Lesson 5: Representing Subtraction

Goals

- Generalize (orally and in writing) that subtracting a number results in the same value as adding the additive inverse.
- Interpret a number line diagram that represents subtracting signed numbers as adding with an unknown addend.
- Use a number line diagram to find the difference of signed numbers, and explain (orally) the reasoning.

Learning Targets

- I can explain the relationship between addition and subtraction of rational numbers.
- I can use a number line to subtract positive and negative numbers.

Lesson Narrative

In this lesson, students represent a subtraction of signed numbers on a number line by relating it to an addition equation with a missing addend. The convention for representing subtraction on the number line fits with the convention for representing addition. When we represent $a + b = c$, we represent a with an arrow starting at zero, b with an arrow starting where the first arrow ends, and c with a point at the end of the second arrow. So when we want to represent $c - a = b$, we represent c with a point, a with an arrow starting at zero, and the difference b is the other arrow that is needed to reach from the end of the first arrow to the point.

At the beginning of the lesson, students see that a subtraction equation like $\text{-}8 - 3 = ?$ can be thought of as the related addition equation $3 + ? = \text{-}8$. After repeatedly calculating differences this way (MP8), students recognize that the answer to each subtraction problem is the same number they would get by adding the opposite of the number. For example, by the end of the lesson, students see that $\text{-}8 - 3 = ?$ can also be thought of as $\text{-}8 + \text{-}3 = ?$

Alignments

Building On

- 1.OA.B.4: Understand subtraction as an unknown-addend problem. For example, subtract $10 - 8$ by finding the number that makes 10 when added to 8.

Addressing

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.

Building Towards

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

> **Student Learning Goals**
>
> Let's subtract signed numbers.

5.1 Equivalent Equations

Warm Up: 5 minutes

The purpose of this warm-up is to refresh students' previous understanding about the relationship between addition and subtraction (MP7) so they can write related equations.

As students work, watch for any students who create a number line diagram to help them generate equations that express the same relationship a different way.

Building On

- 1.OA.B.4

Building Towards

- 7.NS.A.1

Launch

Give students 1 minute of quiet work time. Remind students that each new equation must include only the numbers in the original equation.

Anticipated Misconceptions

If students struggle to come up with other equations, encourage them to represent the relationship using a number line diagram, and then think about other operations they can use to show the same relationship with the same numbers.

> **Student Task Statement**
>
> Consider the equation $2 + 3 = 5$. Here are some more equations, using the same numbers, that express the same relationship in a different way:
>
> $3 + 2 = 5$ $\qquad$ $5 - 3 = 2$ $\qquad$ $5 - 2 = 3$

For each equation, write two more equations, using the same numbers, that express the same relationship in a different way.

1. $9 + (-1) = 8$
2. $-11 + x = 7$

Student Response

1. For $9 + (-1) = 8$
    - $-1 + 9 = 8$
    - $8 - 9 = -1$
    - $8 - (-1) = 9$
2. For $-11 + x = 7$
    - $x + (-11) = 7$
    - $7 - x = -11$
    - $7 - (-11) = x$

Activity Synthesis

Ask selected students to share their equations that express the same relationship a different way. If any students created a number line diagram to explain their thinking, display this for all to see to facilitate connections between addition equations and related subtraction equations. Every addition equation has related subtraction equations and every subtraction equation has related addition equations; these are the most important takeaways from this activity.

5.2 Subtraction with Number Lines

10 minutes

The purpose of this activity is to apply the representation students have used while adding signed numbers, as well as the relationship between addition and subtraction, to begin subtracting signed numbers. Students are given number line diagrams showing one addend and the sum. They are asked to figure out what the other addend would be. Students examine how these addition equations with missing addends can be written using subtraction by analyzing and critiquing the reasoning of others (MP3).

