## Unit 2 Lesson 24: Polynomial Identities (Part 2)

### 1 Revisiting an Old Theorem (Warm up)

#### Student Task Statement

Instructions to make a right triangle:

* Choose two integers.
* Make one side length equal to the sum of the squares of the two integers.
* Make one side length equal to the difference of the squares of the two integers.
* Make one side length equal to twice the product of the two integers.

Follow these instructions to make a few different triangles. Do you think the instructions always produce a right triangle? Be prepared to explain your reasoning.

### 2 Theorems and Identities

#### Student Task Statement

Here are the instructions to make a right triangle from earlier:

* Choose two integers.
* Make one side length equal to the sum of the squares of the two integers.
* Make one side length equal to the difference of the squares of the two integers.
* Make one side length equal to twice the product of the two integers.
1. Using $a$ and $b$ for the two integers, write expressions for the three side lengths.
2. Why do these instructions make a right triangle?

### 3 Identifying Identities (Optional)

#### Student Task Statement

Here is a list of equations. Circle all the equations that are identities. Be prepared to explain your reasoning.

1. $a=-a$
2. $a^{2}+2ab+b^{2}=\left(a+b\right)^{2}$
3. $a^{2}−2ab+b^{2}=\left(a−b\right)^{2}$
4. $a^{2}−b^{2}=\left(a−b\right)\left(a−b\right)$
5. $\left(a+b\right)\left(a^{2}−ab+b^{2}\right)=a^{3}−b^{3}$
6. $\left(a−b\right)^{3}=a^{3}−b^{3}−3ab\left(a+b\right)$
7. $a^{2}\left(a−b\right)^{4}−b^{2}\left(a−b\right)^{4}=\left(a−b\right)^{5}\left(a+b\right)$

### 4 Egyptian Fractions

#### Student Task Statement



In Ancient Egypt, all non-unit fractions were represented as a sum of distinct unit fractions. For example, $\frac{4}{9}$ would have been written as $\frac{1}{3}+\frac{1}{9}$ (and not as $\frac{1}{9}+\frac{1}{9}+\frac{1}{9}+\frac{1}{9}$ or any other form with the same unit fraction used more than once). Let’s look at some different ways we can rewrite $\frac{2}{15}$ as the sum of distinct unit fractions.

1. Use the formula $\frac{2}{d}=\frac{1}{d}+\frac{1}{2d}+\frac{1}{3d}+\frac{1}{6d}$ to rewrite the fraction $\frac{2}{15}$, then show that this formula is an identity.
2. Another way to rewrite fractions of the form $\frac{2}{d}$ is given by the identity $\frac{2}{d}=\frac{1}{d}+\frac{1}{d+1}+\frac{1}{d\left(d+1\right)}$. Use it to re-write the fraction $\frac{2}{15}$, then show that it is an identity.



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