Lesson 2: Circular Grid

Goals

- Comprehend that “a point on the circle” (in written and spoken language) refers to a point that lies on the edge of the circle and not in the circle’s interior.
- Create dilations of polygons using a circular grid given a scale factor and center of dilation.
- Explain (orally) how a dilation affects the size, side lengths and angles of polygons.

Learning Targets

- I can apply dilations to figures on a circular grid when the center of dilation is the center of the grid.

Lesson Narrative

The previous lesson introduced the general idea of a dilation as a method for producing scaled copies of geometric figures. This lesson formally introduces a method for producing dilations. A **dilation** has a center and a *scale factor*. For a dilation with center P and scale factor 2, for example, the center does not move. Meanwhile each point Q stays on ray PQ but its distance from P doubles (because the scale factor is 2).

A circular grid is an effective tool for performing a dilation. A circular grid has circles with radius 1 unit, 2 units, and so on all sharing the same center. Students experiment with dilations on a circular grid, where the center of dilation is the common center of the circles. By using the structure of the grid, they make several important discoveries about the images of figures after a dilation including:

- Each grid circle maps to a grid circle.
- Line segments map to line segments and, in particular, the image of a polygon is a scaled copy of the polygon.

The next several lessons will examine dilations on a rectangular grid and with no grid, solidifying student understanding of the relationship between a polygon and its dilated image. This echoes similar work in the previous unit investigating the relationship between a figure and its image under a rigid transformation.

As with previous geometry lessons, students should have access to geometry toolkits so they can make strategic choices about which tools to use (MP5).

Alignments

Building On

- 3.G.A: Reason with shapes and their attributes.

- 4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Student Learning Goals

Let’s dilate figures on circular grids.

2.1 Notice and Wonder: Concentric Circles

Warm Up: 5 minutes
The goal of this warm-up is to introduce the circular grid which students will examine in greater detail throughout this unit. The circles in the grid all have the same center and the distance between consecutive circles is the same. The circular grid is particularly useful for showing dilations where the center of dilation is the center of the grid.

Students engage in MP7 as they look for structure and relationships between the circles and lines in the picture.

Building On

- 3.G.A
- 4.MD.C.5

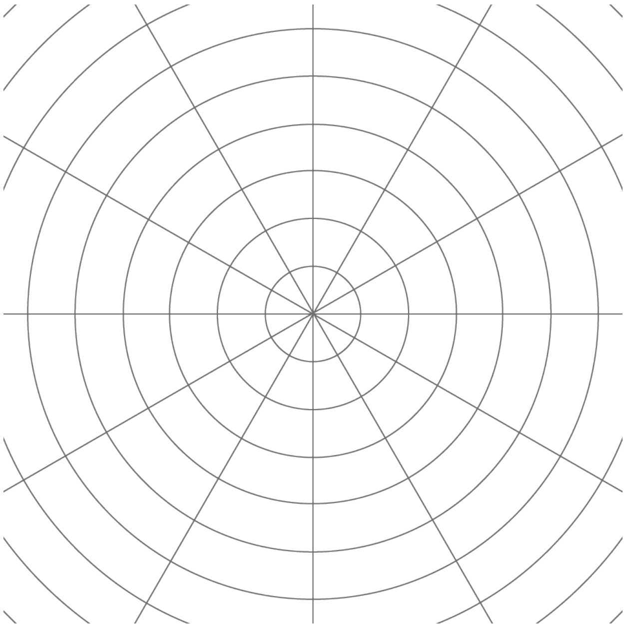
Instructional Routines

- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image, and their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement



What do you notice? What do you wonder?

Student Response

Answers vary. Sample responses:

- Do the circles have the same center?
- Is the center of the circles where the lines meet?
- Why are there 6 lines meeting in the center?
- Is the distance between the consecutive circles the same?
- How many pieces is each circle divided into? (This can be taken two ways depending whether you are talking about the one dimensional or two dimensional object.)

