Lesson 3: Changing Elevation

Goals

- Comprehend the term “opposite” (in spoken and written language) refers to numbers with the same magnitude but different signs.
- Create and interpret equations and diagrams that represent adding signed numbers in the context of elevation.
- Generalize (orally) a method for determining the sum of two signed numbers.

Learning Targets

- I understand how to add positive and negative numbers in general.

Lesson Narrative

In this lesson, students build towards fluency with adding signed numbers. They begin with the concrete context of elevations above and below sea level, but then move to more abstract work. They see that adding a number and its opposite gives a sum of 0. They contrast adding numbers with the same sign with adding numbers with different signs. Using the structure of opposites on the number line, they see that when adding two numbers with different signs, the sign of the sum will match the sign of the addend with the greater magnitude (MP7).

Alignments

Building On

- 6.NS.C: Apply and extend previous understandings of numbers to the system of rational numbers.

Addressing

- 7.NS.A.1.a: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.
- 7.NS.A.1.b: Understand $p + q$ as the number located a distance $|q|$ from p, in the positive or negative direction depending on whether q is positive or negative. Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts.
- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7.NS.A.1.d: Apply properties of operations as strategies to add and subtract rational numbers.

Building Towards

- 7.NS.A.1.a: Describe situations in which opposite quantities combine to make 0. For example, a hydrogen atom has 0 charge because its two constituents are oppositely charged.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR2: Collect and Display
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Receipt tape

Required Preparation

For optional activity, cut one strip of receipt tape, for every 2 students. Each strip of receipt tape should be at least 4 feet long. You may want to prepare some strips that are even longer, in case groups choose extra long objects.

> ### Student Learning Goals
>
> Let's solve problems about adding signed numbers.

3.1 That's the Opposite

Warm Up: 5 minutes
The purpose of this warm up is to think about how we might define opposites in different contexts.

Building On

- 6.NS.C

Building Towards

- 7.NS.A.1.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2 minute of quiet work time followed by 1 minute of partner discussion, then follow with whole-class discussion.

Student Task Statement

1. Draw arrows on a number line to represents these situations:

   a. The temperature was -5 degrees. Then the temperature rose 5 degrees.

   b. A climber was 30 feet above sea level. Then she descended 30 feet.

2. What's the opposite?

   a. Running 150 feet east.

   b. Jumping down 10 steps.

   c. Pouring 8 gallons into a fish tank.

Student Response

1. Answers vary. Sample responses:
   a. A number line with an arrow pointing from 0 to -5 and another from -5 to 0. A point at 0.

   b. A number line with an arrow pointing from 0 to 30 and another from 30 to 0. A point at 0.

2. a. Running 150 feet west.

   b. Jumping up 10 steps.

   c. Pouring 8 gallons out of a fish tank.

Activity Synthesis

Ask a few students to share their responses. Tell students that there are many situations involving changes in quantities where we can represent the opposite action with the opposite value. In this lesson, we are going to investigate several situations like this.

3.2 Cliffs and Caves

15 minutes (there is a digital version of this activity)
The purpose of this activity is to see how to tell, from the equation, whether the sum will be positive, negative, or 0, without having to draw a number line diagram every time.

In this activity, students return to the context of height and depth to continue making sense of adding signed numbers. They use the structure of number line diagrams (MP7) to examine a variety of situations, which include starting in the positives, starting in the negatives, moving away from 0, moving towards 0, crossing over 0, and ending exactly on 0. From this variety of situations, students see that the lengths of the arrows in the number line diagrams give important information about the situation. This opens up the discussion of comparing the magnitude of the addends. When the

addends have opposite signs, the longer arrow (the number with the larger magnitude) determines the sign of the sum. $400 + (-150) = 250$ and $-200 + 150 = -50$.

This activity purposefully gives many problems to figure out, but only asks students to draw a diagram for 3 rows. Watch for students who think they need to draw a number line for every problem. Encourage them to look for and generalize from regularity in the problems (MP8) so that they don't need to always draw a diagram.

