Lesson 6: Subtracting Rational Numbers

Goals

- Compare and contrast (orally) subtraction expressions that have the same numbers in the opposite order.
- Recognize that the "difference" of two numbers can be positive or negative, depending on the order they are listed, while the "distance" between two numbers is always positive.
- Subtract signed numbers, and explain (orally) the reasoning.

Learning Targets

- I can find the difference between two rational numbers.
- I understand how to subtract positive and negative numbers in general.

Lesson Narrative

In this lesson, students see that the *difference* between two numbers can be positive or negative, but the *distance* between two numbers is always positive. Using the geometry of the number line (MP7), they see that if you switch the order in which you subtract two numbers, the difference becomes its opposite.

For example, to find the difference in temperature between +70°C and +32°C we calculate $70 - 32 = 38$, so the difference is 38°C. The distance between these two is also 38°C. On the other hand, to find the difference in temperature between +32°C and +70°C we calculate $32 - 70 = -38$, so the difference is -38°C. The distance is still 38°C. In general, if $a - b = x$, then $b - a = -x$. By observing the outcome of several examples, students may conjecture that this is always true (MP8).

Alignments

Building On

- 6.EE.B: Reason about and solve one-variable equations and inequalities.

Addressing

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports
- Number Talk
- Think Pair Share

Required Materials

Four-function calculators

Required Preparation

Use of calculators is optional. In this lesson, the important insights come from observing the outcome of evaluating expressions. Practice evaluating the expressions is of secondary importance.

> **Student Learning Goals**
>
> Let's bring addition and subtraction together.

6.1 Number Talk: Missing Addend

Warm Up: 5 minutes
The purpose of this number talk is to remind students about reasoning to find a missing addend and to rewrite each addition equation using subtraction. In this case, each problem is presented as an equation to solve. Previously in this unit, we have represented unknown values with question marks. Here, the unknown value is represented with a letter.

It may not be possible to share every possible strategy for the given limited time. Consider gathering only two distinctive strategies per problem.

Building On

- 6.EE.B

Building Towards

- 7.NS.A.1

Instructional Routines

- MLR8: Discussion Supports
- Number Talk

Launch

Display one problem at a time. Give students 30 seconds of quiet think time per problem and ask them to give a signal when they have an answer and a strategy. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Solve each equation mentally. Rewrite each addition equation as a subtraction equation.

$247 + c = 458$

$c + 43.87 = 58.92$

$\frac{15}{8} + c = \frac{51}{8}$

Student Response

-
  - $c = 211$ Possible strategies:
    - Working backwards, I can subtract $458 - 247$ to find the missing addend 211.
    - I can make several guesses in a row until I find the right number:
      If $c = 200$, then $247 + 200 = 447$, which is too small.
      If $c = 210$, then $247 + 210 = 457$, which is still a little bit too small.
      If $c = 211$, then $247 + 211 = 458$, which is just right.
  - $c = 15.05$ Possible strategy:
    - $43.87 + 5 = 48.87$
      $48.87 + 10 = 58.87$
      $58.87 + 0.05 = 58.92$
      And then $5 + 10 + 0.05 = 15.05$
  - $c = \frac{36}{8}$ or equivalent. Possible strategy: $51 - 15 = 36$
-
  - $458 - 247 = 211$ (or $458 - 211 = 247$)
  - $58.92 - 43.87 = 15.05$ (or $58.92 - 15.05 = 43.87$)
  - $\frac{51}{8} - \frac{15}{8} = \frac{36}{8}$ (or $\frac{51}{8} - \frac{36}{8} = \frac{15}{8}$)

Activity Synthesis

Ask students to share their reasoning. Record and display the responses for all to see. If students begin to talk about the distance between the given addend and sum when finding c, it may be helpful to draw a number line to represent their thinking. To involve more students in the conversation, use some of the following questions:

- "Who can restate ___'s reasoning in a different way?"
- "Did anyone find the value of n the same way, but would explain it differently?"
- "Did anyone find the value of n in a different way?"
- "Does anyone want to add on to _____'s strategy?"
- "Do you agree or disagree? Why?"

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

6.2 Expressions with Altitude

10 minutes
In this activity, students return to the familiar context of climbing up and down a cliff to apply what they have learned about subtracting signed numbers. They represent the change in elevation with an expression and then calculate the value of the expression. This activity does not provide a number line diagram or ask students to draw one, but some students may still choose to do so.

