Lesson 7: Adding and Subtracting to Solve Problems

Goals

- Apply addition and subtraction of signed numbers to solve problems in an unfamiliar context, and explain (orally and in writing) the solution method.
- Interpret signed numbers used to represent gains or losses in an unfamiliar context.

Learning Targets

- I can solve problems that involve adding and subtracting rational numbers.

Lesson Narrative

The purpose of this lesson is to put students' knowledge about addition and subtraction of signed numbers to use in real-life contexts. They work with tables that show the change, positive or negative, in quantities such as inventory or energy usage, and must make sense of these tables to answer questions about the context. An optional activity extends students' work with signed numbers on the number line to points in all four quadrants of the coordinate plane.

As students reason about quantities using signed numbers they engage in MP2.

Alignments

Building On

- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Addressing

- 7.NS.A.1.c: Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference, and apply this principle in real-world contexts.
- 7.NS.A.3: Solve real-world and mathematical problems involving the four operations with rational numbers.Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Building Towards

- 7.EE.B.4: Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR6: Three Reads
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's apply what we know about signed numbers to different situations.

7.1 Positive or Negative?

Warm Up: 5 minutes

The purpose of this warm-up is to have students reason about an equation involving positive and negative rational numbers using what they have learned about operations with rational numbers.

Building On

- 7.NS.A.1

Building Towards

- 7.EE.B.4

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet think time and ask them to give a signal when they have an answer and a strategy for the first question. Then have them discuss their reasoning with a partner. Ask for an explanation, and then ask if everyone agrees with that reasoning.

Then give students 30 seconds of quiet think time and ask them to give a signal when they have an answer for the second question. Then have them discuss their reasoning with a partner.

Student Task Statement

Without computing:

1. Is the solution to $-2.7 + x = -3.5$ positive or negative?
2. Select **all** the expressions that are solutions to $-2.7 + x = -3.5$.

    a. $-3.5 + 2.7$

    b. $3.5 - 2.7$

    c. $-3.5 - (-2.7)$

    d. $-3.5 - 2.7$

Student Response

1. The solution is negative.
2. $-3.5 + 2.7$ and $-3.5 - (-2.7)$

Activity Synthesis

Ask several student to share which expressions they chose for the second question. Discuss until everyone is in agreement about the answer to the second question.

7.2 Phone Inventory

10 minutes

Positive and negative numbers are often used to represent *changes* in a quantity. An increase in the quantity is positive, and a decrease in the quantity is negative. In this activity, students see an example of this convention and are asked to make sense of it in the given context.

Addressing

- 7.NS.A.3

Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 30 seconds of quiet work time followed by 1 minute of partner discussion for the first two problems. Briefly, ensure everyone agrees on the interpretation of positive and negative numbers in this context, and then invite students to finish the rest of the questions individually. Follow with whole-class discussion.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Use virtual or concrete manipulatives to connect symbols to concrete objects or values. For example, demonstrate "change" by adding or taking away phones in a whole-class discussion.
Supports accessibility for: Visual-spatial processing; Conceptual processing

---

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. To support small-group discussion, provide sentence frames such as: "The change between Monday and Tuesday is ___ because....". Some students may see the change in the column labeled "change" while others may note that on Monday, there were 18 phones, and on Tuesday, there are only 16. This will help students use the table representation to reason about the inventory.
Design Principle(s): Support sense-making

Student Task Statement

A store tracks the number of cell phones it has in stock and how many phones it sells.

The table shows the inventory for one phone model at the beginning of each day last week. The inventory changes when they sell phones or get shipments of phones into the store.

	inventory	change
Monday	18	-2
Tuesday	16	-5
Wednesday	11	-7
Thursday	4	-6
Friday	-2	20

1. What do you think it means when the change is positive? Negative?
2. What do you think it means when the inventory is positive? Negative?
3. Based on the information in the table, what do you think the inventory will be at on Saturday morning? Explain your reasoning.
4. What is the difference between the greatest inventory and the least inventory?

