Lesson 11: Dividing Rational Numbers

Goals

- Apply multiplication and division of signed numbers to solve problems involving constant speed with direction, and explain (orally) the reasoning.
- Generalize (orally) a method for determining the quotient of two signed numbers.
- Generate a division equation that represents the same relationship as a given multiplication equation with signed numbers.

Learning Targets

- I can divide rational numbers.

Lesson Narrative

In this lesson, students complete their work extending all four operations to signed numbers by studying division. They use the relationship between multiplication and division to develop rules for dividing signed numbers. In preparation for the next lesson on negative rates of change, students look at a context, drilling a well, that is modeled by an equation $y = kx$ where k is a negative number. This builds on their previous work with proportional relationships.

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.

Addressing

- 7.NS.A.2: Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
- 7.NS.A.2.b: Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts.

Building Towards

- 7.EE.B.4.a: Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. For example, the perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width?

Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports
- Think Pair Share

Student Learning Goals

Let's divide signed numbers.

11.1 Tell Me Your Sign

Warm Up: 5 minutes
For this warm-up students use what they have learned about multiplication and division with rational numbers to answer questions about the solution to an equation.

Building On

- 7.NS.A

Building Towards

- 7.EE.B.4.a

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time and ask them to discuss their reasoning with a partner. Follow with a whole-class discussion.

Student Task Statement

Consider the equation: $-27x = -35$

Without computing:

1. Is the **solution** to this equation positive or negative?
2. Are either of these two numbers solutions to the equation?

$\frac{35}{27}$ $\qquad\qquad$ $-\frac{35}{27}$

Student Response

1. Positive
2. The first one

Activity Synthesis

Ask students to share their reasoning.

11.2 Multiplication and Division

10 minutes

The purpose of this activity is to understand that the division facts for rational numbers are simply a consequence of the multiplication done previously. Students work several numerical examples relating multiplication to division, and then articulate a rule for the sign of a quotient based on the signs of the dividend and divisor (MP8).

Monitor for students who identify and describe the rule clearly.

Addressing

- 7.NS.A.2.b

Instructional Routines

- MLR6: Three Reads
- Think Pair Share

Launch

Remind students that we can rearrange division equations to be multiplication equations, and vice versa. It may be useful to demonstrate with positive numbers if students struggle to recall this. For example, ask how we could rewrite $10 \div 2 = 5$ as a multiplication equation. Students can also reference the multiplication table from a previous lesson, if needed.

Arrange students in groups of 2. Give students 4 minute of quiet work time followed by 2 minutes of partner discussion, then follow with whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lesson on signed multiplication to provide an entry point into this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

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Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support students' reading comprehension of the final question. In the first read, students read the text with the goal of understanding what the situation is about (e.g., Han and Clare are walking toward each other). In the second read, ask students to name the important quantities (e.g., Han's velocity, Clare's velocity, each person's distance from 0 in feet, elapsed time in seconds). After the third read, ask students to brainstorm possible strategies to determine where each person will be 10 seconds before they meet up, and when each person will be -10 feet from the meeting place. This will help students connect the language in the word problem with the reasoning needed to solve the problem.
Design Principle(s): Support sense-making

Anticipated Misconceptions

Have students refer to their previous work with signed multiplication to help them with the division statements.

Student Task Statement

1. Find the missing values in the equations

    a. $-3 \cdot 4 = ?$

    b. $-3 \cdot ? = 12$

    c. $3 \cdot ? = 12$

    d. $? \cdot -4 = 12$

    e. $? \cdot 4 = -12$

2. Rewrite the unknown factor problems as division problems.

3. Complete the sentences. Be prepared to explain your reasoning.

    a. The sign of a positive number divided by a positive number is always:

    b. The sign of a positive number divided by a negative number is always:

    c. The sign of a negative number divided by a positive number is always:

    d. The sign of a negative number divided by a negative number is always:

4. Han and Clare walk towards each other at a constant rate, meet up, and then continue past each other in opposite directions. We will call the position where they meet up 0 feet and the time when they meet up 0 seconds.

- Han's velocity is 4 feet per second.
- Clare's velocity is -5 feet per second.

a. Where is each person 10 seconds before they meet up?

b. When is each person at the position -10 feet from the meeting place?

Student Response

1. a. -12

   b. -4

   c. 4

   d. -3

   e. -3

2. Answers vary. Possible responses:
   a. NA

   b. $12 \div (-3) = -4$

   c. $12 \div 4 = 3$

   d. $12 \div (-4) = -3$

   e. $-12 \div 4 = -3$

3. a. positive

   b. negative

   c. negative

   d. positive

4. a. Han was at -40 feet and Clare was at 50 feet.

   b. Han was there at -2.5 seconds and Clare was there at 2 seconds.