Activity Synthesis

Ask students to share their responses, highlighting these features of the picture:

- The circles share the same center
- The center of the circles is the point where the lines meet

- The distance from one circle to the next is always the same (the radius of each successive circle is one unit more than its predecessor)

Students may also notice that the angle made by successive rays from the center is always 30 degrees. Some things students may wonder include

- When is this grid useful?
- Why are the circles equally spaced?
- Why are the lines there?

2.2 A Droplet on the Surface

15 minutes (there is a digital version of this activity)
The purpose of this activity is to begin to think of a dilation with a scale factor as a rule or operation on points in the plane. Students work on a circular grid with center of dilation at the center of the grid. They examine what happens to different points on a given circle when the dilation is applied and observe that these points all map to another circle whose radius is scaled by the scale factor of the dilation. For example, if the scale factor is 3 and the points lie on a circle whose radius is 2 grid units, then the dilated points will all lie on a circle whose radius is 6 units. Students need to explain their reasoning for finding the scale factor (MP3).

Students discover that the circular grid is a powerful tool for representing dilations and they will continue to use the circular grid as they study what happens when dilations are applied to shapes other than grid circles.

In the digital activity, students encounter some new tools and the circular grid.

Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch

Ask students if they have ever seen a pebble dropped in a still pond, and select students to describe what happens. (The pebble becomes the center of a sequence of circular ripples.) Display the image from the task statement, and ask students to think about how it is like a pebble dropped in a still pond. Demonstrate that distance on the circular grid is measured by counting units along one of the rays that start at the center, P. Use MLR 8 (Discussion Supports) to draw students' attention to a few important words in the task:

- "When we say 'on the circle,' we mean on the curve or on the edge. (We do not mean the circle's interior.)"

- “Remember that a ray starts at a point and goes forever in one direction. Their rays should start at P and be drawn to the edge of the grid.”

If using the digital activity, you may want to demonstrate dilating a point before having students begin the task.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following terms and maintain the display for reference throughout the unit: circular grid, “on the circle,” ray, scale factor, and distance. Include a circular grid on the display, give examples, and label the features.
Supports accessibility for: Conceptual processing; Language

Anticipated Misconceptions

For question 5, students might think the scale factor is 4, because the distance between the smaller and larger circle for each point increases by 4. If this happens, ask students how many grid units circle c is from the center (2) and how many grid units circle d is from the center (6). Then remind them that *scale factor* means a number you multiply by.

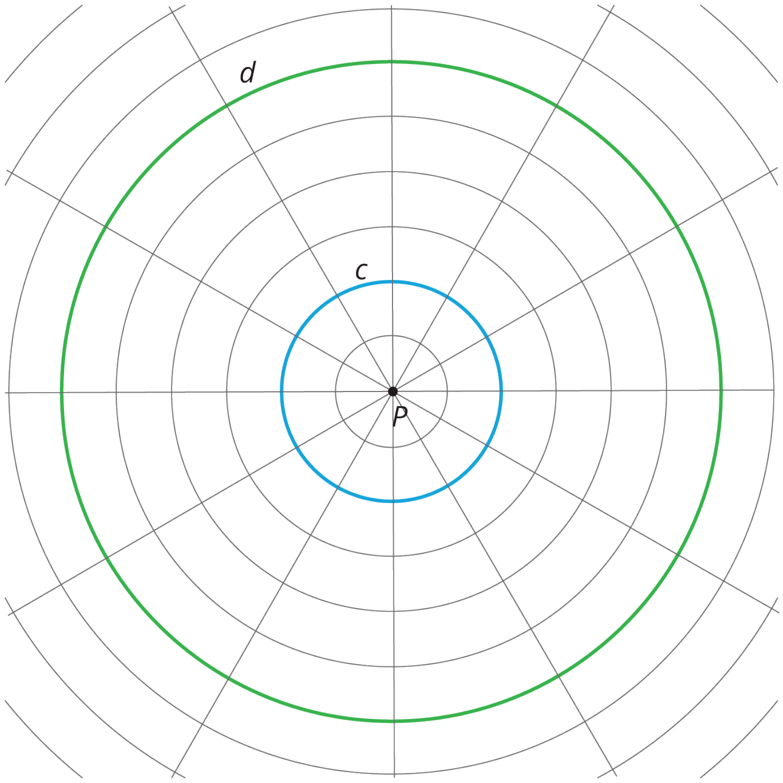
Student Task Statement

The larger Circle d is a **dilation** of the smaller Circle c. P is the **center of dilation**.