Student Response

Answers vary. Sample responses:

1. The inventory increases; the inventory decreases.
2. There are phones in the store that people can buy; someone ordered a phone but they are waiting for one to come into the store.
3. Sample response: I think the inventory will be 18, because the inventory one day is the sum of the inventory and the change on the previous day.

4. The difference is 20: $18 - (-2) = 20$

Activity Synthesis

Tell students that we often use positive and negative to represent changes in a quantity. Typically, an increase in the quantity is positive, and a decrease in the quantity is negative.

Ask students what they answered for the second question and record their responses. Highlight one or two that describe the situation clearly.

Ask a few students to share their answer to the third question, and discuss any differences. Then discuss the answer to the last question.

7.3 Solar Power

15 minutes

It is common to use positive numbers to represent credit and negative numbers to represent debts on a bill. This task introduces students to this convention and asks them to solve addition and subtraction questions in that context. Note that whether a number should be positive or negative is often a choice, which means one must be very clear about explaining the interpretation of a signed number in a particular context (MP6).

For the second question, monitor for students who find the amount each week and sum those, and students who sum the value of the electricity used and the value of the electricity generated separately, and then find the sum of those.

Addressing

- 7.NS.A.3

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR6: Three Reads
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 4 minutes of quiet work time followed by partner discussion. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Access for Perception. Read Han's electricity bill situation aloud. Students who both listen to and read the information will benefit from extra processing time. Check in with students to see if they have any questions about the context of the situation.
Supports accessibility for: Language

Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension of this word problem, without solving it for students. In the first read, read the problem with the goal of comprehending the situation (e.g., This problem is about a house with solar panels that generate energy.). In the second read, ask students to analyze the mathematical quantities (e.g., used $83.56 worth of electricity, generated $6.75 worth of electricity, current charges are $83.56, Solar Credit is -$6.75 and the amount due is $74.81). In the third read, ask students to brainstorm possible strategies to answer the questions. This helps students connect the language in the word problem and the reasoning needed to solve the problem, while still keeping the intended level of cognitive demand in the task.
Design Principle(s): Support sense-making

Student Task Statement

Han's family got a solar panel. Each month they get a credit to their account for the electricity that is generated by the solar panel. The credit they receive varies based on how sunny it is.

Here is their electricity bill from January.

Current charges: $83.56
Solar Credit: -$6.75
Amount due: $76.81

In January they used $83.56 worth of electricity and generated $6.75 worth of electricity.

1. In July they were traveling away from home and only used $19.24 worth of electricity. Their solar panel generated $22.75 worth of electricity. What was their amount due in July?

2. The table shows the value of the electricity they used and the value of the electricity they generated each week for a month. What amount is due for this month?

	used ($)	generated ($)
week 1	13.45	-6.33
week 2	21.78	-8.94
week 3	18.12	-7.70
week 4	24.05	-5.36

3. What is the difference between the value of the electricity generated in week 1 and week 2? Between week 2 and week 3?

Student Response

1. -$3.51
2. $49.07
3. Between week 1 and week 2 it is $2.61. Between week 2 and week 3 it is -$1.24.

Are You Ready for More?

While most rooms in any building are all at the same level of air pressure, hospitals make use of "positive pressure rooms" and "negative pressure rooms." What do you think it means to have negative pressure in this setting? What could be some uses of these rooms?

Student Response

Here the pressure of a room is being measured relative to the air pressure outside of the room, by taking the quantity

$$(\text{air pressure inside}) - (\text{air pressure outside})$$

So a positive pressure room is one where there air pressure inside the room is greater than the air pressure outside the room, and the reverse for negative pressure rooms. In positive pressure rooms, air does not naturally flow into the room, so it is a good place to keep patients who have a weakened immune system and are very susceptible to getting infected by airborne diseases. In negative pressure rooms, air does not naturally flow out of the room, so it is a good place to keep patients who are highly contagious.

Activity Synthesis

Ask one or more students to share their answer to the first question and resolve any discrepancies.