In this activity students are introduced to the idea that to find the difference between two values, we subtract one from the other. They use the context to make sure the order of the numbers in the subtraction expression correct. In the next activity, they attend to this explicitly in the abstract.

In this activity, no scaffolding is given, and students are free to use any strategy to find the differences.

Addressing

- 7.NS.A.1.c

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Give them 3 minutes of quiet work time, then have them check their progress with their partner. After students have come to agreement about the first few, they should finish the remainder. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students have access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Some students may wonder why they are being asked to solve the same problems that they already have. Point out that they have learned new ways of representing these problems than what they did previously.

Student Task Statement

A mountaineer is changing elevations. Write an expression that represents the difference between the final elevation and beginning elevation. Then write the value of the change. The first one is done for you.

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change
+400	+900	$900 - 400$	+500
+400	+50		
+400	-120		
-200	+610		
-200	-50		
-200	-500		
-200	0		

Student Response

beginning elevation (feet)	final elevation (feet)	difference between final and beginning	change
+400	+900	$900 - 400$	+500
+400	+50	$50 - 400$	-350
+400	-120	$-120 - 400$	-520
-200	+610	$610 - (-200)$	+810
-200	-50	$-50 - (-200)$	+150
-200	-500	$-500 - (-200)$	-300
-200	0	$0 - (-200)$	200

Are You Ready for More?

Fill in the table so that every row and every column sums to 0. Can you find another way to solve this puzzle?

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

	-12	0		5
0			-18	25
25		-18	5	-12
-12				-18
	-18	25	-12	

Student Response

Answers vary. Sample responses:

-18	-12	0	25	5
0	5	-12	-18	25
25	0	-18	5	-12
-12	25	5	0	-18
5	-18	25	-12	0

-18	-12	0	25	5
0	-7	0	-18	25
25	0	-18	5	-12
-12	37	-7	0	-18
5	-18	25	-12	0

Activity Synthesis

Students should understand that to find the difference between two numbers, we subtract. Be sure they attend to the order in which the numbers appear in the subtraction expressions: the final elevation always comes first because the question asked for the difference between the final and the beginning elevations. Also reinforce the notion that to subtract a number, we can add its opposite. For familiar problems like $900 - 400$, this isn't necessary. But for problems like $610 - (\text{-}200)$ it is easier for some people than going through the process of reasoning about it as an addition problem like $? + \text{-}200 = 610$, although this is always an option, and it is good to reinforce that we get the same answer whenever students choose to solve it this way.

Draw attention to the final three lines in the table, which all involve subtracting a negative number. Make sure that students see that subtracting a negative results in the same answer as adding its opposite.

Access for English Language Learners

Writing, Conversing: MLR3 Clarify, Critique, Correct. Present an incorrect statement that reflects a possible misunderstanding from the class. For example, "If the final elevation is -50 feet and the starting elevation is -200 feet, then the difference is $-50 - 200$". Invite students to clarify and then critique the reasoning, and to write an improved statement. Listen for ways students use the words, "negative," "difference," "subtract," "opposite," and clarify any cases where they are used incorrectly. This helps students evaluate, and improve on, the written mathematical work of others.
Design Principle(s): Maximize meta-awareness; Cultivate conversation

6.3 Does the Order Matter?

10 minutes
In this activity, students see that if you reverse the order of the two numbers in a subtraction expression, you get the same magnitude with the opposite sign (MP8).

For students who might overly struggle to evaluate the expressions, consider providing access to a calculator and showing them how to enter a negative value. The important insight here is the outcome of evaluating the expressions. Practice evaluating the expressions is of lesser importance.

In this activity, no supports are given or suggested and students are free to use any strategy to find the differences.

Addressing

- 7.NS.A.1

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Before working with the subtraction expressions in the task statement, consider telling students to close their books or devices and display these addition expressions for all to see. Discuss whether the order of the addends matters when adding signed numbers.

A	B
$3 + 2$	$2 + 3$
$5 + (-9)$	$(-9) + 5$
$(-11) + 2$	$2 + (-11)$
$(-6) + (-3)$	$(-3) + (-6)$
$(-1.2) + (-3.6)$	$(-3.6) + (-1.2)$
$(-2\frac{1}{2}) + (-3\frac{1}{2})$	$(-3\frac{1}{2}) + (-2\frac{1}{2})$

Arrange students in groups of 2. Give students quiet work time followed by partner and whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Display a list of familiar strategies students can choose from to find the value of each subtraction expression.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

Some students may try to interpret each subtraction expression as an addition equation with a missing addend and struggle to calculate the correct answer. Remind them that we saw another way to evaluate subtraction is by adding the additive inverse. Consider demonstrating how one of the subtraction expressions can be rewritten (e.g. $-11 - 2 = -11 + (-2)$).

