ears, paws, and tails in the room.

2. Complete each statement:

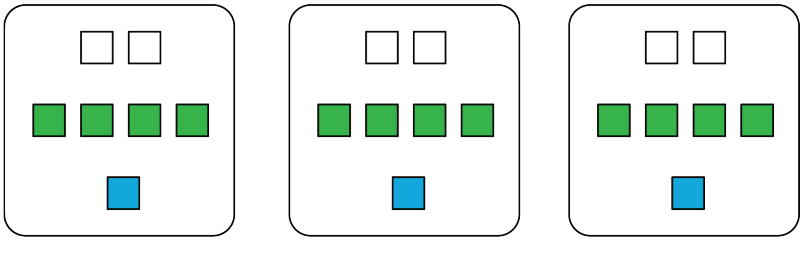
   a. The ratio of _____ to _____ to _____ is _____ : _____ : _____.

   b. There are _____ paws for every tail.

   c. There are _____ paws for every ear.

### Student Response

1. Answers vary. Sample response:



2. a. The ratio of ears to paws to tails is $6 : 12 : 3$.

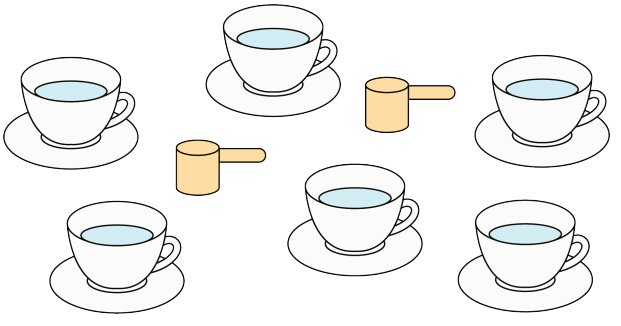
   b. There are 4 paws for every tail.

   c. There are 4 paws for every 2 ears. This means that there are 2 paws for every ear.

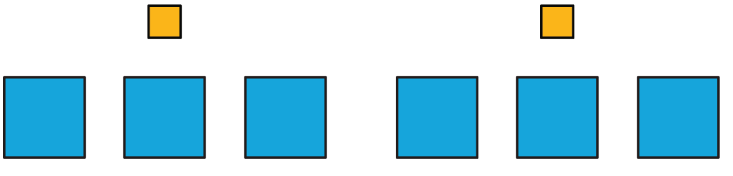
## Student Lesson Summary

Ratios can be represented using diagrams. The diagrams do not need to include realistic details. For example, a recipe for lemonade says, “Mix 2 scoops of lemonade powder with 6 cups of water.”

Instead of this:



We can draw something like this:



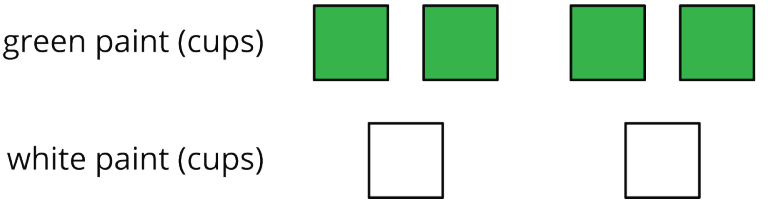
This diagram shows that the ratio of cups of water to scoops of lemonade powder is 6 to 2. We can also see that for every scoop of lemonade powder, there are 3 cups of water.

# Lesson 2 Practice Problems

## Problem 1

### Statement

Here is a diagram that describes the cups of green and white paint in a mixture.




Select **all** the statements that correctly describe this diagram

A. The ratio of cups of white paint to cups of green paint is 2 to 4.

B. For every cup of green paint, there are two cups of white paint.

C. The ratio of cups of green paint to cups of white paint is $4 : 2$.

D. For every cup of white paint, there are two cups of green paint.

E. The ratio of cups of green paint to cups of white paint is $2 : 4$.

## Solution

["A", "C", "D"]

## Problem 2

### Statement

To make a snack mix, combine 2 cups of raisins with 4 cups of pretzels and 6 cups of almonds.