For students using the digital version of the materials, be sure they do not use the applet to represent and answer every question and bypass good thinking. Encourage students to figure out an answer for each question first, and then using the applet to check their work.

In preparation for dealing with subtraction of signed numbers, some of the problems involve determining how much the temperature changed, given the initial and final temperatures.

Addressing

- 7.NS.A.1.a
- 7.NS.A.1.b

Instructional Routines

- MLR1: Stronger and Clearer Each Time

Launch

Explain that a *mountaineer* is someone who climbs mountains, and a *spelunker* is someone who explores caves and caverns. Arrange students in groups of 2. Give students 3 minute of quiet work time followed by partner discussion after the first question. Then have students work on the second question and follow with whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Represent the same information through different modalities by using physical objects to represent abstract concepts. Create an interactive display that allows students to experience physically moving an object (mountaineer) up and down a number line. Highlight connections between the act of physically moving the object up and down with arrows on a number line diagram.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Anticipated Misconceptions

Some students may struggle with working backwards to fill in the change when given the final elevation. Ask them "What do you do to 400 to get to 50?" for example.

Student Task Statement

1. A mountaineer is climbing on a cliff. She is 400 feet above the ground. If she climbs up, this will be a positive change. If she climbs down, this will be a negative change.

   a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	+400	300 up	
B	+400	150 down	
C	+400	400 down	
D	+400		+50

   b. Write an addition equation and draw a number line diagram for B. Include the starting elevation, change, and final elevation in your diagram.

2. A spelunker is down in a cave next to the cliff. If she climbs down deeper into the cave, this will be a negative change. If she climbs up, whether inside the cave or out of the cave and up the cliff, this will be a positive change.

   a. Complete the table.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	-200	150 down	
B	-200	100 up	
C	-200	200 up	
D	-200	250 up	
E	-200		-500

b. Write an addition equation and draw a number line diagram for C and D. Include the starting elevation, change, and final elevation in your diagram.

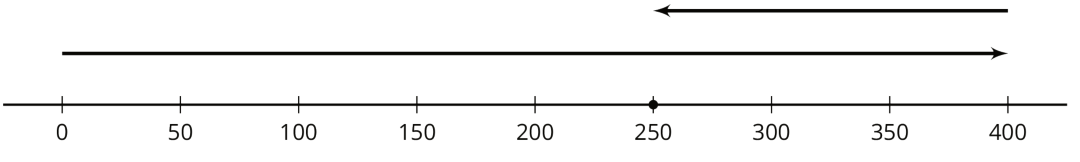
c. What does the expression $-75 + 100$ tell us about the spelunker? What does the value of the expression tell us?

Student Response

1. a.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	+400	300 up	+700
B	+400	150 down	+250
C	+400	400 down	0
D	+400	350 down	+50

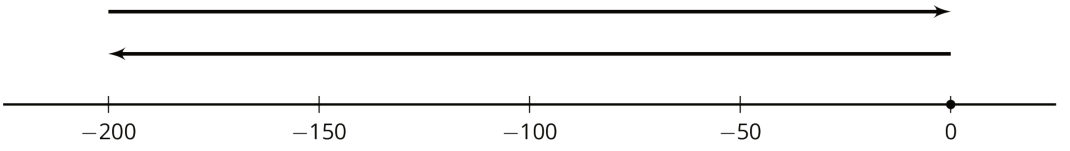
b. $400 + (-150) = 250$.



2. a.

	starting elevation (feet)	change (feet)	final elevation (feet)
A	-200	150 down	-350
B	-200	100 up	-100
C	-200	200 up	0
D	-200	250 up	+50
E	-200	300 down	-500

b. C: $-200 + 200 = 0$.



D: $-200 + 250 = 50$. A number line diagram with an arrow from 0 to -200, an arrow from -200 to 50, and a point at 50.

c. The spelunker was at an elevation of -75 feet, then went 100 feet up. The spelunker is now at an elevation of 25 feet.