1. Draw four points *on* the smaller circle (not inside the circle!), and label them E, F, G, and H.

2. Draw the rays from P through each of those four points.

3. Label the points where the rays meet the larger circle E', F', G', and H'.



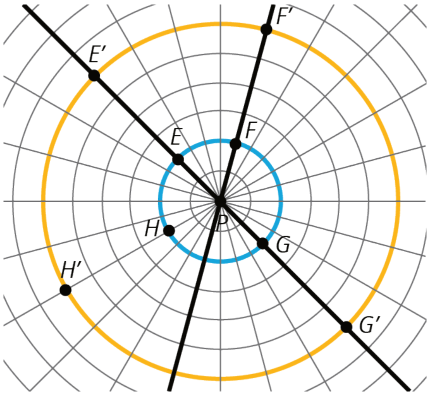
4. Complete the table. In the row labeled c, write the distance between P and the point on the smaller circle in grid units. In the row labeled d, write the distance between P and the corresponding point on the larger circle in grid units.

	E	F	G	H
c				
d				

5. The center of dilation is point P. What is the *scale factor* that takes the smaller circle to the larger circle? Explain your reasoning.

Student Response

1–3. Answers vary. Sample response:



4.

	E	F	G	H
c	2	2	2	2
d	6	6	6	6

5. The scale factor is 3 because the distance for the small circle is multiplied by 3 to find the distance in the large circle.

Activity Synthesis

Ask students if they made a strategic choice of points, such as points that lie on the grid lines coming from the center point P. Why are these points good choices for dilating?

Ask students what they think would happen if a circle were dilated about its center with a scale factor of 2 or 4. (The result would be a circle with twice the radius and 4 times the radius, respectfully, all sharing the same center.)

Two important observations coming from the lesson are:

1. The scale factor for this dilation is 3 so distances from the center of the circles triple when the dilation is applied.
2. The large circle is the dilation of the small circle, that is each point on the circle with radius 6 units is the dilated image of a point on the circle of radius 2 units. (To find which one, draw the line from the point to the center and see where it intersects the circle of radius 2 units.)

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. As students share the scale factor that takes the smaller circle to the larger circle, press for details in students' reasoning by asking how they know the scale factor is 3. Listen for students' explanations that reference the table with distances between the center of dilation and points on the circles. Amplify statements that use precise language such as, "The distance between point P and any point on the smaller circle is multiplied by a scale factor of 3 to get the distance between point P and the corresponding point on the larger circle." This will a support rich and inclusive discussion about how the scale factor affects the distance between the center of dilation and points on the circle.
Design Principle(s): Support sense-making

2.3 Quadrilateral on a Circular Grid

15 minutes (there is a digital version of this activity)
This activity continues studying dilations on a circular grid, this time focusing on what happens to points lying on a polygon. Students first dilate the vertices of a polygon as in the previous activity. Then they examine what happens to points on the sides of the polygon. They discover that when these points are dilated, they all lie on a side of another polygon. Just as the image of a grid circle is another circle, so the dilation of a polygon is another polygon. Moreover the dilated polygon is a scaled copy of the original polygon. These important properties of dilations are not apparent in the definition.

Monitor for students who notice that the sides of the scaled polygon $A'B'C'D'$ are parallel to the sides of $ABCD$ and that $A'B'C'D'$ is a scaled copy of $ABCD$ with scale factor 2. Also monitor for students who notice the same structure for $EFGH$ except this time the scale factor is $\frac{1}{2}$. Invite these students to share during the discussion.