Are You Ready for More?

It is possible to make a new number system using *only* the numbers 0, 1, 2, and 3. We will write the symbols for multiplying in this system like this: $1 \otimes 2 = 2$. The table shows some of the products.

$\otimes$	0	1	2	3
0	0	0	0	0
1		1	2	3
2			0	2
3				

1. In this system, $1 \otimes 3 = 3$ and $2 \otimes 3 = 2$. How can you see that in the table?
2. What do you think $2 \otimes 1$ is?
3. What about $3 \otimes 3$?
4. What do you think the solution to $3 \otimes n = 2$ is?
5. What about $2 \otimes n = 3$?

Student Response

1. The table shows $1 \otimes 3 = 3$ in the cell in the "1" row and the "3" column. The table shows $2 \otimes 3 = 2$ in the cell in the "2" row and the "3" column.
2. It could be anything, but if multiplication is commutative, then $2 \otimes 1 = 2$.
3. It could be anything, but if we fill out the table assuming multiplication is commutative, we can get almost all of the entries:

$\otimes$	0	1	2	3
0	0	0	0	0
1	0	1	2	3
2	0	2	0	2
3	0	3	2	

   Looking at the table, setting $3 \otimes 3 = 1$ would make things symmetrical.

   A more sophisticated argument requires us to know how to add these numbers, which we saw in an earlier extension. If the distributive property holds and $1 \oplus 2 = 3$, then $3 \otimes 3 = (1 \oplus 2) \otimes 3 = (1 \otimes 3) \oplus (2 \otimes 3) = 3 \oplus 2$. In the earlier extension we suggested that $3 \oplus 2 = 1$.

4. $2 \otimes 3 = 2$ so if multiplication is commutative, then $3 \otimes 2 = 2$, so $n = 2$.
5. Looking at all of the $2 \otimes$, there is no value of n that makes this equation true.

Activity Synthesis

Select students who can articulate and explain what they have learned about the sign of a quotient based on the signs of the dividend and divisor (note that students do not need to use these words, they can just give examples). Highlight that the division facts are simply a consequence of the multiplication done previously. Moving forward, we are going to take these facts to be true for all rational numbers.

11.3 Drilling Down

10 minutes (there is a digital version of this activity)
The purpose of this activity is to use the new skills of multiplying and dividing rational numbers to represent and solve problems in a new context (MP4). In this activity students are using using multiplication and division of negatives and working with a proportional relationship with a negative constant of proportionality. Students should use what they know about proportional relationships to help them (for example; points lie on a straight line, line passes through zero, there is a point $(1, k)$ that lies on the line).

Addressing

- 7.NS.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Remind students that we can model positions below the surface with negative values, so drilling 30 feet down is represented with -30 feet.

Ask students what they remember about proportional relationships. Examples:

- Often represented with an equation in the form $y = kx$.
- The constant of proportionality, often called k, is the change in y for a change by 1 in x.
- A graph representing a proportional relationship is a line through (0,0) and $(1, k)$

Arrange students in groups of 3 during the discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide appropriate reading accommodations and supports to ensure students have access to written directions, word problems and other text-based content.
Supports accessibility for: Language; Conceptual processing

Anticipated Misconceptions

Some students may struggle to get the length of time out of the phrase "one full day of continuous use," wondering whether the drill was running at night. Let them know the drill has been going for 24 hours.

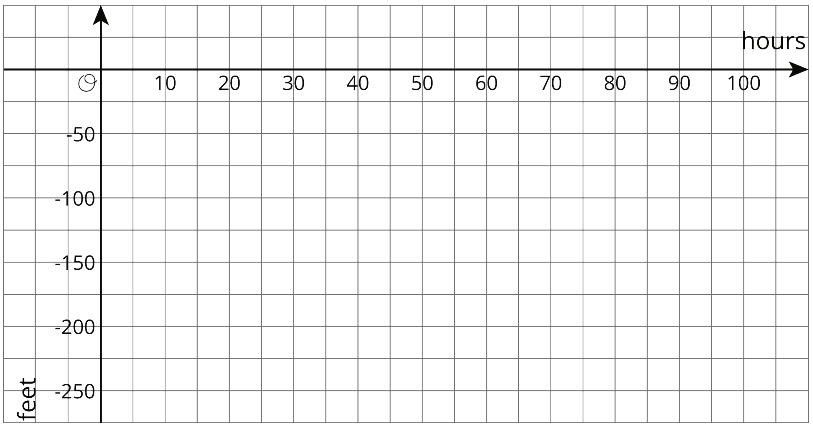
Student Task Statement

A water well drilling rig has dug to a height of -60 feet after one full day of continuous use.

