## Lesson 14: Solving Exponential Equations

* Let’s solve equations using logarithms.

### 14.1: A Valid Solution?

To solve the equation , Lin wrote the following:

Is her solution valid? Be prepared to explain what she did in each step to support your answer.

### 14.2: Natural Logarithm

1. Complete the table with equivalent equations. The first row is completed for you.

|  | * exponential form | * logarithmic form |
| --- | --- | --- |
| * a. |  |  |
| * b. |  |  |
| * c. |  |  |
| * d. |  |  |
| * e. |  |  |

1. Solve each equation by expressing the solution using notation. Then, find the approximate value of the solution using the “ln” button on a calculator.

### 14.3: Solving Exponential Equations

Without using a calculator, solve each equation. It is expected that some solutions will be expressed using log notation. Be prepared to explain your reasoning.

#### Are you ready for more?

1. Solve the equations and . Express your answers as logarithms.
2. What is the relationship between these two solutions? Explain how you know.

### Lesson 14 Summary

So far we have solved exponential equations by

* finding whole number powers of the base (for example, the solution of is 5)
* estimation (for example, the solution of is between 2 and 3)
* using a logarithm and approximating its value on a calculator (for example, the solution of is )

Sometimes solving exponential equations takes additional reasoning. Here are a couple of examples.

In the first example, the power of 10 is multiplied by 5, so to find the value of that makes this equation true each side was divided by 5. From there, the equation was rewritten as a logarithm, giving an exact value for .

In the second example, the expressions on each side of the equation were rewritten as powers of 10: . This means that the exponent on one side and the 3 on the other side must be equal, and leads to a simpler expression to solve where we don't need to use a logarithm.

How do we solve an exponential equation with base , such as ? We can express the solution using the **natural logarithm**, the logarithm for base . Natural logarithm is written as , or sometimes as . Just like the equation can be rewritten, in logarithmic form, as , the equation and be rewritten as . Similarly, can be rewritten as .

All this means that we can solve by rewriting the equation as . This says that is the exponent to which base is raised to equal 5.

To estimate the size of , remember that is about 2.7. Because 5 is greater than , this means that is greater than 1. is about or 7.3. Because 5 is less than , this means that is less than 2. This suggests that is between 1 and 2. Using a calculator we can check that .



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