Addressing

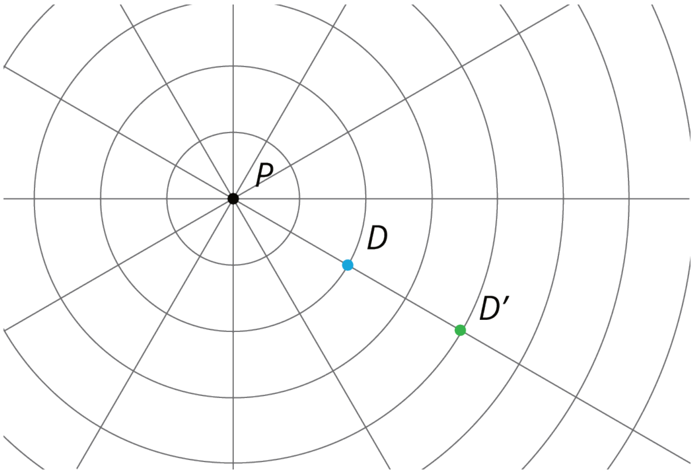
- 8.G.A

Instructional Routines

- MLR2: Collect and Display

Launch

Provide access to geometry toolkits. Tell students that they are going to dilate some points. Before they begin, demonstrate the mechanics of dilating a point using a center of dilation and a scale factor. Tell students, "In the previous activity, each point was dilated to its image using a scale factor of 3. The dilated point was three times as far from the center as the original point. When we dilate point D using P as the center of dilation and a scale factor of 2, that means we're going to take the distance from P to D and place a new point on the ray PD twice as far away from P." Display for all to see:



If using the digital activity, demonstrate the mechanics of dilating using the applet. You can also use the measurement tool to confirm.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of the process of dilating a point using a center of dilation and a scale factor to support connections between new situations and prior understandings. Consider using these prompts: "How does this build on the previous activity in which the main task was to find distances and scale factor?" or "How does the point D' correspond to the points D and P?"
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Access for English Language Learners

Conversing, Reading: MLR2 Collect and Display. Circulate and listen to students as they make observations about the polygon with a scale factor of 2 and the polygon with a scale factor of $\frac{1}{2}$. Write down the words and phrases students use to compare features of the new polygons to the original polygon. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "the new polygon is the same as the original polygon but bigger" can be clarified with the phrase "the new polygon is a scaled copy with scale factor 2 of the original polygon." A phrase such as "the polygons have the same angles" can be clarified with the phrase "each angle in the original polygon is the same as the corresponding angle in the new polygon." This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

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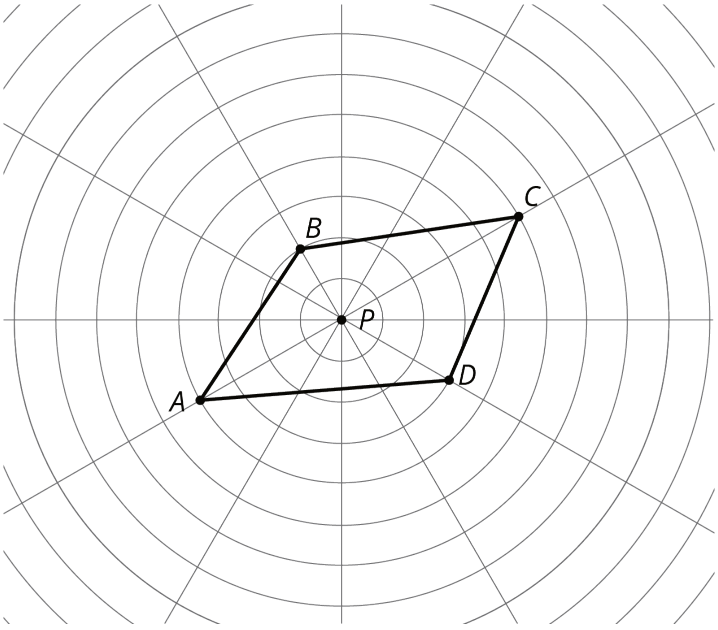
Anticipated Misconceptions

Students may think only grid points can be dilated. In fact, any point can, but they may have to measure or estimate the distances from the center. Grid points are convenient because you can measure by counting.

Student Task Statement

Here is a polygon $ABCD$.