Activity Synthesis

The most important thing for students to get out of this activity is how to tell from the equation whether the sum will be positive, negative, or 0, without having to draw a number line diagram every time.

- Write all of the equations where the two addends have the same sign. The ask, "What happens when the two addends have the same sign?"
- Write all of the equations where the two addends have the opposite sign. The ask, "What happens when the two addends have opposite signs and . . .
  - the number with the larger magnitude is positive?"
  - the number with the larger magnitude is negative?"
- Ask, "How can you tell when the sum will be zero?"

Access for English Language Learners

Speaking, Listening, Writing: MLR1 Stronger and Clearer Each Time. To provide students with an opportunity to generalize about the sum of two addends, ask students to draft an initial response to the question "How can you tell from the equation whether the sum will be positive, negative, or 0?". Ask students to meet with 2–3 partners for feedback. Provide students with prompts for feedback that will help each other strengthen their ideas and clarify their language (e.g., "What if the signs of both numbers are the same/different?" or "How do you know if the sum will be positive/negative?", etc.). Students can borrow ideas and language from each partner to refine their explanation. This will help students to use mathematical language to generalize about the sums of rational numbers.
Design Principle(s): Optimize output (for generalization)

3.3 Adding Rational Numbers

10 minutes

In this activity, no scaffolding is given and students use any strategy to find the sums. Monitor for students who reason in different ways about the sums.

Addressing

- 7.NS.A.1.d

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. 2 minutes of quiet work time, followed by partner and whole-class discussion.

Student Task Statement

Find the sums.

1. $-35 + (30 + 5)$
2. $-0.15 + (-0.85) + 12.5$
3. $\frac{1}{2} + (-\frac{3}{4})$

Student Response

1. 0
2. 11.5
3. $-\frac{1}{4}$

Are You Ready for More?

Find the sum without a calculator.

$$10 + 21 + 32 + 43 + 54 + (-54) + (-43) + (-32) + (-21) + (-10)$$

Student Response

0

Activity Synthesis

Select students to share their solutions. Sequence beginning with diagrams, then with more abstract reasoning. Help students connect the different reasoning strategies.

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Demonstrate possible approaches to finding the sums by thinking aloud and talking through your reasoning as you calculate the value of each expression. For the first problem, highlight adding $30 + 5 = 35$ (i.e., opposite of -35), and how recognizing opposites can help to find the value of the expression. Ask students, "Who can restate my reasoning in a different way?". This helps invite more student participation and meta-awareness of language and reasoning.
Design Principle(s): Support sense-making; Maximize meta-awareness

3.4 School Supply Number Line

Optional: 10 minutes
The purpose of this activity is to understand that even without knowing the actual numbers, knowing how the *signs* and *magnitudes* of two numbers compare is enough to determine whether their sum will be positive or negative.

In this activity, students use the relative position of numbers on the number line to compare them (MP7). Instead of working with specific given numbers, they start with two different, arbitrary lengths. They use these lengths to label various points on a number line. Students see that even though each group may have worked with different lengths, they should still get the same answers to the final set of questions, because everyone started with $a > b$. Students use their new insights to explain comparisons between signed numbers (MP3).

Identify a group that has a small difference between the lengths of their two objects, as well as a group that has a large difference between the lengths of their two objects, to contrast their number lines during the whole-class discussion.

