Lesson 8: Position, Speed, and Direction

Goals

- Explain (orally and in writing) how signed numbers can be used to represent positions and speeds in opposite directions.
- Generalize (orally) that the product of a negative number and a positive number is negative.
- Write a multiplication equation to represent a situation involving constant speed with direction.

Learning Targets

- I can multiply a positive number with a negative number.
- I can use rational numbers to represent speed and direction.

Lesson Narrative

In this lesson, students are introduced to multiplying a negative number with a positive number, using the context of velocity, time, and position. In the next lesson, they multiply two negative numbers.

The context of elevation is an example of using signed numbers to represent the position of an object along a line relative to a reference position (sea level in the case of elevation). In the general case, zero represents the reference position, positive numbers represent positions on one side of the reference position, and negative numbers represent positions on the other side. In this lesson, students see that signed numbers can also be used to represent *speed with direction*. Scientists use the term *velocity* to describe the speed of an object in a specified direction. If one object is moving with a positive velocity, then any object moving in the opposite direction will have a negative velocity.

In previous units, students solved problems about moving objects, using the fact that the product of the (positive) speed and the (positive) travel time gives the (positive) distance traveled. In this lesson, students use several examples in the context of moving along a line to see that the product of a *negative* velocity and a positive travel time results in a *negative* position relative to the starting point.

Alignments

Addressing

- 7.NS.A.2.a: Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.

- 7.RP.A: Analyze proportional relationships and use them to solve real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports

> **Student Learning Goals**
>
> Let's use signed numbers to represent movement.

8.1 Distance, Rate, Time

Warm Up: 5 minutes

This activity reminds students of previous work they have done with constant speed situations, using $d = rt$ for the relationship between distance, rate, and time. This prepares students for representing movement in opposite directions using signed numbers in the rest of this lesson.

Students may fall back to earlier methods to make sense of these problems and come up with a solution, like creating a double number line or a table of equivalent ratios relating distance and time. These approaches are fine. In the discussion, though, ensure everyone understands using $d = rt$ to represent the relationship between distance traveled, elapsed time, and rate of travel for constant speed situations.

Addressing

- 7.RP.A

Launch

Ask students what they remember about problems involving distance, rate, and time. They might offer that distances traveled and elapsed time creates a set of equivalent ratios, or that the elapsed time can be multiplied by the speed to give the distance traveled. Give students 1 minute of quiet work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may struggle knowing whether they should multiply or divide the numbers in each problem situation. Remind them of the equation $d = rt$.

> **Student Task Statement**
>
> 1. An airplane moves at a constant speed of 120 miles per hour for 3 hours. How far does it go?
> 2. A train moves at constant speed and travels 6 miles in 4 minutes. What is its speed in miles per minute?

3. A car moves at a constant speed of 50 miles per hour. How long does it take the car to go 200 miles?

Student Response

1. 360 miles, because $120 \cdot 3 = 360$
2. $\frac{3}{2}$ or equivalent miles per minute, because $6 \div 4 = \frac{3}{2}$
3. 4 hours, because $200 \div 50 = 4$

Activity Synthesis

The most important thing for students to remember is that the equation $d = rt$ can be used to solve problems involving movement at a constant speed.

- To find the distance traveled, you can multiply the rate of travel (or speed) by the elapsed time.
- To find the rate of travel (or speed), you can divide the distance by the elapsed time.
- To find the elapsed time, you can divide the distance traveled by the rate of travel (or speed).

Consider drawing a table to facilitate the discussion of each problem and to remind students of the strategies they used while working with proportional relationships, such as using a scale factor or calculating the constant of proportionality. When relating distance and time in a constant speed situation, the speed is the constant of proportionality.

8.2 Going Left, Going Right

10 minutes
The purpose of this activity is to understand that a rate of travel at a constant speed (defined as velocity) can indicate the direction of travel, by using a negative or positive value to describe travel to the left or to the right of a location taken to be 0.

Students use their earlier understanding of a chosen zero point and describe their movement left (negative) or right (positive) along the number line, with different speeds. This will produce negative or positive end points depending on if they are moving to the left or the right. This will lead to students describing negative numbers multiplied by positive in the next activity.

