## Lesson 7: Interpreting and Using Exponential Functions

* Let’s explore the ages of ancient things.

### 7.1: Halving and Doubling

1. A colony of microbes doubles in population every 6 hours. Explain why we could say that the population grows by a factor of $\sqrt[6]{2}$ every hour.
2. A bacteria population decreases by a factor of $\frac{1}{2}$ every 4 hours. Explain why we could also say that the population decays by a factor of $\sqrt[4]{\frac{1}{2}}$ every hour.

### 7.2: Radiocarbon Dating

Carbon-14 is used to find the age of certain artifacts and fossils. It has a half-life of 5,730 years, so if an object has carbon-14, it loses half of it every 5,730 years.

1. At a certain point in time, a fossil had 3 picograms (a trillionth of a gram) of carbon-14. Complete the table with the missing mass of carbon-14 and years.

| * number of years after fossil had3 picograms of carbon-14
 | * mass of carbon-14in picograms
 |
| --- | --- |
| * 0
 | * 3
 |
| * 1,910
 | *
 |
| * 5,730
 | *
 |
| *
 | * 0.75
 |

1. A scientist uses the expression $\left(2.5\right)⋅\left(\frac{1}{2}\right)^{\frac{t}{5,730}}$ to model the number of picograms of carbon-14 remaining in a different fossil $t$ years after 20,000 BC.
	1. What do the 2.5, $\frac{1}{2}$, and 5,730 mean in this situation?
	2. Would more or less than 0.1 picogram of carbon-14 remain in this fossil today? Explain how you know.

### 7.3: Old Manuscripts

The half-life of carbon-14 is about 5,730 years.

1. Pythagoras lived between 600 BCE and 500 BCE. Explain why the age of a papyrus from the time of Pythagoras is about half of a carbon-14 half-life.
2. Someone claims they have a papyrus scroll written by Pythagoras. Testing shows the scroll has 85% of its original amount of carbon-14 remaining. Explain why the scroll is likely a fake.

#### Are you ready for more?

A copy of the Gutenberg Bible was made around 1450. Would more or less than 90% of the carbon-14 remain in the paper today? Explain how you know.

### Lesson 7 Summary

Some substances change over time through a process called radioactive decay, and their rate of decay can be measured or estimated. Let’s take sodium-22 as an example.

Suppose a scientist finds 4 nanograms of sodium-22 in a sample of an artifact. (One nanogram is 1 billionth, or $10^{-9}$, of a gram.) Approximately every 3 years, half of the sodium-22 decays. We can represent this change with a table.

| number of years after firstbeing measured | mass of sodium-22in nanograms |
| --- | --- |
| 0 | 4 |
| 3 | 2 |
| 6 | 1 |
| 9 | 0.5 |

This can also be represented by an equation. If the function $f$ gives the number of nanograms of sodium remaining after $t$ years then $f\left(t\right)=4⋅\left(\frac{1}{2}\right)^{\frac{t}{3}}$

The 4 represents the number of nanograms in the sample when it was first measured, while the $\frac{1}{2}$ and 3 show that the amount of sodium is cut in half every 3 years, because if you increase $t$ by 3, you increase the exponent by 1.

How much of the sodium remains after one year? Using the equation, we find $f\left(1\right)=4⋅\left(\frac{1}{2}\right)^{\frac{1}{3}}$. This is about 3.2 nanograms.

About how many years after the first measurement will there be about 0.015 nanogram of sodium-22? One way to find out is by extending the table and multiplying the mass of sodium-22 by $\frac{1}{2}$ each time. If we multiply 0.5 nanogram (the mass of sodium-22 9 years after first being measured) by $\frac{1}{2}$ five more times, the mass is about 0.016 nanogram. For sodium-22, five half-lives means 15 years, so 24 years after the initial measurement, the amount of sodium-22 will be about 0.015 nanogram.

Archaeologists and scientists use exponential functions to help estimate the ages of ancient things.



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