Monitor for students who are using a consistent structure to analyze the diagrams to generalize and write related addition and subtraction equations (MP8). A template for this work might look something like:

$$a + ? = b$$
$$b - a = ?$$

Addressing

- 7.NS.A.1.c

Instructional Routines

- MLR8: Discussion Supports

Launch

It may be useful to remind students how they represented addition on a number line in previous lessons. In particular, it is helpful to keep in mind that the two addends in an addition equation are drawn "tip-to-tail." You might use any number line diagrams created in the previous activity as an illustration of this idea.

Ask students to complete the questions for the first diagram and pause for discussion. Then, give students quiet work time to complete the remaining problems, followed by whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference previous activities where students used a representation while adding signed numbers to provide an entry point for this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

Access for English Language Learners

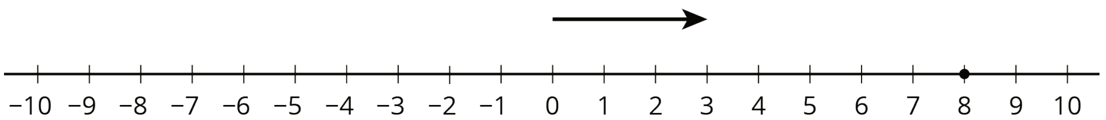
Speaking, Listening: MLR8 Discussion Support. To support students in producing statements about Mai's and Tyler's equations, provide sentence frames such as: "I agree/disagree with Mai/ Tyler because....".
Design Principle(s): Support sense-making; Optimize output (for critiques)

Anticipated Misconceptions

Some students may say they disagree with Tyler's equations for the number lines. Use fact families to help students see that subtraction equations are a valid way to represent problems involving finding a missing addend given a sum. It may help to remind them of the work they did in the warm-up.

Student Task Statement

1. Here is an unfinished number line diagram that represents a sum of 8.

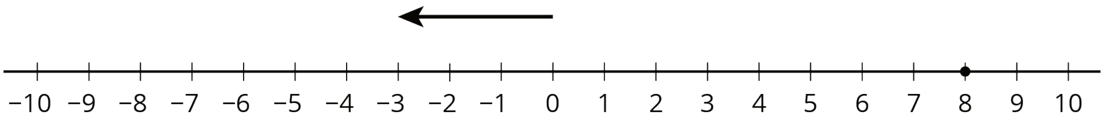


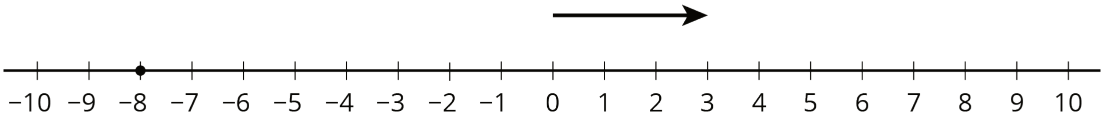
   a. How long should the other arrow be?

b. For an equation that goes with this diagram, Mai writes $3 + ? = 8$. Tyler writes $8 - 3 = ?$. Do you agree with either of them?

c. What is the unknown number? How do you know?

2. Here are two more unfinished diagrams that represent sums.





For each diagram:

a. What equation would Mai write if she used the same reasoning as before?

b. What equation would Tyler write if he used the same reasoning as before?

c. How long should the other arrow be?

d. What number would complete each equation? Be prepared to explain your reasoning.

3. Draw a number line diagram for $(-8) - (-3) = ?$ What is the unknown number? How do you know?

Student Response

1.
   a. 5 units

   b. Answers vary. Sample response: I agree with Mai because I want to know what to add to 3 to get 8 because 8 is the end point. I also agree with Tyler because the equations are equivalent.

   c. +5. Sample explanation: The unknown number is 5 because to get from 3 to 8 you add on 5.

2. First Image:
   a. Mai would write $(-3) + ? = 8$

   b. Tyler would write $8 - (-3) = ?$

c. The other arrow should be 11 units long.

d. The number is 11, because the other arrow is 11 units long and pointing to the right.