Ask selected students to share their reasoning for the second questions. Discuss the relative merits of different approaches to solving the problem.

Finish by going over the solution to the third question. Point out that the bill will reflect a negative number in the amount due section, but we can interpret this to mean that the family receives a credit, and it will be applied to their next bill.

7.4 Differences and Distances

Optional: 15 minutes (there is a digital version of this activity)
In grade 6, students practiced finding the horizontal or vertical distance between points on a coordinate plane. In this activity, students see that this can be done by subtracting the x or y-coordinates for the points (MP7). Students continue to work with the distinction between *distance* (which is unsigned) and *difference* (which is signed) (MP6). This prepares them finding the slope of a line and the diagonal distance between points in grade 8.

Addressing

- 7.NS.A.1.c
- 7.NS.A.3

Instructional Routines

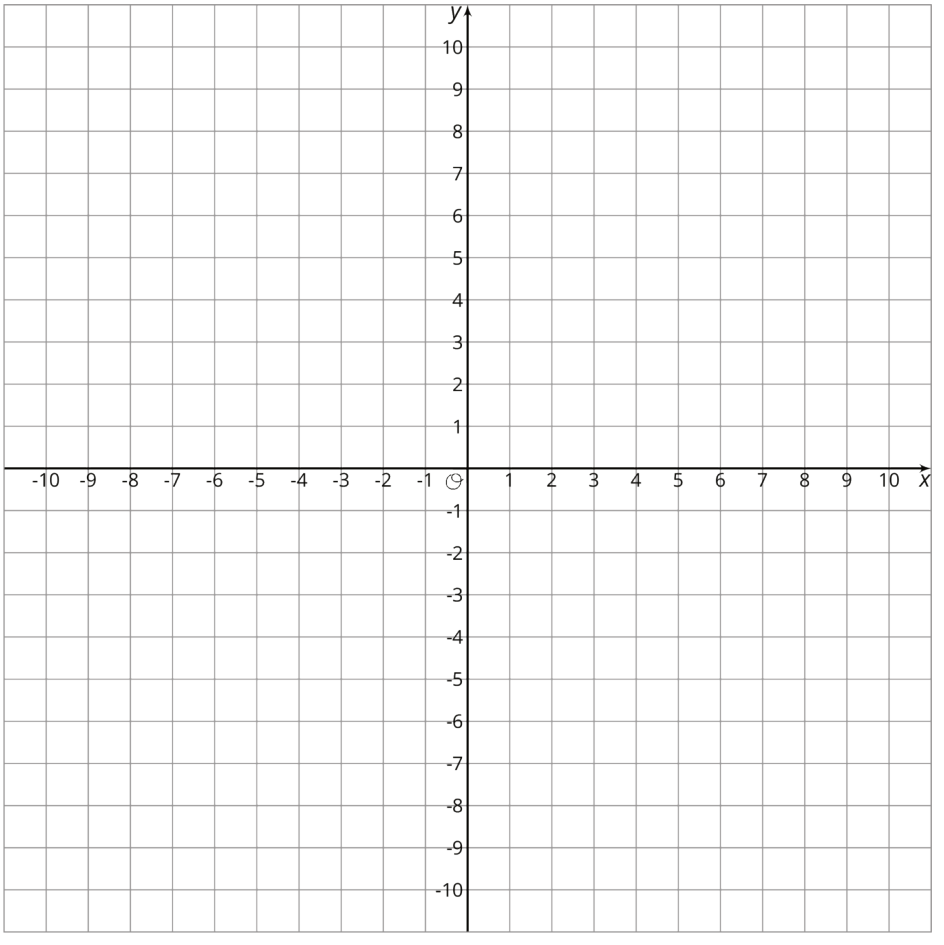
- MLR8: Discussion Supports
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 3 minutes of quiet work time followed by partner discussion. Follow with a whole-class discussion.