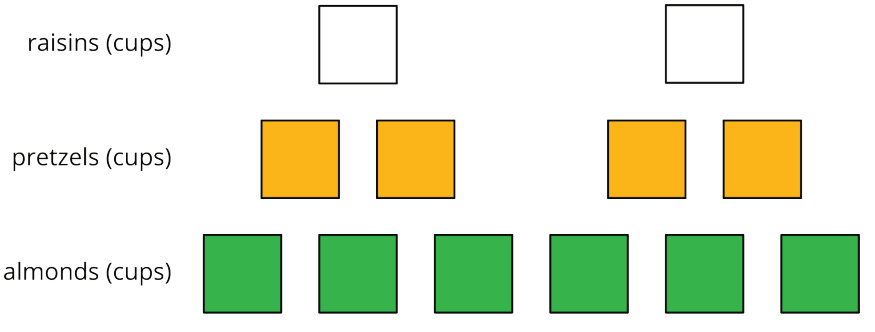
a. Create a diagram to represent the quantities of each ingredient in this recipe.

b. Use your diagram to complete each sentence.

- The ratio of _____ to _____ to _____ is _____ : _____ : _____.
- There are _____ cups of pretzels for every cup of raisins.
- There are _____ cups of almonds for every cup of raisins.

## Solution

a. Answers vary. Sample response:




b. Statements:

- Answers vary. Sample response: cups of raisins, cups of pretzels, cups of almonds, 2, 4, 6
- 2
- 3

## Problem 3

### Statement

a. A square is 3 inches by 3 inches. What is its area?

b. A square has a side length of 5 feet. What is its area?

c. The area of a square is 36 square centimeters. What is the length of each side of the square?

## Solution

a. 9 square inches ($3 \cdot 3 = 9$)

b. 25 square feet ($5 \cdot 5 = 25$)

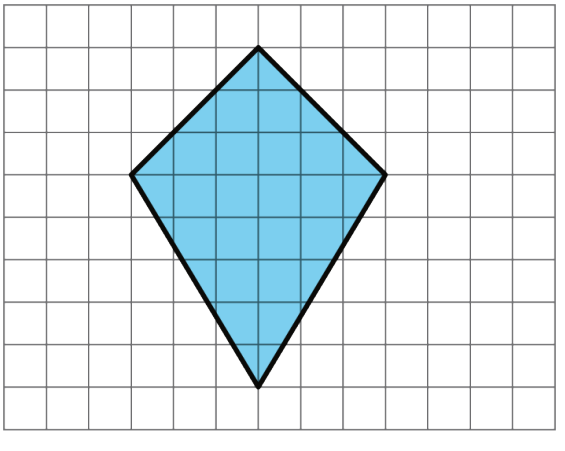
c. 6 centimeters ($6 \cdot 6 = 36$)

(From Unit 1, Lesson 17.)

## Problem 4

### Statement

Find the area of this quadrilateral. Explain or show your strategy.



## Solution

24 square units. Possible strategy: Decompose the quadrilateral into two triangles with a horizontal cut. The top triangle has a base of 6 units and a height of 3 units. Its area is 9 square units, as $(6 \cdot 3) \div 2 = 9$. The bottom triangle has a base of 6 units and a height of 5 units. Its area is 15 square units, as $(6 \cdot 5) \div 2 = 15$. $9 + 15 = 24$. The area of the quadrilateral is then 24 square units.

(From Unit 1, Lesson 11.)

## Problem 5

### Statement

Complete each equation with a number that makes it true.

- $\frac{1}{8} \cdot 8 =$ _____
- $\frac{1}{8} \cdot 7 =$ _____
- $\frac{3}{8} \cdot 8 =$ _____
- $\frac{3}{8} \cdot 7 =$ _____

## Solution

a. $1$ (or equivalent)

b. $3$ (or equivalent)

c. $\frac{7}{8}$ (or equivalent)

d. $\frac{21}{8}$ (or equivalent, $2\frac{5}{8}$ for example)

(From Unit 2, Lesson 1.)