Addressing

- 7.NS.A.2.a

Instructional Routines

- MLR2: Collect and Display

Launch

Remind students we have seen in earlier lessons that we can pick a location to represent zero, and then locations on either side are positive or negative.

Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, use the same color to highlight important connections between the number line and changes in position in the table.
Supports accessibility for: Visual-spatial processing

Access for English Language Learners

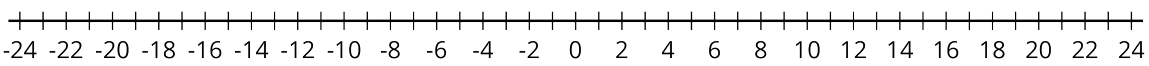
Conversing: MLR2 Collect and Display. As students discuss their expressions with a partner, listen for and collect the language students use to identify and describe the direction of travel in their expressions. Write the students' words and phrases on a visual display and update it throughout the remainder of the lesson. Remind students to borrow language from the display as needed. This will help students read and use mathematical language during their paired and whole-group discussions.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Anticipated Misconceptions

Students may want to use the number line to help them with the position changes in the table.

Have students write in words how they calculated ending positions for the left and right if they get stuck trying to write an expression with variables.

Student Task Statement



1. After each move, record your location in the table. Then write an expression to represent the ending position that uses the starting position, the speed, and the time. The first row is done for you.

starting position	direction	speed (units per second)	time (seconds)	ending position (units)	expression
0	right	5	3	+15	$0 + 5 \cdot 3$
0	left	4	6		
0	right	2	8		
0	right	6	2		
0	left	1.1	5		

2. How can you see the *direction* of movement in the expression?

3. Using a starting position p, a speed s, and a time t, write two expressions for an ending position. One expression should show the result of moving right, and one expression should show the result of moving left.

Student Response

1.

starting position	direction	speed (units per second)	time (seconds)	ending position	expression
0	right	5	3	+15	$0 + 5 \cdot 3$
0	left	4	6	-24	$0 - 4 \cdot 6$
0	right	2	8	+16	$0 + 2 \cdot 8$
0	right	6	2	+12	$0 + 6 \cdot 2$
0	left	1.1	5	-5.5	$0 - 1.1 \cdot 5$

2. The direction of movement is represented by whether you add to or subtract from 0.

3. Moving right $p + s \cdot t$. Moving left $p - s \cdot t$.

Activity Synthesis

Ask students to share how they could see the direction of travel in their expressions. It has to do with whether they added or subtracted the product st from zero. Ensure that everyone understands why $p + st$ represents final positions to the right of zero and $p - st$ represents positions to the left of zero.

We saw, earlier in this unit, that subtracting a positive is the same as adding a negative. So let's write a *single* expression, $p + vt$ where instead of speed (which is always a positive number) we use a signed number for *speed plus direction* and we call this quantity *velocity*. Velocities when moving to the right will be represented by positive numbers, and velocities when moving to the left will be represented by negative numbers.

8.3 Velocity

15 minutes

The purpose of this activity is for students to encounter a concrete situation where multiplying two positive numbers results in a positive number, and multiplying a positive and a negative number results in a negative number.

Students use their earlier understanding of a chosen zero point, location relative to this as a positive or negative quantity and description of movement left (negative) or right (positive) along the number line, with different speeds. They extend their understanding to movement with positive and negative velocity and different times. This will produce negative or positive end points depending on if the velocity is negative or positive. Looking at a number of different examples will help students describe rules for identifying the sign of the product of a negative number with a positive number (MP8).

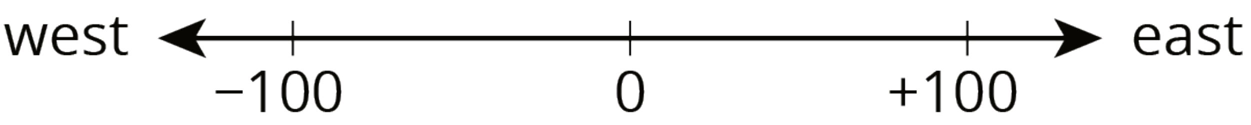
Addressing

- 7.NS.A.2.a

Instructional Routines

- MLR8: Discussion Supports

Launch



Display the image to remind students of west (left, negative) and east (right, positive) from the previous activity. Describe that we can talk about speed in a direction by calling it velocity and using a sign, so negative velocities represent movement west, and positive velocities represent movement east.

