## Lesson 14: Solving Exponential Equations

* Let’s solve equations using logarithms.

### 14.1: A Valid Solution?

To solve the equation $5⋅e^{3a}=90$, Lin wrote the following:

$\begin{matrix}5⋅e^{3a}&=90\\e^{3a}&=18\\3a&=log\_{e}18\\a&=\frac{log\_{e}18}{3}\end{matrix}$

Is her solution valid? Be prepared to explain what she did in each step to support your answer.

### 14.2: Natural Logarithm

1. Complete the table with equivalent equations. The first row is completed for you.

| *
 | * exponential form
 | * logarithmic form
 |
| --- | --- | --- |
| * a.
 | * $e^{0}=1$
 | * $ln1=0$
 |
| * b.
 | * $e^{1}=e$
 | *
 |
| * c.
 | * $e^{-1}=\frac{1}{e}$
 | *
 |
| * d.
 | *
 | * $ln\frac{1}{e^{2}}=-2$
 |
| * e.
 | * $e^{x}=10$
 | *
 |

1. Solve each equation by expressing the solution using $ln$ notation. Then, find the approximate value of the solution using the “ln” button on a calculator.
	1. $e^{m}=20$
	2. $e^{n}=30$
	3. $e^{p}=7.5$

### 14.3: Solving Exponential Equations

Without using a calculator, solve each equation. It is expected that some solutions will be expressed using log notation. Be prepared to explain your reasoning.

1. $10^{x}=10,​000$
2. $5⋅10^{x}=500$
3. $10^{\left(x+3\right)}=10,​000$
4. $10^{2x}=10,​000$
5. $10^{x}=315$
6. $2⋅10^{x}=800$
7. $10^{\left(1.2x\right)}=4,​000$
8. $7⋅10^{\left(0.5x\right)}=70$
9. $2⋅e^{x}=16$
10. $10⋅e^{3x}=250$

#### Are you ready for more?

1. Solve the equations $10^{n}=16$ and $10^{n}=2$. Express your answers as logarithms.
2. What is the relationship between these two solutions? Explain how you know.

### Lesson 14 Summary

So far we have solved exponential equations by

* finding whole number powers of the base (for example, the solution of $10^{x}=100,​000$ is 5)
* estimation (for example, the solution of $10^{x}=300$ is between 2 and 3)
* using a logarithm and approximating its value on a calculator (for example, the solution of $10^{x}=300$ is $log300≈2.48$)

Sometimes solving exponential equations takes additional reasoning. Here are a couple of examples.

$\begin{matrix}5⋅10^{x}&=45\\5⋅10^{x}&=45\\10^{x}&=9\\x&=log9\end{matrix}$

$\begin{matrix}10^{\left(0.2t\right)}&=1,​000\\10^{\left(0.2t\right)}&=10^{3}\\0.2t&=3\\t&=\frac{3}{0.2}\\t&=15\end{matrix}$

In the first example, the power of 10 is multiplied by 5, so to find the value of $x$ that makes this equation true each side was divided by 5. From there, the equation was rewritten as a logarithm, giving an exact value for $x$.

In the second example, the expressions on each side of the equation were rewritten as powers of 10: $10^{\left(0.2t\right)}=10^{3}$. This means that the exponent $0.2t$ on one side and the 3 on the other side must be equal, and leads to a simpler expression to solve where we don't need to use a logarithm.

How do we solve an exponential equation with base $e$, such as $e^{x}=5$? We can express the solution using the **natural logarithm**, the logarithm for base $e$. Natural logarithm is written as $ln$, or sometimes as $log\_{e}$. Just like the equation $10^{2}=100$ can be rewritten, in logarithmic form, as $log\_{10}100=2$, the equation $e^{0}=1$ and be rewritten as $ln1=0$. Similarly, $e^{-2}=\frac{1}{e^{2}}$ can be rewritten as $ln\frac{1}{e^{2}}=-2$.

All this means that we can solve $e^{x}=5$ by rewriting the equation as $x=ln5$. This says that $x$ is the exponent to which base $e$ is raised to equal 5.

To estimate the size of $ln5$, remember that $e$ is about 2.7. Because 5 is greater than $e^{1}$, this means that $ln5$ is greater than 1. $e^{2}$ is about $\left(2.7\right)^{2}$ or 7.3. Because 5 is less than $e^{2}$, this means that $ln5$ is less than 2. This suggests that $ln5$ is between 1 and 2. Using a calculator we can check that $ln5≈1.61$.



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