1. Assuming the rig drilled at a constant rate, what was the height of the drill after 15 hours?

2. If the rig has been running constantly and is currently at a height of -147.5 feet, for how long has the rig been running?

3. Use the coordinate grid to show the drill's progress.



4. At this rate, how many hours will it take until the drill reaches -250 feet?

Student Response

1. The drill drills $-60 \div 24$ or -2.5 feet in one hour. After 15 hours it has drilled -37.5 feet, because $-2.5 \cdot 15 = -37.5$.
2. The drill has been running for 59 hours, because $-147.5 \div (-2.5)$ is 59.
3. A ray starting at $(0, 0)$ and passing through $(24, -60)$ and $(59, -147.5)$.
4. 100 hours because $-250 \div (-2.5) = 100$

Activity Synthesis

Have students to share their solutions with each other in groups of three and work to come to agreement.

To wrap up, emphasize that because we can now complete any calculation with any rational number, we can extend constants of proportionality k to include negative values. Because the operations are now well-defined, we can answer questions using such a constant.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Invite select groups to share their solution and reasoning with the class. After each group presents, ask students to restate and/or revoice what they heard using mathematical language. Consider providing students time to restate what they heard to a partner, before selecting one or two students to share with the class. Ask the original group if this accurately reflects their thinking. This will provide more students with an opportunity to describe what they have learned about proportional relationships with a negative constant of proportionality.
Design Principle(s): Support sense-making

Lesson Synthesis

Key takeaways:

- Recall that we can write a division problem as a multiplication problem.
- A positive divided by a negative is negative.
- A negative divided by a positive is negative.
- A negative divided by a negative is positive.

Discussion questions:

- What kind of number do you get when you divide a negative number by a positive number? Use a multiplication equation to explain why this makes sense.
- What kind of number do you get when you divide a negative number by a negative number? Use a multiplication equation to explain why this makes sense.

11.4 Matching Division Expressions

Cool Down: 5 minutes
Addressing

- 7.NS.A.2.b

Student Task Statement

Match each expression with its value.

1. $15 \div 12$
2. $12 \div (-15)$
3. $12 \div 15$
4. $15 \div (-12)$

- -0.8
- 0.8
- -1.25
- 1.25

Student Response

1. $15 \div 12 = 1.25$
2. $12 \div (-15) = -0.8$
3. $12 \div 15 = 0.8$
4. $15 \div (-12) = -1.25$

Student Lesson Summary

Any division problem is actually a multiplication problem:

- $6 \div 2 = 3$ because $2 \cdot 3 = 6$
- $6 \div -2 = -3$ because $-2 \cdot -3 = 6$
- $-6 \div 2 = -3$ because $2 \cdot -3 = -6$
- $-6 \div -2 = 3$ because $-2 \cdot 3 = -6$

Because we know how to multiply signed numbers, that means we know how to divide them.

- The sign of a positive number divided by a negative number is always negative.
- The sign of a negative number divided by a positive number is always negative.
- The sign of a negative number divided by a negative number is always positive.

A number that can be used in place of the variable that makes the equation true is called a **solution** to the equation. For example, for the equation $x \div -2 = 5$, the solution is -10, because it is true that $-10 \div -2 = 5$.

Glossary

- solution to an equation

Lesson 11 Practice Problems

Problem 1

Statement

Find the quotients:

a. $24 \div -6$

b. $-15 \div 0.3$

c. $-4 \div -20$

Solution

-4, -50, 0.2

Problem 2

Statement

Find the quotients.

a. $\frac{2}{5} \div \frac{3}{4}$

b. $\frac{9}{4} \div \frac{-3}{4}$

c. $\frac{-5}{7} \div \frac{-1}{3}$

d. $\frac{-5}{3} \div \frac{1}{6}$

Solution

a. $\frac{8}{15}$

b. -3

c. $\frac{15}{7}$

d. -10

Problem 3

Statement

Is the solution positive or negative?

a. $2 \cdot x = 6$

b. $-2 \cdot x = 6.1$

c. $2.9 \cdot x = -6.04$

d. $-2.473 \cdot x = -6.859$

Solution

a. Positive

b. Negative

c. Negative

d. Positive

Problem 4

Statement

Find the solution mentally.

a. $3 \cdot -4 = a$

b. $b \cdot (-3) = -12$

c. $-12 \cdot c = 12$

d. $d \cdot 24 = -12$

Solution

a. $a = -12$

b. $b = 4$

c. $c = -1$

d. $d = \frac{-1}{2}$

Problem 5

Statement

In order to make a specific shade of green paint, a painter mixes $1\frac{1}{2}$ quarts of blue paint, 2 cups of green paint, and $\frac{1}{2}$ gallon of white paint. How much of each color is needed to make 100 cups of this shade of green paint?