Second Image:

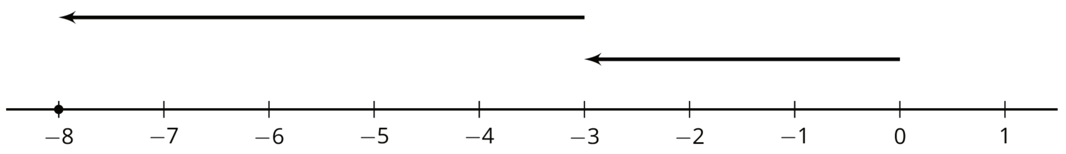
a. Mai would write $3 + ? = (-8)$

b. Tyler would write $(\text{-}8) - (3) = ?$

c. The other arrow should be 11 units long.

d. The number is -11, because the other arrow is 11 units long but pointing to the left.

3. Answers vary. Sample response:



The unknown number is -5, because the arrow is 5 units long and pointing left.

Activity Synthesis

The most important things for students to understand is that subtraction equations can be written as addition equations with a missing addend and number line diagrams can help students figure out what the missing addend is. Students need to be comfortable with this way of representing subtraction for the next activity.

Ask at least one student to share their missing addend for each problem. Ask students to share their reasoning until they come to an agreement. Display two related equations for all to see and use as a reference in the following activity. They might look something like this, or you might choose to use numbers in a specific example rather than letters in a general example.

$$a + ? = b$$
$$b - a = ?$$

5.3 We Can Add Instead

15 minutes

In this activity, students begin to see that subtracting a signed number is equivalent to adding its opposite. First, students match expressions and number line diagrams. Then they add and subtract numbers to see that subtracting a number is the same as adding its opposite (MP8).

Monitor for students who see and can articulate the pattern that adding a number is the same as subtracting its opposite.

Addressing

- 7.NS.A.1.c

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct

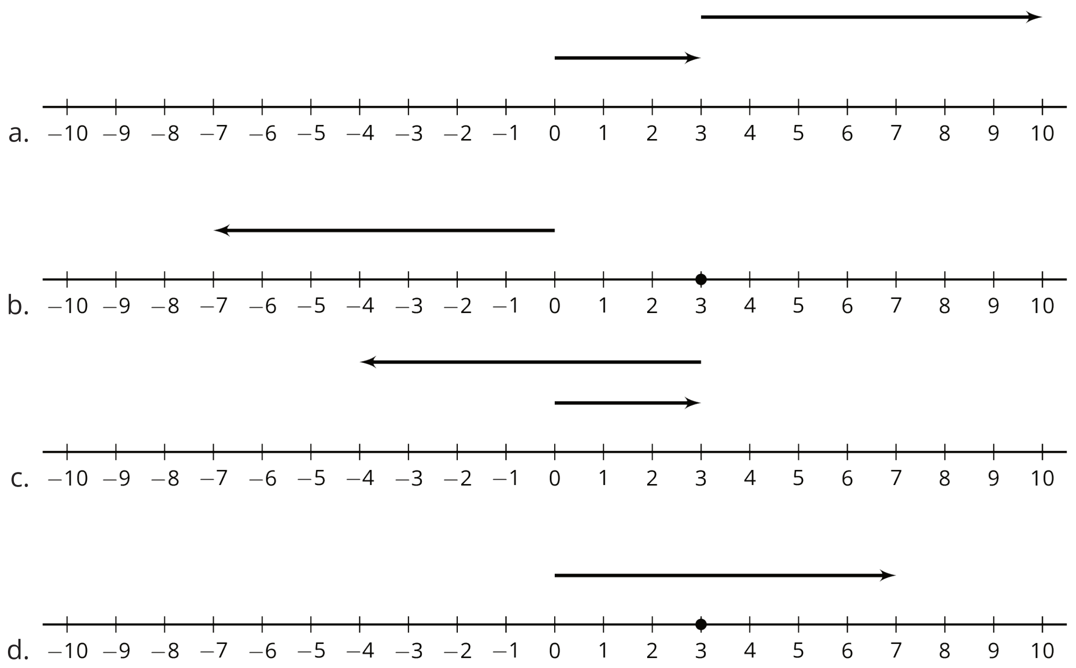
Launch

Arrange students in groups of 2. Give students 3 minutes of quiet work time, then have them check in with a partner. Have them continue to complete the activity, and follow with a whole-group discussion.

