## Lesson 12: The Number $e$

* Let’s learn about the number $e$.

### 12.1: Matching Situations and Equations

Match each equation to a situation it represents. Be prepared to explain how you know. Not all equations have a match.

$f\left(t\right)=400⋅\left(0.5\right)^{0.1t}$

$g\left(t\right)=400⋅\left(1.25\right)^{0.1t}$

$h\left(t\right)=400⋅\left(0.75\right)^{0.1t}$

$j\left(t\right)=400⋅\left(2\right)^{10t}$

$k\left(t\right)=400⋅\left(2\right)^{0.1t}$

1. A scientist begins an experiment with 400 bacteria in a petri dish. The population doubles every 10 hours. The function gives the number of bacteria $t$ hours since the experiment began.
2. A patient takes 400 mg of a medicine. The amount of medicine in her bloodstream decreases by 25% every 10 hours. The function gives the amount of medicine left in her bloodstream after $t$ hours of taking the medicine.
3. The half-life of a radioactive element is 10 years. There are 400 g of the element in a sample when it is first studied. The function gives the amount of the element remaining $t$ years later.
4. In a lake, the population of a species of fish is 400. The population is expected to grow by 25% in the next decade. The function gives the number of fish in the lake $t$ years after it was 400.

### 12.2: Notice and Wonder: Moldy Growth

A spot of mold is found on a basement wall. Its area is about 10 square centimeters. Here are three representations of a function that models how the mold is growing.

| time (weeks) | area of mold (sq cm) |
| --- | --- |
| 0 | 10 |
| 1 | 27 |
| 2 | 74 |
| 3 | 201 |
| 4 | 546 |

$a\left(t\right)=10⋅e^{t}$



What do you notice? What do you wonder?

### 12.3: $\left(1+tiny\right)^{huge}$

1. Here are some functions. For each function, describe, in words, the outputs for very tiny, positive values of $x$ and for very large values of $x$.
* $a\left(x\right)=1^{x}$
* $b\left(x\right)=-x$
* $d\left(x\right)=\frac{1}{x}$
* $f\left(x\right)=\left(\frac{1}{x}\right)^{x}$
* $g\left(x\right)=\left(1+\frac{1}{x}\right)^{x}$
* $h\left(x\right)=e^{x}$
* $k\left(x\right)=1+x$
*
1. Remember that $e≈2.718$. What does the function $g$ have to do with the number $e$?
2. What do you notice about the relationship between $h$ and $k$ for very small, positive values of $x$?

#### Are you ready for more?

Complete the table to show the value of each expression to the nearest hundred-thousandth. Two entries have already been completed as an example.

| $x$ | $2^{x}$ | $e^{x}$ | $3^{x}$ |
| --- | --- | --- | --- |
| 0.1 | 1.07177 | 1.10517 | $$ |
| 0.01 |   |   |   |
| 0.001 |   |   |   |
| 0.0001 |   |   |   |

What do you notice about the values in the table?

### Lesson 12 Summary

Scientists, economists, engineers, and others often use the number $e$ in their mathematical models. What is $e$?

$e$ is an important constant in mathematics, just like the constant $π$, which is important in geometry. The value of $e$ is approximately 2.718. Just like $π$, the number $e$ is irrational, so it can’t be represented as a fraction, and its decimal representation never repeats or terminates. The number is named after the 18th-century mathematician Leonhard Euler and is sometimes called *Euler’s number*.

$e$ has many useful properties and it arises in situations involving exponential growth or decay, so $e$ often appears in exponential functions. In upcoming lessons, we will work with functions that are expressed using $e$.



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