1. Dilate each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of 2. Label the image of A as A', and label the images of the remaining three vertices as B', C', and D'.
2. Draw segments between the dilated points to create polygon $A'B'C'D'$.
3. What are some things you notice about the new polygon?



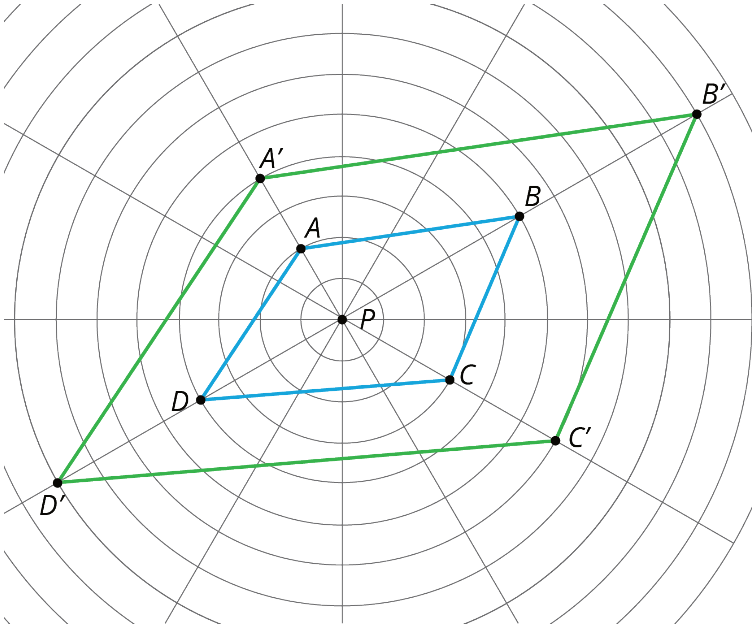
4. Choose a few more points on the sides of the original polygon and transform them using the same dilation. What do you notice?

5. Dilate each vertex of polygon $ABCD$ using P as the center of dilation and a scale factor of $\frac{1}{2}$. Label the image of A as E, the image of B as F, the image of C as G and the image of D as H.

6. What do you notice about polygon $EFGH$?

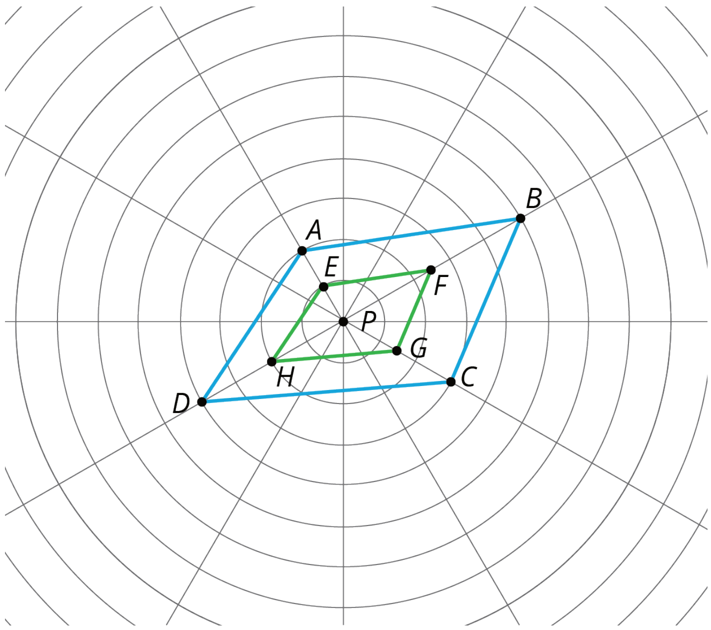
Student Response

1–5.



6. Answers vary. Possible responses:

- The new polygon is a scaled copy of the original polygon.
- Each side of the new polygon is parallel to the corresponding side on the original polygon.
- Each angle in the original figure is congruent to the corresponding angle in the dilated figure.
- Each side of the new polygon is twice the length of the corresponding side in the original polygon.



Are You Ready for More?