Addressing

- 7.NS.A.1.b
- 7.NS.A.1.c

Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2. Provide each group a blank strip of receipt tape. The length of tape has to be longer than four times the length of their longer object. Ensure that each group has access to two objects of appropriate length.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Display or provide charts with symbols and meanings. For example, display the symbols <, >, and = and their respective meanings "less than," "greater than," and "equal to." Remind students that they can refer to the display to answer the last question in the activity.
Supports accessibility for: Conceptual processing; Memory

Access for English Language Learners

Conversing, Representing: MLR2 Collect and Display. Use this routine to connect the length of given objects to variable expressions with number line representations. Listen for and collect vocabulary and phrases students use to describe how they measure and label lengths on the number line (e.g., "-b is located on the left side of the number line") and how they compare the values (e.g., "-a is less than -b because..."). This will help students use mathematical language as they reason on a number line.
Design Principle(s): Optimize output (for justification); Maximize meta-awareness

Anticipated Misconceptions

Some students might forget which symbol means greater than or less than.

Some students may be confused between comparing the value of the expression and the magnitude of the expression. Explain that the number to the left on a number line has the lesser value, even if it may have the greater magnitude (farther away from zero).

Student Task Statement

Your teacher will give you a long strip of paper.

Follow these instructions to create a number line.

1. Fold the paper in half along its length and along its width.
2. Unfold the paper and draw a line along each crease.
3. Label the line in the middle of the paper 0. Label the right end of the paper + and the left end of the paper −.
4. Select two objects of different lengths, for example a pen and a gluestick. The length of the longer object is a and the length of the shorter object is b.
5. Use the objects to measure and label each of the following points on your number line.

a	$2b$	$-b$
b	$a + b$	$a + -b$
$2a$	$-a$	$b + -a$

6. Complete each statement using <, >, or =. Use your number line to explain your reasoning.

   a. a ____ b

   b. $-a$ ____ $-b$

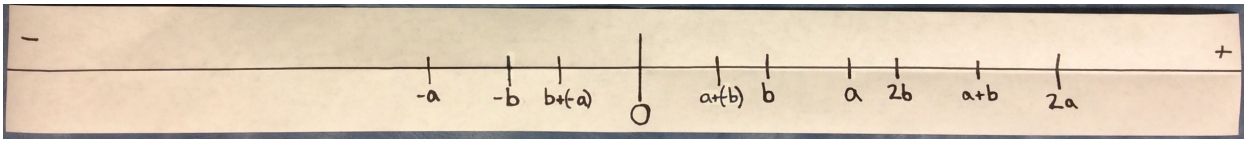
   c. $a + -a$ ____ $b + -b$

   d. $a + -b$ ____ $b + -a$

   e. $a + -b$ ____ $-a + b$

Student Response

1–5. Answers vary. Sample response:



1. a. $a > b$. Both values are positive and a is the value of the longer object.

   b. $-a < -b$. Both numbers are negative but the same distance away from zero as a and b.

   c. $a + (-a) = b + (-b)$. Both sides are zero.

   d. $a + (-b) > b + (-a)$. The expression on the left is greater than zero and the one on the right is less than zero.

   e. $a + (-b) > -a + b$. Since $b + (-a)$ is the same as $-a + b$, the same reason applies as before.

Activity Synthesis

The most important thing for students to understand is that even without knowing the actual numbers, knowing how the signs and magnitudes of two numbers compare is enough to determine whether their sum will be positive or negative.

Display the number lines from the selected groups (one with a small difference between a and b and one with a large difference).

Discuss:

- Which points are in the same relative position? (For example, $a + b$ is greater than $2b$ and less than $2a$ in both cases.)

- Which points are in different relative positions? (For example, $2b$ may be greater than a on one diagram but less than a on the other.)

For each part of the last question, ask students to indicate whether they used <, >, or =. Have students explain their reasoning until they come to an agreement. It may be helpful to point out the different positions on the number lines that students refer to during their explanations.

If desired, you can extend the discussion by highlighting the fact that all the comparisons in the task statement have the same answer for every group, but this didn't have to be the case. Ask students to invent another comparison that would have a different answer for some of the groups than others. For students who are struggling, ask them to connect positive and negative numbers and addition and subtraction to the previous contexts using MLR 7 (Compare and Connect).