Access for Students with Disabilities

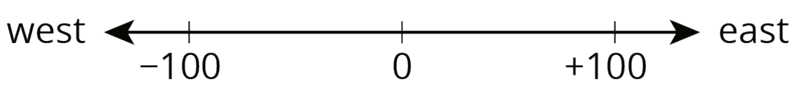
Representation: Internalize Comprehension. Begin with a physical demonstration of positive and negative velocity and different number times using the given number line to support connections between new situations and prior understandings. Consider using these prompts—“What does this demonstration have in common with moving left or right in the previous activity?” or “How is the direction of velocity related to the positive or negative sign?”
Supports accessibility for: Conceptual processing; Visual-spatial processing

Anticipated Misconceptions

Encourage students who get stuck to use the provided number line to represent each situation.

Student Task Statement

A traffic safety engineer was studying travel patterns along a highway. She set up a camera and recorded the speed and direction of cars and trucks that passed by the camera. Positions to the east of the camera are positive, and to the west are negative.



Vehicles that are traveling towards the east have a positive velocity, and vehicles that are traveling towards the west have a negative velocity.

1. Complete the table with the position of each vehicle if the vehicle is traveling at a constant speed for the indicated time period. Then write an equation.

velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation describing the position
+25	+10	+250	$25 \cdot 10 = 250$
-20	+30		
+32	+40		
-35	+20		
+28	0		

2. If a car is traveling east when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply two positive numbers, is the result positive or negative?

3. If a car is traveling west when it passes the camera, will its position be positive or negative 60 seconds after it passes the camera? If we multiply a negative and a positive number, is the result positive or negative?

Student Response

velocity (meters per second)	time after passing the camera (seconds)	ending position (meters)	equation describing position
+25	+10	+250	$25 \cdot 10 = 250$
-20	+30	-600	$(-20) \cdot 30 = -600$
+32	+40	+1280	$32 \cdot 40 = 1280$
-35	+20	-700	$-35 \cdot 20 = -700$
+28	0	0	$28 \cdot 0 = 0$

2. A car traveling east will be at a positive position 60 seconds after passing the camera—this means that if we multiply two positive numbers the result is positive.

3. A car traveling west will be at a negative position 60 seconds after passing the camera—this means that if we multiply a negative and a positive number the result is negative.

Are You Ready for More?

In many contexts we can interpret negative rates as "rates in the opposite direction." For example, a car that is traveling -35 miles per hour is traveling in the opposite direction of a car that is traveling 40 miles per hour.

1. What could it mean if we say that water is flowing at a rate of -5 gallons per minute?
2. Make up another situation with a negative rate, and explain what it could mean.

Student Response

1. Answers vary. Sample responses: A tank is draining out 5 gallons of water per minute. Due to tides, a creek is flowing upstream at a rate of 5 gallons per minute.
2. Answers vary.

Activity Synthesis

The most important thing for students to understand from this activity is that if we multiply two positive numbers the result is positive and that if we multiply a positive and a negative number the result is negative.

Ask a student to share their rationale about each problem. Display the number line from the launch, and place the negative answers in the context of the problem (to the west). Make sure the distinction is made between the velocity (the direction of movement) and the position. Then, ensure students see that at least in this case, it appears that when we multiply two positive values, the product is positive. But when we multiply a positive by a negative value, the product is negative. We are going to take this to be true moving forward, even if the numbers represent other things.

Access for English Language Learners

Speaking: MLR8: Discussion Supports. As students make sense of the ending position of the vehicles, provide sentence frames such as: “If we multiply two positive numbers, the result is ___ because....” and “If we multiply a positive and a negative number, the result is ___ because....”. This helps students use mathematical language to generalize about multiplying positive and negative numbers.
Design Principle(s): Support sense-making; Optimize output (for generalization)

Lesson Synthesis

Main takeaways:

- We can choose a zero point and then positive and negative numbers can represent positions to the right or left of this zero point.
- Signed numbers can also be used to represent speed in opposite directions. This is called velocity.
- A negative number multiplied by a positive gets a negative product.

