## Lesson 16: Graphing from the Vertex Form

* Let’s graph equations in vertex form.

### 16.1: Which Form to Use?

Expressions in different forms can be used to define the same function. Here are three ways to define a function $f$.

$f\left(x\right)=x^{2}−4x+3$

(standard form)

$f\left(x\right)=\left(x−3\right)\left(x−1\right)$

(factored form)

$f\left(x\right)=\left(x−2\right)^{2}−1$

(vertex form)

Which form would you use if you want to find the following features of the graph of $f$? Be prepared to explain your reasoning.

1. the $x$-intercepts
2. the vertex
3. the $y$-intercept

### 16.2: Sharing a Vertex

Here are two equations that define quadratic functions.

$p\left(x\right)=-\left(x−4\right)^{2}+10q\left(x\right)=\frac{1}{2}\left(x−4\right)^{2}+10$

1. The graph of $p$ passes through $\left(0,-6\right)$ and $\left(4,10\right)$, as shown on the coordinate plane.
* Find the coordinates of another point on the graph of $p$. Explain or show your reasoning. Then, use the points to sketch and label the graph.
* 
1. On the same coordinate plane, identify the vertex and two other points that are on the graph of $q$. Explain or show your reasoning. Sketch and label the graph of $q$.
2. Priya says, "Once I know the vertex is $\left(4,10\right)$, I can find out, without graphing, whether the vertex is the maximum or the minimum of function $p$. I would just compare the coordinates of the vertex with the coordinates of a point on either side of it."
* Complete the table and then explain how Priya might have reasoned about whether the vertex is the minimum or maximum.

|  |  |  |  |
| --- | --- | --- | --- |
| * $x$
 | * 3
 | * 4
 | * 5
 |
| * $p\left(x\right)$
 |  | * 10
 |  |

#### Are you ready for more?

1. Write a the equation for a quadratic function whose graph has the vertex at $\left(2,3\right)$ and contains the point $\left(0,-5\right)$.
2. Sketch a graph of your function.

### 16.3: Card Sort: Matching Equations with Graphs

Your teacher will give you a set of cards. Each card contains an equation or a graph that represents a quadratic function. Take turns matching each equation to a graph that represents the same function.

* For each pair of cards that you match, explain to your partner how you know they belong together.
* For each pair of cards that your partner matches, listen carefully to their explanation. If you disagree, discuss your thinking and work to reach an agreement.
* Once all the cards are matched, record the equation, the label and a sketch of the corresponding graph, and write a brief note or explanation about how you knew they were a match.

Equation:



Explanation:

Equation:



Explanation:

Equation:



Explanation:

Equation:



Explanation:

Equation:



Explanation:

Equation:



Explanation:

### Lesson 16 Summary

We saw that vertex form is especially helpful for finding the vertex of a graph of a quadratic function. For example, we can tell that the function $p$ given by $p\left(x\right)=\left(x−3\right)^{2}+1$ has a vertex at $\left(3,1\right)$.

We also noticed that, when the squared expression $\left(x−3\right)^{2}$ has a positive coefficient, the graph opens upward. This means that the vertex $\left(3,1\right)$ represents the minimum function value.



But why does the function $p$ take on its minimum value when $x$ is 3?

Here is one way to explain it: When $x=3$, the squared term $\left(x−3\right)^{2}$ equals 0, as $\left(3−3\right)^{2}=0^{2}=0$. When $x$ is any other value besides 3, the squared term $\left(x−3\right)^{2}$ is a positive number greater than 0. (Squaring any number results in a positive number.) This means that the output when $x\ne 3$ will always be greater than the output when $x=3$, so the function $p$ has a minimum value at $x=3$.

This table shows some values of the function for some values of $x$. Notice that the output is the least when $x=3$ and it increases both as $x$ increases and as it decreases.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| $x$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| $\left(x−3\right)^{2}+1$ | 10 | 5 | 2 | 1 | 2 | 5 | 10 |

The squared term sometimes has a negative coefficient, for instance: $h\left(x\right)=-2\left(x+4\right)^{2}$. The $x$ value that makes $\left(x+4\right)^{2}$ equal 0 is -4, because $\left(-4+4\right)^{2}=0^{2}=0$. Any other $x$ value makes $\left(x+4\right)^{2}$ greater than 0. But when $\left(x+4\right)^{2}$ is multiplied by a negative number (-2), the resulting expression, $-2\left(x+4\right)^{2}$, ends up being negative. This means that the output when $x\ne -4$ will always be less than the output when $x=-4$, so the function $h$ has its maximum value when $x=-4$.



Remember that we can find the $y$-intercept of the graph representing any function we have seen. The $y$-coordinate of the $y$-intercept is the value of the function when $x=0$. If $g$ is defined by $g\left(x\right)=\left(x+1\right)^{2}−5$, then the $y$-intercept is $\left(0,-4\right)$ because $g\left(0\right)=\left(0+1\right)^{2}−5=-4$. Its vertex is at $\left(-1,-5\right)$.  Another point on the graph with the same $y$-coordinate is located the same horizontal distance from the vertex but on the other side.





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