Lesson Synthesis

Main learning points:

- To add two numbers with the same sign, we add their magnitudes (because the arrows point in the same direction) and keep the same sign for the sum.
- To add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and use the sign of the number with the larger magnitude for the sum.
- When we add a number and its opposite, the sum is zero. These are called additive inverses.

Discussion questions:

- What is the opposite of 5? of -8? of $\frac{1}{3}$? of -0.6?
- What is the sum of a number and its opposite?
- Explain how to add two numbers with the same sign. With different signs.

3.5 Add 'Em Up

Cool Down: 5 minutes

Addressing

- 7.NS.A.1.b

Student Task Statement

Find each sum.

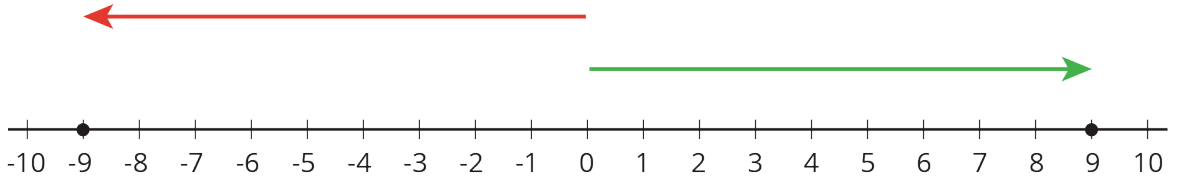
1. $56 + (-56)$
2. $-240 + 370$
3. $-5.7 + (-4.2)$

Student Response

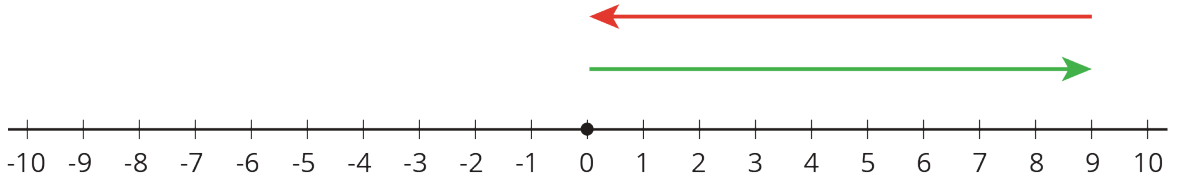
1. 0
2. 130
3. -9.9

Student Lesson Summary

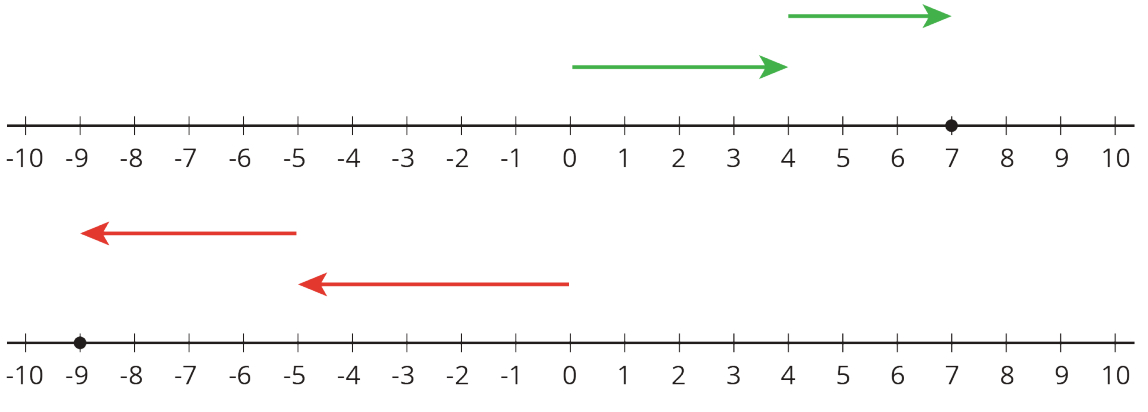
The opposite of a number is the same distance from 0 but on the other side of 0.