Student Task Statement

1. Match each diagram to one of these expressions:

$3 + 7$ $\quad$ $3 - 7$ $\quad$ $3 + (-7)$ $\quad$ $3 - (-7)$



2. Which expressions in the first question have the same value? What do you notice?

3. Complete each of these tables. What do you notice?

expression	value
$8 + (-8)$	
$8 - 8$	
$8 + (-5)$	
$8 - 5$	
$8 + (-12)$	
$8 - 12$	

expression	value
$-5 + 5$	
$-5 - (-5)$	
$-5 + 9$	
$-5 - (-9)$	
$-5 + 2$	
$-5 - (-2)$	

Student Response

1. a. $3 + 7$

   b. $3 - (-7)$

   c. $3 + (-7)$

   d. $3 - 7$

2. $3 + 7$ and $3 - (-7)$ have the same value. $3 + (-7)$ and $3 - 7$ have the same value. I notice that subtracting a number is the same as adding its opposite.

3. I notice that subtracting a number is the same as adding its opposite.

expression	value
$8 + (-8)$	0
$8 - 8$	0
$8 + (-5)$	3
$8 - 5$	3
$8 + (-12)$	-4
$8 - 12$	-4

expression	value
$-5 + 5$	0
$-5 - (-5)$	0
$-5 + 9$	4
$-5 - (-9)$	4
$-5 + 2$	-3
$-5 - (-2)$	-3

Are You Ready for More?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for adding and subtracting in this system like this: $2 \oplus 1 = 3$ and $2 \ominus 1 = 1$. The table shows some of the sums.

$\oplus$	0	1	2	3
0	0	1	2	3
1	1	2	3	0
2	2	3	0	1
3				

1. In this system, $1 \oplus 2 = 3$ and $2 \oplus 3 = 1$. How can you see that in the table?
2. What do you think $3 \oplus 1$ should be?
3. What about $3 \oplus 3$?
4. What do you think $3 \ominus 1$ should be?
5. What about $2 \ominus 3$?
6. Can you think of any uses for this number system?

Student Response

1. $1 \oplus 2 = 3$ is in the third row and fourth column and $2 \oplus 3 = 1$ is in the fourth row and fifth column.
2. Any responses are allowed. However, if we want $\oplus$ to satisfy the commutative and associative properties, then $3 \oplus 1 = 0$.
3. Assuming commutativity and associativity, $3 \oplus 3 = 2$.
4. Assuming $\ominus$ is the inverse of $\oplus$, $3 \ominus 1 = 2$.
5. Assuming $\ominus$ is the inverse of $\oplus$, $2 \ominus 3 = 3$.
6. Answers vary. For example, think about a dial which can only make 90° clockwise turns. The fact that making three such turns followed by two more turns is the same as making only one turn (since making four of those five 90° turns leaves you in the same position) could be represented by the statement $3 \oplus 2 = 1$. If we thought of the $\ominus$ operation as turning the dial counterclockwise, then the statement $2 \ominus 3 = 3$ reflects that two clockwise turns followed by three counterclockwise turns has the same effect as doing 3 clockwise turns.

Activity Synthesis

The most important takeaway is that subtracting a number gets the same answer as adding its opposite.

Select students to share what patterns they noticed. If no student mentions it, point out that subtracting a number is the same as adding its opposite. Ask students to help you list all of the pairs that show this.

Then write this expression: $3 - 7$. Ask how it could be written as a sum? $3 + (-7)$. What numbers are both of these expressions equal to?

$3 - 7 = -4$

$3 + (-7) = -4$

For students who are ready to explore how knowing how to solve a one-step equation involving addition or subtraction (from grade 6) helps us show that subtracting a number is the same as adding its opposite, continue.