Some students may struggle with deciding whether to add or subtract the magnitudes of the numbers in the problem. Prompt them to sketch a number line diagram and notice how the arrows compare.

Student Task Statement

1. Find the value of each subtraction expression.

A	B
$3 - 2$	$2 - 3$
$5 - (-9)$	$(-9) - 5$
$(-11) - 2$	$2 - (-11)$
$(-6) - (-3)$	$(-3) - (-6)$
$(-1.2) - (-3.6)$	$(-3.6) - (-1.2)$
$(-2\frac{1}{2}) - (-3\frac{1}{2})$	$(-3\frac{1}{2}) - (-2\frac{1}{2})$

2. What do you notice about the expressions in Column A compared to Column B?

3. What do you notice about their values?

Student Response

1.

A	B
1	-1
14	-14
-13	13
-3	3
2.4	-2.4
1	-1

2. Answers vary. Sample response: The numbers are the same, but they are subtracted in the opposite order.

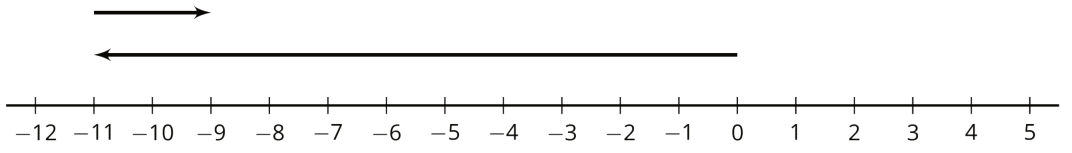
3. Answers vary. Sample reponse: They are opposites (additive inverses).

Activity Synthesis

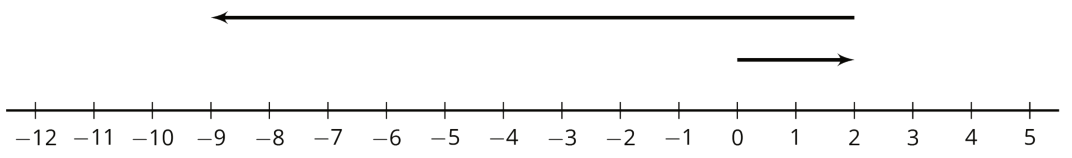
The most important thing for students to understand is that changing the order of the two numbers being subtracted will give the additive inverse of the original difference: $a - b = -(b - a)$. The two differences have the same magnitude but opposite signs. On a number line diagram, the arrows are the same length but pointing in opposite directions.

Consider displaying these unfinished number line diagrams as specific examples that students can refer to during the whole-class discussion:

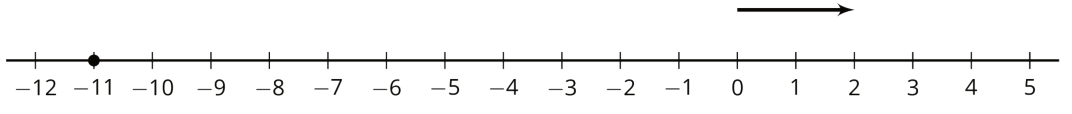
$-11 + 2$



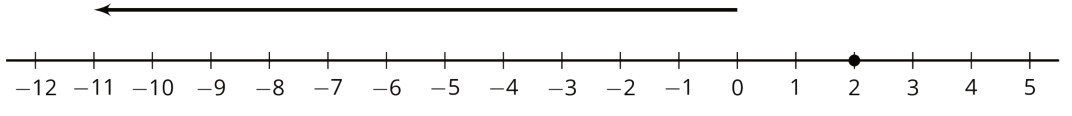
$2 + (-11)$



$-11 - 2$



$2 - (-11)$



Discuss:

- Does changing the order of the numbers in an addition expression change the value? Why?
- Does changing the order of the numbers in a subtraction expression change the value? Why?