Solution

Blue: $37\frac{1}{2}$ cups, green: $12\frac{1}{2}$ cups, white: 50 cups. There are 4 cups in a quart so $1\frac{1}{2}$ quarts is 6 cups. There are 16 cups in a gallon so $\frac{1}{2}$ gallons is 8 cups. So the ratio of cups of blue to cups of green to cups of white is 6 to 2 to 8 or, equivalently, 3 to 1 to 4. $3 + 1 + 4 = 8$. $100 \div 8 = 12.5$ So, the painter needs 37.5 cups of blue because $3 \cdot (12.5) = 37.5$. The painter needs 12.5 cups of green because $1 \cdot (12.5) = 12.5$. And the painter needs 50 cups of white because $4 \cdot (12.5) = 50$.

(From Unit 4, Lesson 2.)

Problem 6

Statement

Here is a list of the highest and lowest elevation on each continent.

	highest point (m)	lowest point (m)
Europe	4,810	-28
Asia	8,848	-427
Africa	5,895	-155
Australia	4,884	-15
North America	6,198	-86
South America	6,960	-105
Antartica	4,892	-50

a. Which continent has the largest difference in elevation? The smallest?

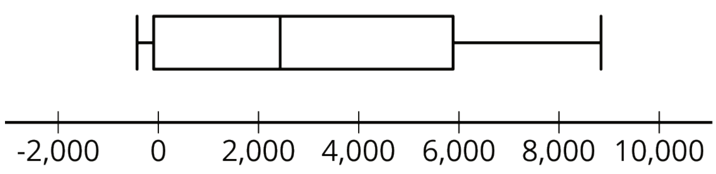
b. Make a display (dot plot, box plot, or histogram) of the data set and explain why you chose that type of display to represent this data set.

Solution

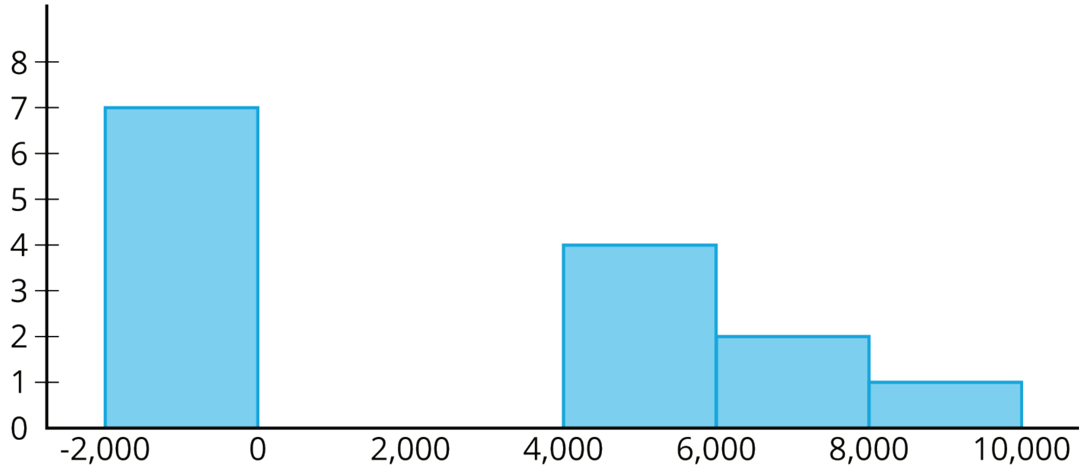
a. Asia has the largest difference in elevation. $8,848 - (\text{-}427) = 9,275$. Europe has the smallest difference in elevation. $4{,}810 - (\text{-}28) = 4{,}838$

b. Answers vary. Possible solutions:

- Box plot:



- Histogram:



I chose to make a histogram because it can show the gap in the middle of the data better than a box plot can. Also, this data set would be hard to see on a dot plot because some of the elevations are very close to each other, but none of them are exactly the same.

(From Unit 5, Lesson 3.)