This is also true when solving equations. Write this equation:

$$x = 3 - 7$$

Ask how it can be written as a sum. Record students' responses. If no student writes

$$x + 7 = 3$$

then write that. Then point out that we can add -7 to each side:

$$\begin{aligned} x + 7 + -7 &= 3 + -7 \\ x &= 3 + -7 \end{aligned}$$

There is nothing special about these numbers, because a number and its opposite always make a sum of 0. So subtracting a number is always the same as adding its opposite.

Access for English Language Learners

Writing, Conversing: MLR3 Clarify, Critique, Correct. Present an incorrect statement about adding and subtracting numbers that reflects a possible misunderstanding from the class. Display the following for all to see: "$(-5) - 3$ has the same value as $5 + (-3)$, because subtracting is the same as adding the opposite". Invite students to discuss this argument with a partner. Ask, "Do you agree with the statement? Why or why not?" Invite students to clarify and then critique the reasoning, and to write an improved response. This will help students use the language of justification to critique the reasoning related to subtraction of signed numbers.
Design Principle(s): Maximize meta-awareness; Cultivate conversation

Lesson Synthesis

Main takeaways:

- You can think of subtraction as addition with a missing addend: What number do I need to add to get from b to a?

- You can evaluate a subtraction expression by adding the opposite: $a - b = a + (\text{-}b)$. This works regardless of the sign for a or b.

Discussion questions

- How could we rewrite the expression $\text{-}5 - 3$ using addition? ($3 + ? = \text{-}5$, or more simply $\text{-}5 + (\text{-}3)$)
- Does this work for all numbers?

5.4 Same Value

Cool Down: 5 minutes

Addressing

- 7.NS.A.1.c

Student Task Statement

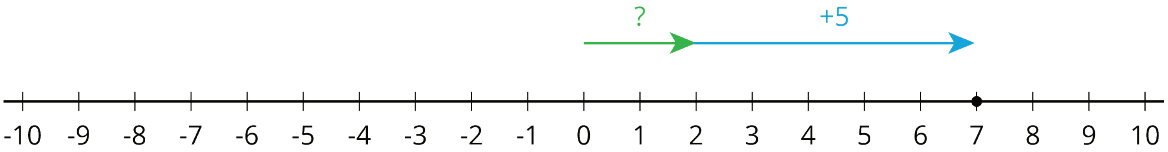
1. Which other expression has the same value as $(\text{-}14) - 8$? Explain your reasoning.
   a. $(\text{-}14) + 8$
   b. $14 - (\text{-}8)$
   c. $14 + (\text{-}8)$
   d. $(\text{-}14) + (\text{-}8)$
2. Which other expression has the same value as $(\text{-}14) - (\text{-}8)$? Explain your reasoning.
   a. $(\text{-}14) + 8$
   b. $14 - (\text{-}8)$
   c. $14 + (\text{-}8)$
   d. $(\text{-}14) + (\text{-}8)$

Student Response

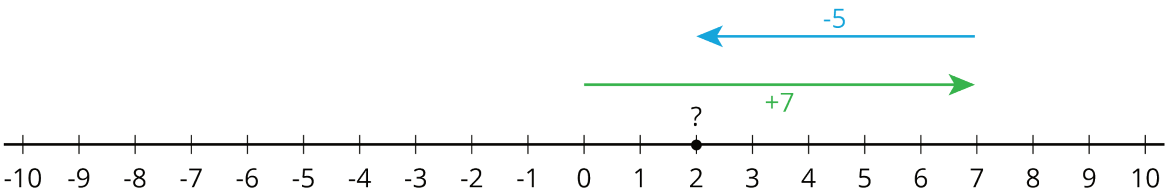
1. $(\text{-}14) + (\text{-}8)$. Sample explanation: because adding -8 is the same as subtracting 8
2. $(\text{-}14) + 8$. Sample explanation: because subtracting -8 is the same as adding 8

Student Lesson Summary

The equation $7 - 5 = ?$ is equivalent to $? + 5 = 7$. The diagram illustrates the second equation.