Student Task Statement

Plot these points on the coordinate grid: $A = (5, 4), B = (5, -2), C = (-3, -2), D = (-3, 4)$



1. What shape is made if you connect the dots in order?
2. What are the side lengths of figure $ABCD$?
3. What is the difference between the x-coordinates of B and C?
4. What is the difference between the x-coordinates of C and B?
5. How do the differences of the coordinates relate to the distances between the points?

Student Response

1. a rectangle
2. 6 and 8
3. 8
4. -8
5. The absolute value of the difference is the distance.

Activity Synthesis

Main learning points:

- When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their x-coordinates.
- When two points in the coordinate plane lie on a vertical line, you can find the distance between them by subtracting their y-coordinates.
- The distance between two numbers is independent of the order, but the difference depends on the order.

Discussion questions:

- Explain what makes the distance between two points and the difference between two points distinct.
- Explain how you would find the vertical or horizontal distance between two points.
- Explain how you would find the vertical or horizontal difference between two points.

Access for English Language Learners

Representing, Speaking: MLR8 Discussion Supports. Before the whole-class discussion, invite students to discuss and prepare their responses to the discussion questions listed in the synthesis. Display the questions for all to see, and provide sentence frames that students can use to explain their reasoning. For example, "To find the vertical (or horizontal) distance between two points, first we _____, and then we _____.", and "To find the vertical (or horizontal difference) between two points, first we _____, and then we _____." Listen for and amplify the ways students describe the distinction between distance (which is unsigned) and difference (which is signed). This opportunity to prepare in advance will provide students with additional opportunities to clarify their thinking, and to consider how they will communicate their reasoning.
Design Principle(s): Support sense-making

Lesson Synthesis

What are some situations where adding and subtracting rational numbers can help us solve problems?

7.5 Coffee Shop Cups

Cool Down: 5 minutes
Addressing

- 7.NS.A.3

Student Task Statement

Here is some record keeping from a coffee shop about their paper cups. Cups are delivered 2,000 at a time.

day	change
Monday	+2000
Tuesday	-125
Wednesday	-127
Thursday	+1719
Friday	-356
Saturday	-782
Sunday	0

1. Explain what a positive and negative number means in this situation
2. Assume the starting amount of coffee cups is 0. How many paper cups are left at the end of the week?
3. How many cups do you think were used on Thursday? Explain how you know.

Student Response

1. Interpretations vary. Positive might mean number of cups delivered or delivered minus used, negative might mean used up.
2. 2329
3. 281. Explanations vary. Sample explanation: It looks like some were delivered and some were used. Since they are delivered 2,000 at a time, then $2{,}000 - 1{,}719$ would be the number used.

Student Lesson Summary

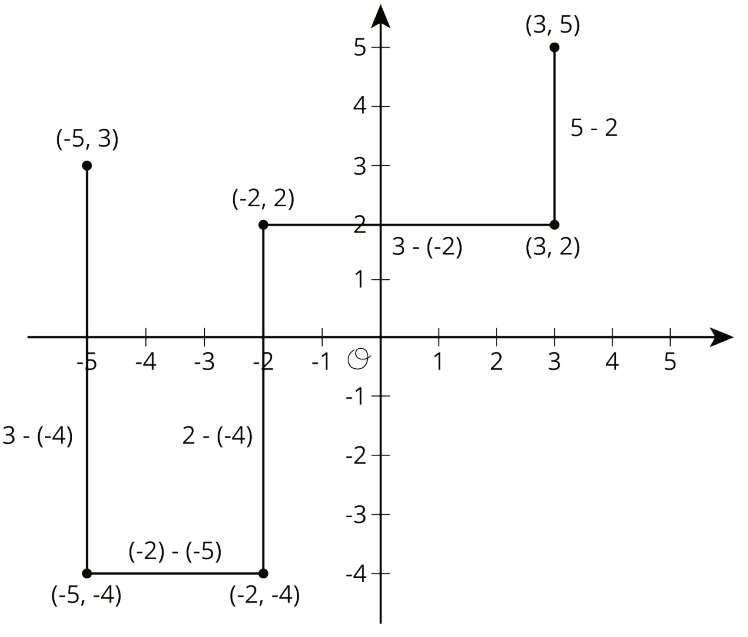
Sometimes we use positive and negative numbers to represent quantities in context. Here are some contexts we have studied that can be represented with positive and negative numbers:

- temperature
- elevation
- inventory
- an account balance
- electricity flowing in and flowing out

In these situations, using positive and negative numbers, and operations on positive and negative numbers, helps us understand and analyze them. To solve problems in these situations, we just have to understand what it means when the quantity is positive, when it is negative, and what it means to add and subtract them.

When two points in the coordinate plane lie on a horizontal line, you can find the distance between them by subtracting their x-coordinates.

When two points in the coordinate plane lie on a vertical line, you can find the distance between them by subtracting their y-coordinates.



Remember: the *distance* between two numbers is independent of the order, but the *difference* depends on the order.

Lesson 7 Practice Problems

Problem 1

Statement

The table shows four transactions and the resulting account balance in a bank account, except some numbers are missing. Fill in the missing numbers.

	transaction amount	account balance
transaction 1	360	360
transaction 2	-22.50	337.50
transaction 3		182.35
transaction 4		-41.40

Solution

	transaction amount	account balance
transaction 1	360	360
transaction 2	-22.50	337.50
transaction 3	-155.15	182.35
transaction 4	-223.75	-41.40

Problem 2

Statement

The *departure from the average* is the difference between the actual amount of rain and the average amount of rain for a given month. The historical average for rainfall in Albuquerque, NM for June, July, and August is shown in the table.

June	July	August
0.67	1.5	1.57

a. Last June only 0.17 inches of rain fell all month. What is the difference between the average rainfall and the actual rainfall for last June?

b. The departure from the average rainfall last July was -0.36 inches. How much rain fell last July?

c. How much rain would have to fall in August so that the total amount of rain equals the average rainfall for these three months? What would the departure from the average be in August in that situation?

Solution

1. 0.5 inches. $0.67 - 0.17 = 0.50$
2. 1.14 inches. The departure from average was negative, the actual rainfall needed to be 0.36 inches less than the average rainfall.
3. 2.43 inches; 0.86 inches. The departure from the average in June was -0.5 and for July was -0.36, so for those two months it was $-0.5 + (-0.36) = -0.86$. So it will have to rain 0.86 more inches in August than usual to make that up and the departure from the average will be 0.86.
$0.86 + 1.57 = 2.43$.

Problem 3

Statement

a. How much higher is 500 than 400 m?

b. How much higher is 500 than -400 m?

c. What is the change in elevation from 8,500 m to 3,400 m?

d. What is the change in elevation between 8,500 m and -300 m?

e. How much higher is -200 m than 450 m?

Solution

a. 100 m, because $400 + 100 = 500$

b. 900 m, because $-400 + 900 = 500$

c. -5,100 m, because $8{,}500 + (-5{,}100) = 3{,}400$

d. -8,800 m, because $8{,}500 + (-8{,}800) = -300$

e. -650 m, because $450 + (-650) = -200$

(From Unit 5, Lesson 6.)

Problem 4

Statement

Tyler orders a meal that costs $15.

a. If the tax rate is 6.6%, how much will the sales tax be on Tyler's meal?

b. Tyler also wants to leave a tip for the server. How much do you think he should pay in all? Explain your reasoning.

Solution

a. $0.99 because $15 \cdot 0.066 = 0.99$

b. Answers vary. Sample response: I think Tyler should pay $19 in all, because that would cover his meal, the sales tax, and a 20% tip for the server.

(From Unit 4, Lesson 10.)

Problem 5

Statement

In a video game, a character is healed at a constant rate as long as they are standing in a certain circle. Complete the table.

time in circle (seconds)	health gained (points)
4	100
10	
3	
	1,000

Solution

seconds in circle	health gained
4	100
10	250
3	75
40	1,000

(From Unit 2, Lesson 3.)