Suppose P is a point not on line segment $\overline{WX}$. Let $\overline{YZ}$ be the dilation of line segment $\overline{WX}$ using P as the center with scale factor 2. Experiment using a circular grid to make predictions about whether each of the following statements must be true, might be true, or must be false.

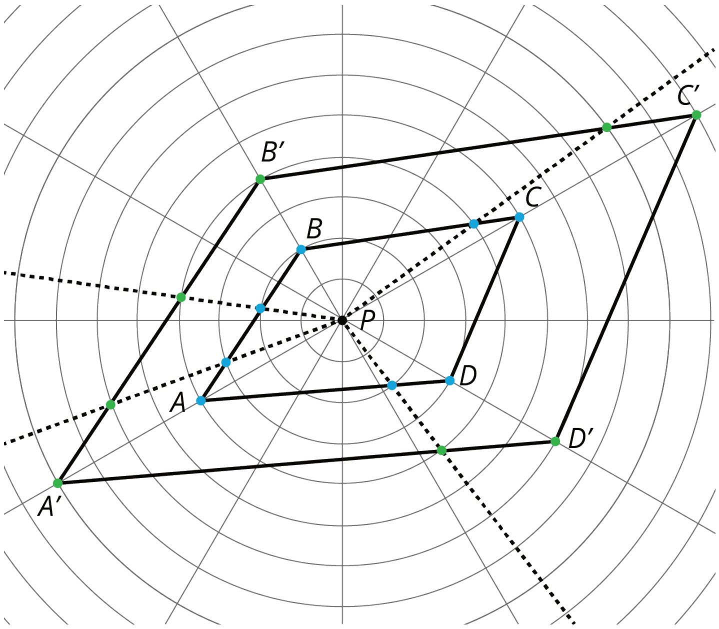
1. $\overline{YZ}$ is twice as long $\overline{WX}$.
2. $\overline{YZ}$ is five units longer than $\overline{WX}$.
3. The point P is on $\overline{YZ}$.
4. $\overline{YZ}$ and $\overline{WX}$ intersect.

Student Response

1. Must be true.
2. Might be true. (True if $\overline{WX}$ has length 5)
3. Must be false.
4. Might be true. (True, for example, if $\overline{WX}$ and $\overline{YZ}$ are both on the same line)

Activity Synthesis

Display the original figure and its image under dilation with scale factor 2 and center P.



Ask selected students to share what they notice about the new polygon. Ensure that the following observations are made. Encourage students to verify each assertion using geometry tools like tracing paper, a ruler, or a protractor.

- The new figure is a scaled copy of the original figure.
- The sides of the new figure are twice the length of the sides of the original figure.
- The corresponding segments are parallel.
- The corresponding angles are congruent.

Ask students what happened to the additional points they dilated on polygon $ABCD$. Note that a good *strategic* choice for these points are points where $ABCD$ meets one of the circles: in these cases, it is possible to double the distance from that point to the center without measuring. The additional points should have landed on a side of the dilated polygon (because of measurement error, this might not always occur exactly). The important takeaway from this observation is that dilating the polygon's vertices, and then connecting them, gives the image of the entire polygon under the dilation.

2.4 A Quadrilateral and Concentric Circles

Optional: 10 minutes (there is a digital version of this activity)
This activity continues work on dilations of polygons on a circular grid. The new twist in this activity is that the radial lines from the center of the circular grid have been removed. This means that when they dilate each point, students will need to use a ruler or other straightedge to connect that point to the center of the circular grid. If there is extra time, they can experiment dilating points other than the vertices and check that the dilation of a side of the polygon is still a line segment (though there may be small deviations due to measurement error).

