## Lesson 9: Equations of Lines

* Let’s investigate equations of lines.

### 9.1: Remembering Slope



The slope of the line in the image is $\frac{8}{15}$. Explain how you know this is true.

### 9.2: Building an Equation for a Line

1. The image shows a line.
* 
	1. Write an equation that says the slope between the points $\left(1,3\right)$ and $\left(x,y\right)$ is 2.
	2. Look at this equation: $y−3=2\left(x−1\right)$
	How does it relate to the equation you wrote?
1. Here is an equation for another line: $y−7=\frac{1}{2}\left(x−5\right)$
	1. What point do you know this line passes through?
	2. What is the slope of this line?
2. Next, let’s write a general equation that we can use for any line. Suppose we know a line passes through a particular point $\left(h,k\right)$.
	1. Write an equation that says the slope between point $\left(x,y\right)$ and $\left(h,k\right)$ is $m$.
	2. Look at this equation: $y−k=m\left(x−h\right)$. How does it relate to the equation you wrote?

### 9.3: Using Point-Slope Form

1. Write an equation that describes each line.
	1. the line passing through point $\left(-2,8\right)$ with slope $\frac{4}{5}$
	2. the line passing through point $\left(0,7\right)$ with slope $-\frac{7}{3}$
	3. the line passing through point $\left(\frac{1}{2},0\right)$ with slope -1
	4. the line in the image
	* 
2. Using the structure of the equation, what point do you know each line passes through? What’s the line’s slope?
	1. $y−5=\frac{3}{2}\left(x+4\right)$
	2. $y+2=5x$
	3. $y=-2\left(x−\frac{5}{8}\right)$

#### Are you ready for more?

Another way to describe a line, or other graphs, is to think about the coordinates as changing over time. This is especially helpful if we’re thinking tracing an object’s movement. This example describes the $x$- and $y$-coordinates separately, each in terms of time, $t$.







1. On the first grid, create a graph of $x=2+5t$ for $-2\leq t\leq 7$ with $x$ on the vertical axis and $t$ on the horizontal axis.
2. On the second grid, create a graph of $y=3−4t$ for $-2\leq t\leq 7$ with $y$ on the vertical axis and $t$ on the horizontal axis.
3. On the third grid, create a graph of the set of points $\left(2+5t,3−4t\right)$ for $-2\leq t\leq 7$ on the $xy$-plane.

### Lesson 9 Summary

The line in the image can be defined as the set of points that have a slope of 2 with the point $\left(3,4\right)$. An equation that says point $\left(x,y\right)$ has slope 2 with $\left(3,4\right)$ is $\frac{y−4}{x−3}=2$. This equation can be rearranged to look like $y−4=2\left(x−3\right)$.



The equation is now in **point-slope form**, or $y−k=m\left(x−h\right)$, where:

* $\left(x,y\right)$ is any point on the line
* $\left(h,k\right)$ is a particular point on the line that we choose to substitute into the equation
* $m$ is the slope of the line

Other ways to write the equation of a line include slope-intercept form, $y=mx+b$, and standard form, $Ax+By=C$.

To write the equation of a line passing through $\left(3,1\right)$ and $\left(0,5\right)$, start by finding the slope of the line. The slope is $-\frac{4}{3}$ because $\frac{5−1}{0−3}=-\frac{4}{3}$. Substitute this value for $m$ to get $y−k=-\frac{4}{3}\left(x−h\right)$. Now we can choose any point on the line to substitute for $\left(h,k\right)$. If we choose $\left(3,1\right)$, we can write the equation of the line as $y−1=-\frac{4}{3}\left(x−3\right)$.

We could also use $\left(0,5\right)$ as the point, giving $y−5=-\frac{4}{3}\left(x−0\right)$. We can rearrange the equation to see how point-slope and slope-intercept forms relate, getting $y=-\frac{4}{3}x+5$. Notice $\left(0,5\right)$ is the $y$-intercept of the line. The graphs of all 3 of these equations look the same.



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