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Support. To support whole-class discussion, display the sentence frames, "Changing the order of the numbers in an addition expression does/does not change the value because . . ." and "Changing the order of the numbers in a subtraction expression does/does not change the value because . . ." In addition, to encourage students to respond to each other's ideas, invite students to use the frames, "I agree with ____ because . . ." and "I disagree with ____ because . . ."
Design Principle(s): Support sense-making; Optimize output (for explanation)

Lesson Synthesis

Main takeaways:

- The difference between two numbers can be positive or negative, depending on their order: $(a - b) = -(b - a)$.
- The distance between two numbers is always positive. It does not depend on their order, because it is the magnitude of the difference: $|a - b| = |b - a|$.

Discussion questions:

- What is the difference between 12 and 10? $(12 - 10 = 2)$
- What is the difference between 10 and 12? $(10 - 12 = -2)$
- What is the distance between 12 and 10? $(|2| = 2)$
- What is the distance between 10 and 12? $(|-2| = 2)$

6.4 A Subtraction Expression

Cool Down: 5 minutes

Addressing

- 7.NS.A.1.c

Student Task Statement

Select all of the choices that are equal to $(-5) - (-12)$.

1. -7
2. 7
3. The difference between -5 and -12.
4. The difference between -12 and -5.
5. $(-5) + 12$
6. $(-5) + (-12)$

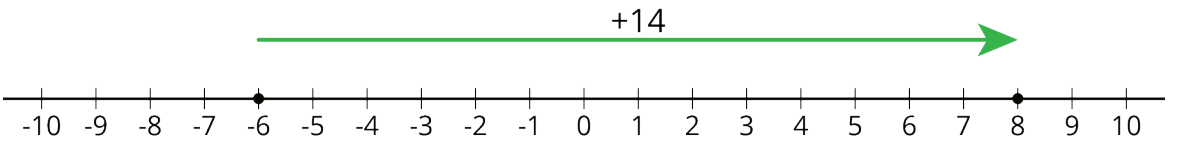
Student Response

7; The difference between -5 and -12; $-5 + 12$

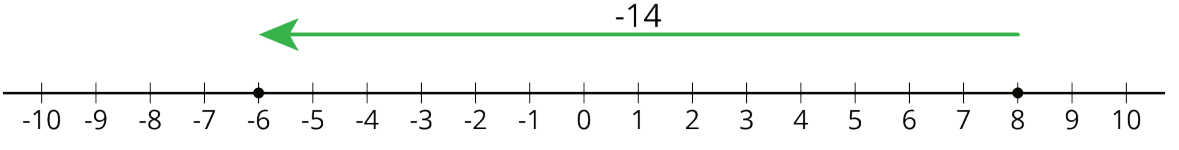
Student Lesson Summary

When we talk about the difference of two numbers, we mean, “subtract them.” Usually, we subtract them in the order they are named. For example, the difference of +8 and -6 is $8 - (-6)$.

The difference of two numbers tells you how far apart they are on the number line. 8 and -6 are 14 units apart, because $8 - (-6) = 14$:



Notice that if you subtract them in the opposite order, you get the opposite number:



$$(-6) - 8 = -14$$

In general, the distance between two numbers a and b on the number line is $|a - b|$. Note that the *distance* between two numbers is always positive, no matter the order. But the *difference* can be positive or negative, depending on the order.

Lesson 6 Practice Problems

Problem 1

Statement

Write a sentence to answer each question:

a. How much warmer is 82 than 40?

b. How much warmer is 82 than -40?

Solution

Answers vary. Possible responses:

a. 82 is 42 degrees warmer than 40.

b. 82 is 122 degrees warmer than -40.

Problem 2

Statement

a. What is the difference in height between 30 m up a cliff and 87 m up a cliff? What is the distance between these positions?

b. What is the difference in height between an albatross flying at 100 m above the surface of the ocean and a shark swimming 30 m below the surface? What is the distance between them if the shark is right below the albatross?

Solution

a. The difference in height is -57 m, because $30 - 87 = -57$. The distance is 57 m, because $|-57| = 57$.

b. The difference in height is 130 m, because $100 - (-30) = 130$. The distance is 130 m, because $|130| = 130$.

Problem 3

Statement

A company produces screens of different sizes. Based on the table, could there be a relationship between the number of pixels and the area of the screen? If so, write an equation representing the relationship. If not, explain your reasoning.

square inches of screen	number of pixels
6	31,104
72	373,248
105	544,320
300	1,555,200

Solution

It is a proportional relationship which can be represented as $p = 5{,}184 \cdot a$ where p represents the number of pixels and a is the area of the screen in square inches.

(From Unit 2, Lesson 8.)

Problem 4

Statement

Find each difference.

