## Unit 2 Lesson 19: End Behavior of Rational Functions

### 1 Different Divisions, Revisited (Warm up)

#### Student Task Statement

Complete all three representations of the polynomial division following the forms of the integer division.



$\begin{matrix}2x^{2}\\x+12x^{3}+7x^{2}+7x+5\end{matrix}$

$2775=11\left(252\right)+3$

$2x^{3}+7x^{2}+7x+5=$

$\frac{2775}{11}=252+\frac{3}{11}$

$\frac{2x^{3}+7x^{2}+7x+5}{x+1}=$

### 2 Combined Fuel Economy

#### Student Task Statement

In 2000, the Environmental Protection Agency (EPA) reported a combined fuel efficiency for cars that assumes 55% city driving and 45% highway driving. The expression for the combined fuel efficiency of a car that gets $x$ mpg in the city and $h$ mpg on the highway can be written as $\frac{100xh}{55x+45h}$.

1. Several conventional cars have a fuel economy for highway driving that is about 10 mpg higher than for city driving. That is, $h=x+10$. Write a function $f$ that represents the combined fuel efficiency for cars like these in terms of $x$.
2. Rewrite $f$ in the form $q\left(x\right)+\frac{r\left(x\right)}{b\left(x\right)}$ where $q\left(x\right)$, $r\left(x\right)$, and $b\left(x\right)$ are polynomials.

### 3 Exploring End Behavior

#### Student Task Statement

| function | degreeof num. | degreeof den. | rewritten in the form of$q\left(x\right)+\frac{r\left(x\right)}{b\left(x\right)}$ | end behavior |
| --- | --- | --- | --- | --- |
| $g\left(x\right)=-\frac{5}{x+2}$ |  |  |  |  |
| $h\left(x\right)=\frac{7x−5}{x+2}$ |  |  |  |  |
| $j\left(x\right)=\frac{3x^{2}+7x−5}{x+2}$ |  |  |  |  |
| $k\left(x\right)=\frac{2x^{3}+3x^{2}+7x−5}{x+2}$ |  |  |  |  |
| $m\left(x\right)=\frac{x+2}{2x^{3}+3x^{2}+7x−5}$ |  |  |  |  |

1. Complete the table to explore the end behavior for rational functions.
2. What do you notice about the end behavior of different types of rational functions?



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