Discussion questions:

- How can we represent a position to the left or right of a starting point without using direction words?
- How can we represent how fast something is moving to the left or right from a starting point? What word do we use to represent speed with a direction?
- What kind of number do you get when you multiply a negative number by a positive number?

8.4 Multiplication Expressions

Cool Down: 5 minutes
Addressing

- 7.NS.A.2.a

Student Task Statement

Four runners start at the same point; Lin, Elena, Diego, Andre. For each runner write a multiplication equation that describes their journey.

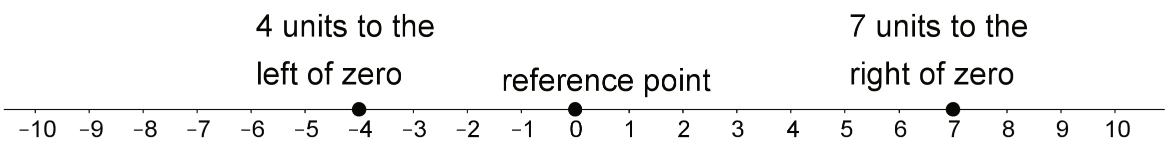
1. Lin runs for 25 seconds at 8.2 meters per second. What is her finish point?
2. Elena runs for 28 seconds and finishes at 250 meters. What is her velocity?
3. Diego runs for 32 seconds at -8.1 meters per second. What is his finish point?
4. Andre runs for 35 seconds and finishes at -285 meters. What is his velocity?

Student Response

1. $25 \cdot 8.2 = ?$
2. $28 \cdot ? = 250$
3. $32 \cdot (-8.1) = ?$
4. $35 \cdot ? = -285$

Student Lesson Summary

We can use signed numbers to represent the position of an object along a line. We pick a point to be the reference point, and call it zero. Positions to the right of zero are positive. Positions to the left of zero are negative.



When we combine speed with direction indicated by the sign of the number, it is called *velocity*. For example, if you are moving 5 meters per second to the right, then your velocity is +5 meters per second. If you are moving 5 meters per second to the left, then your velocity is -5 meters per second.

If you start at zero and move 5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the right of zero. In other words, $5 \cdot 10 = 50$.

If you start at zero and move -5 meters per second for 10 seconds, you will be $5 \cdot 10 = 50$ meters to the *left* of zero. In other words,

$$-5 \cdot 10 = -50$$

In general, a negative number times a positive number is a negative number.

Lesson 8 Practice Problems

Problem 1

Statement

A number line can represent positions that are north and south of a truck stop on a highway. Decide whether you want positive positions to be north or south of the truck stop. Then plot the following positions on a number line.

a. The truck stop

b. 5 miles north of the truck stop

c. 3.5 miles south of the truck stop

Solution

Either choice is fine as long as students are consistent in the next part.

Problem 2

Statement

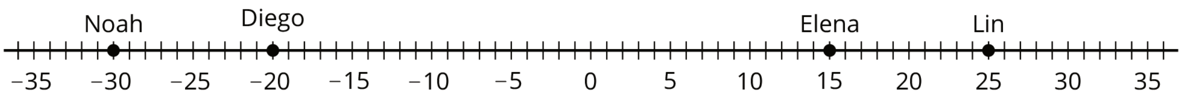
a. How could you distinguish between traveling west at 5 miles per hour and traveling east at 5 miles per hour without using the words "east" and "west"?

b. Four people are cycling. They each start at the same point. (0 represents their starting point.) Plot their finish points after five seconds of cycling on a number line

  - Lin cycles at 5 meters per second
  - Diego cycles at -4 meters per second
  - Elena cycles at 3 meters per second
  - Noah cycles at -6 meters per second

Solution

a. By giving the velocities opposite signs.

b. 