- $(-5) - 6$
- $35 - (-8)$
- $\frac{2}{5} - \frac{3}{5}$
- $-4\frac{3}{8} - (-1\frac{1}{4})$

Solution

a. -11

b. 43

c. $-\frac{1}{5}$

d. $-3\frac{1}{8}$

Problem 5

Statement

A family goes to a restaurant. When the bill comes, this is printed at the bottom of it:

Gratuity Guide For Your Convenience:
15% would be $4.89
18% would be $5.87
20% would be $6.52

How much was the price of the meal? Explain your reasoning.

Solution

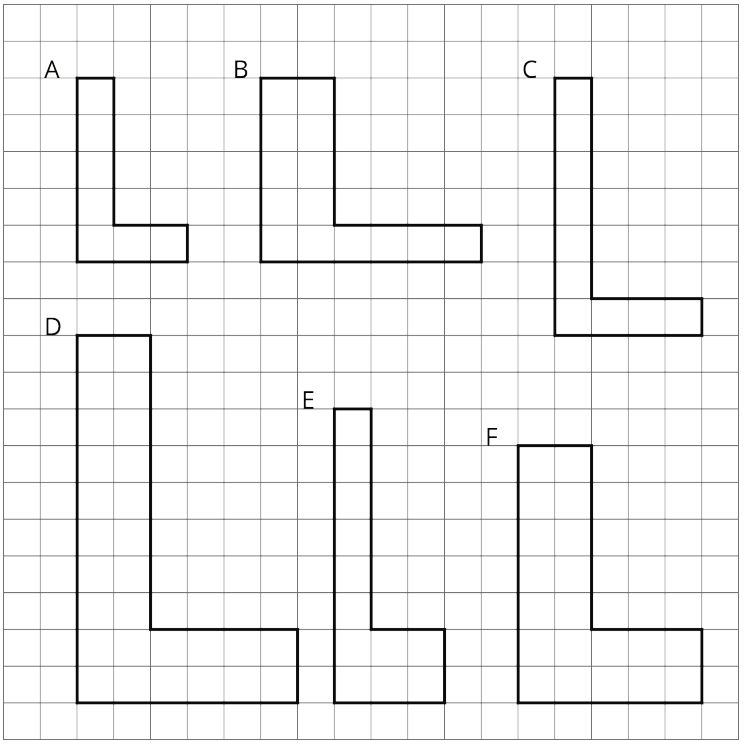
The bill was close to $32.60. We can't tell the exact amount because the suggested dollar amounts have been rounded to the hundredths place. $4.89 \div 0.15 = 32.60$, $5.87 \div 0.18 = 32.61$, and $6.52 \div 0.2 = 32.60$. The first and third quotients are exact while the middle quotient is rounded to the nearest hundredth.

(From Unit 4, Lesson 10.)

Problem 6

Statement

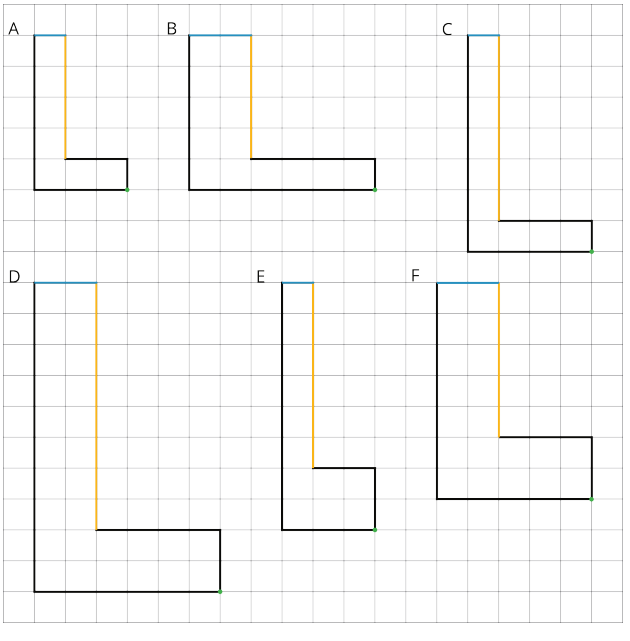
Which is a scaled copy of Polygon A? Identify a pair of corresponding sides and a pair of corresponding angles. Compare the areas of the scaled copies.



Solution

Polygon D is a scaled copy of Polygon A. This is true because all of the side lengths are doubled. The area of Polygon D is 4 times the area of Polygon A.

Answers vary. Sample markings:



(From Unit 1, Lesson 2.)