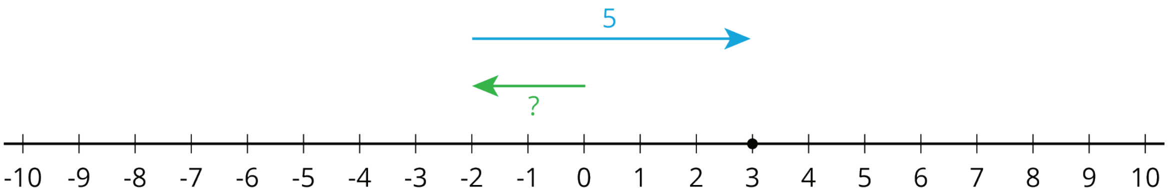


Notice that the value of $7 + (\text{-}5)$ is 2.

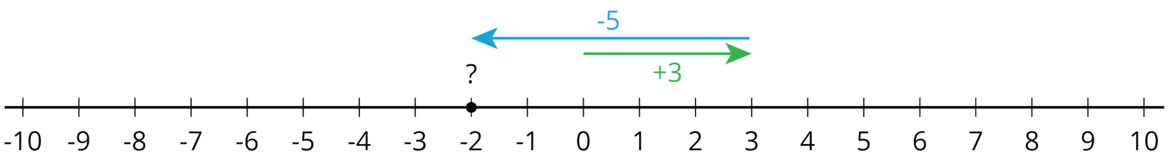


We can solve the equation $? + 5 = 7$ by adding -5 to both sides. This shows that $7 - 5 = 7 + (\text{-}5)$

Likewise, $3 - 5 = ?$ is equivalent to $? + 5 = 3$.



Notice that the value of $3 + (\text{-}5)$ is -2.



We can solve the equation $? + 5 = 3$ by adding -5 to both sides. This shows that $3 - 5 = 3 + (\text{-}5)$

In general:

$$a - b = a + (\text{-}b)$$

If $a - b = x$, then $x + b = a$. We can add $\text{-}b$ to both sides of this second equation to get that $x = a + (\text{-}b)$

Lesson 5 Practice Problems

Problem 1

Statement

Write each subtraction equation as an addition equation.

a. $a - 9 = 6$

b. $p - 20 = -30$

c. $z - (-12) = 15$

d. $x - (-7) = -10$

Solution

a. $a = 6 + 9$ (or $a = 9 + 6$)

b. $p = -30 + 20$ (or $p = 20 + -30$)

c. $z = 15 + (-12)$ (or $z = (-12) + 15$)

d. $x = -10 + (-7)$ (or $x = -7 + (-10)$)

Problem 2

Statement

Find each difference. If you get stuck, consider drawing a number line diagram.

a. $9 - 4$

b. $4 - 9$

c. $9 - (-4)$

d. $-9 - (-4)$

e. $-9 - 4$

f. $4 - (-9)$

g. $-4 - (-9)$

h. $-4 - 9$

Solution

a. 5

b. -5

c. 13

d. -5

e. -13

f. 13

g. 5

h. -13

Problem 3

Statement

A restaurant bill is \$59 and you pay \$72. What percentage gratuity did you pay?

Solution

22%, because $13 \div 59 \approx 0.22$.

(From Unit 4, Lesson 10.)

Problem 4

Statement

Find the solution to each equation mentally.

a. $30 + a = 40$

b. $500 + b = 200$

c. $-1 + c = -2$

d. $d + 3{,}567 = 0$

Solution

a. $a = 10$

b. $b = -300$

c. $c = -1$

d. $d = -3{,}567$

Problem 5

Statement

One kilogram is 2.2 pounds. Complete the tables. What is the interpretation of the constant of proportionality in each case?

pounds	kilograms
2.2	1
11	
5.5	
1	

______ kilogram per pound

kilograms	pounds
1	2.2
7	
30	
0.5	

______ pounds per kilogram

Solution

Complete the tables. What is the interpretation of the constant of proportionality in each case?

a.

pounds	kilograms
2.2	1
11	5
5.5	2.5
1	0.45

0.45 kilogram per pound

b.

kilograms	pounds
1	2.2
7	15.4
30	66
0.5	1.1

2.2 pounds per kilogram

(From Unit 2, Lesson 3.)