The opposite of -9 is 9. When we add opposites, we always get 0. This diagram shows that $9 + -9 = 0$.

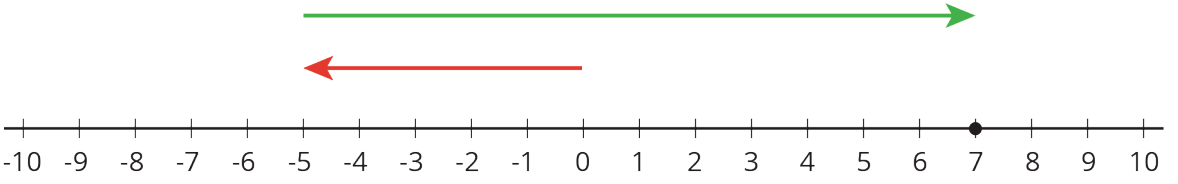


When we add two numbers with the same sign, the arrows that represent them point in the same direction. When we put the arrows tip to tail, we see the sum has the same sign.



To find the sum, we add the magnitudes and give it the correct sign. For example, $(-5) + (-4) = -(5 + 4)$.

On the other hand, when we add two numbers with different signs, we subtract their magnitudes (because the arrows point in the opposite direction) and give it the sign of the number with the larger magnitude. For example, $(-5) + 12 = +(12 - 5)$.



Lesson 3 Practice Problems

Problem 1

Statement

What is the final elevation if

a. A bird starts at 20 m and changes 16 m?

b. A butterfly starts at 20 m and changes -16 m?

c. A diver starts at 5 m and changes -16 m?

d. A whale starts at -9 m and changes 11 m?

e. A fish starts at -9 meters and changes -11 meters?

Solution

a. 36 m because $20 + 16 = 36$

b. 4 m because $20 + (-16) = 4$

c. -11 m because $5 + (-16) = -11$

d. 2 m because $-9 + 11 = 2$

e. -20 m because $-9 + (-11) = -20$

Problem 2

Statement

One of the particles in an atom is called an electron. It has a charge of -1. Another particle in an atom is a proton. It has charge of +1. The charge of an atom is the sum of the charges of the electrons and the protons. A carbon atom has an overall charge of 0, because it has 6 electrons and 6 protons and $-6 + 6 = 0$. Find the overall charge for the rest of the elements on the list.

	charge from electrons	charge from protons	overall charge
carbon	-6	+6	0
neon	-10	+10	
oxide	-10	+8	
copper	-27	+29	
tin	-50	+50	

Solution

Carbon: $-6 + 6 = 0$

Neon: $-10 + 10 = 0$

Oxide: $-10 + 8 = -2$

Copper: $-27 + 29 = +2$

Tin: $-50 + 50 = 0$

Problem 3

Statement

Add.

$14.7 + 28.9$ $-9.2 + 4.4$ $-81.4 + (-12)$ $51.8 + (-0.8)$

Solution

a. 43.6

b. -4.8

c. -93.4

d. 51

Problem 4

Statement

Last week, the price, in dollars, of a gallon of gasoline was g. This week, the price of gasoline per gallon increased by 5%. Which expressions represent this week's price, in dollars, of a gallon of gasoline? Select **all** that apply.

A. $g + 0.05$

B. $g + 0.05g$

C. $1.05g$

D. $0.05g$

E. $(1 + 0.05)g$

Solution

["B", "C", "E"]

(From Unit 4, Lesson 8.)

Problem 5

Statement

Decide whether each table could represent a proportional relationship. If the relationship could be proportional, what would be the constant of proportionality?

a. Annie's Attic is giving away $5 off coupons.

original price	sale price
$15	$10
$25	$20
$35	$30

b. Bettie's Boutique is having a 20% off sale.

original price	sale price
$15	$12
$25	$20
$35	$28

Solution

a. Not proportional.

b. Proportional. The constant of proportionality is 0.8 sale dollars per original dollar.

(From Unit 2, Lesson 7.)