Problem 3

Statement

Find the value of each expression.

a. $16.2 + -8.4$

b. $\frac{2}{5} - \frac{3}{5}$

c. $-9.2 + -7$

d. $-4\frac{3}{8} - (-1\frac{1}{4})$

Solution

a. 7.8

b. $\frac{-1}{5}$

c. -16.2

d. $-3\frac{1}{8}$

(From Unit 5, Lesson 6.)

Problem 4

Statement

For each equation, write two more equations using the same numbers that express the same relationship in a different way.

a. $3 + 2 = 5$

b. $7.1 + 3.4 = 10.5$

c. $15 - 8 = 7$

d. $\frac{3}{2} + \frac{9}{5} = \frac{33}{10}$

Solution

For each question, students should have 2 of the 3 equations listed.

a. $5 - 3 = 2$; $5 - 2 = 3$; $2 + 3 = 5$

b. $10.5 - 3.4 = 7.1$; $10.5 - 7.1 = 3.4$; $3.4 + 7.1 = 10.5$

c. $15 - 7 = 8$; $8 + 7 = 15$; $7 + 8 = 15$

d. $\frac{33}{10} - \frac{3}{2} = \frac{9}{5}$; $\frac{33}{10} - \frac{9}{5} = \frac{3}{2}$; $\frac{9}{5} + \frac{3}{2} = \frac{33}{10}$

(From Unit 5, Lesson 5.)

Problem 5

Statement

A shopper bought a watermelon, a pack of napkins, and some paper plates. In his state, there is no tax on food. The tax rate on non-food items is 5%. The total for the three items he bought was \$8.25 before tax, and he paid \$0.19 in tax. How much did the watermelon cost?

Solution

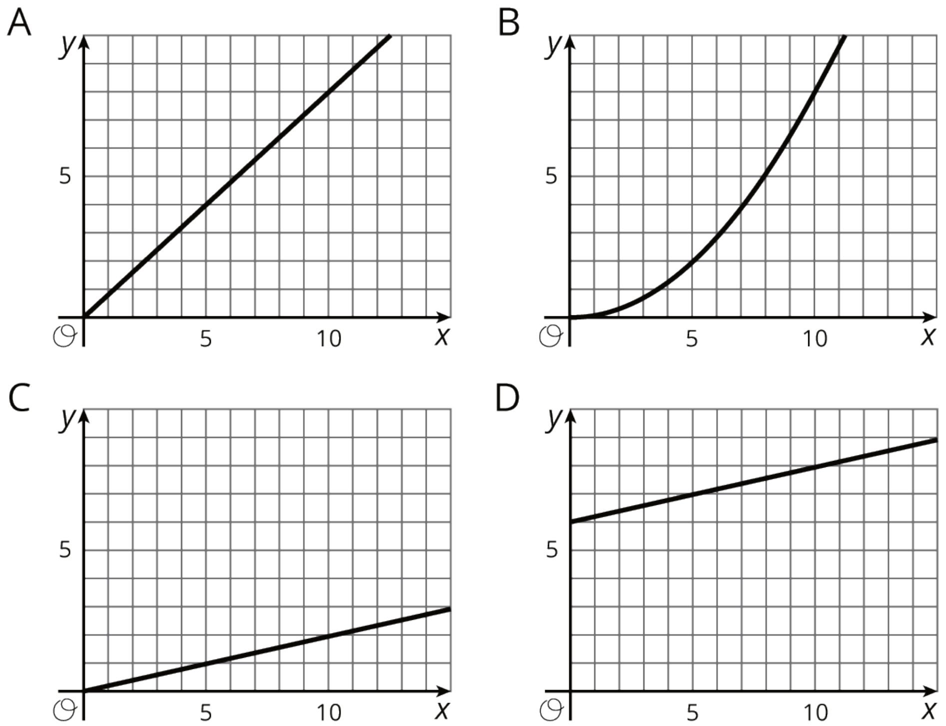
\$4.45. The non-food items cost $0.19 \div 0.05 = 3.8$. The watermelon cost $8.25 - 3.8 = 4.45$.

(From Unit 4, Lesson 10.)

Problem 6

Statement

Which graphs could not represent a proportional relationship? Explain how you decided.



Solution

B and D are graphs that do not represent proportional relationships. B is not a straight line and D does not go through the origin.

(From Unit 2, Lesson 10.)