Addressing

- 8.G.A

Launch

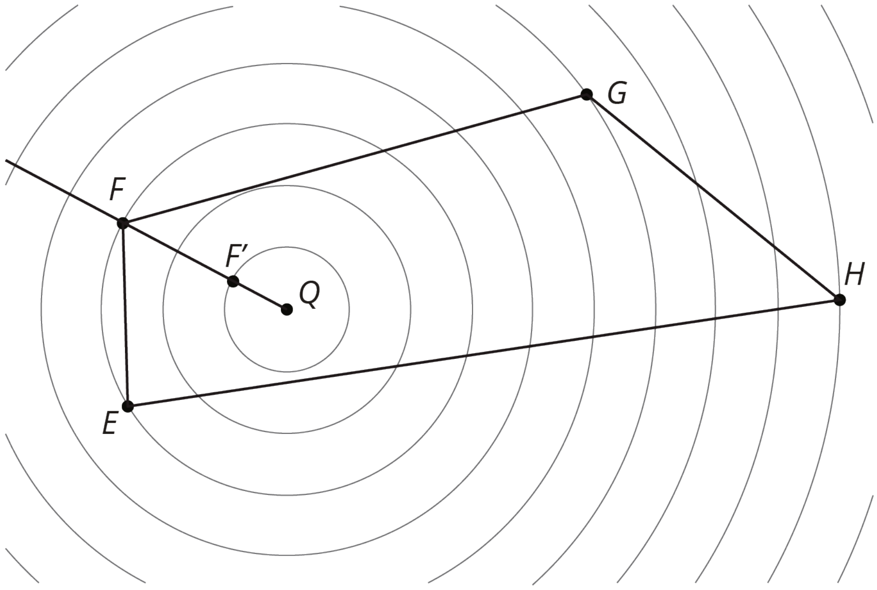
Ask students to quietly read the problem, and then ask them how this problem is alike and different from the previous one. It is alike because it shows a quadrilateral and concentric circles, and we are asked to dilate the quadrilateral using the center of the circles as the center of dilation. It is different because there is only one radial line through the center, because the scale factor is now $\frac{1}{3}$, and because one of the points is already dilated.

Tell students to study how the location of F' was determined, and then to dilate the remaining points.

Anticipated Misconceptions

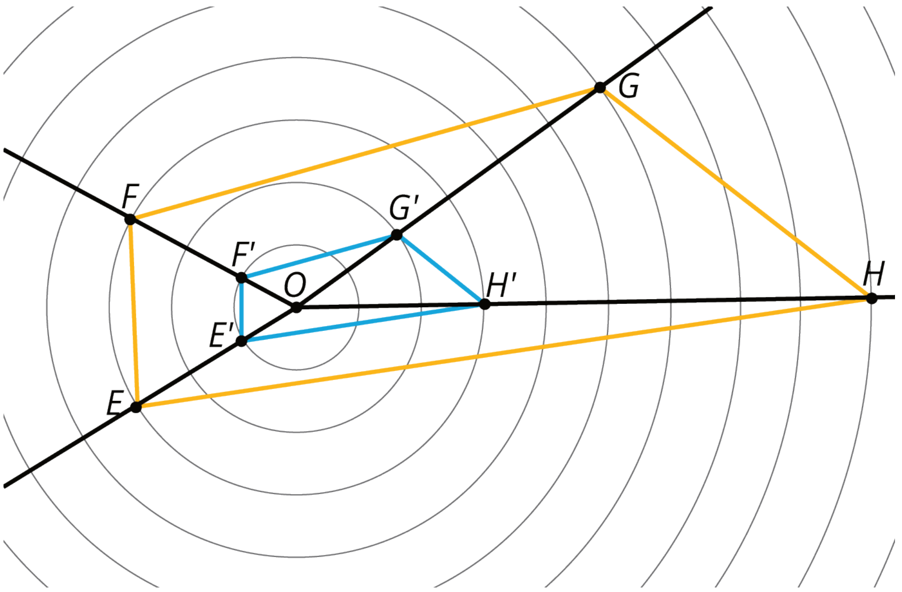
Students may be bothered because the dilated quadrilateral looks off-center and the distance between corresponding sides of the quadrilaterals depends on the side. Ensure them that the image is correct and ask them to focus on the parallel corresponding sides of the shapes or ask them if the dilated quadrilateral appears to be a scaled copy of the original (it does).

Student Task Statement



Dilate polygon $EFGH$ using Q as the center of dilation and a scale factor of $\frac{1}{3}$. The image of F is already shown on the diagram. (You may need to draw more rays from Q in order to find the images of other points.)

Student Response



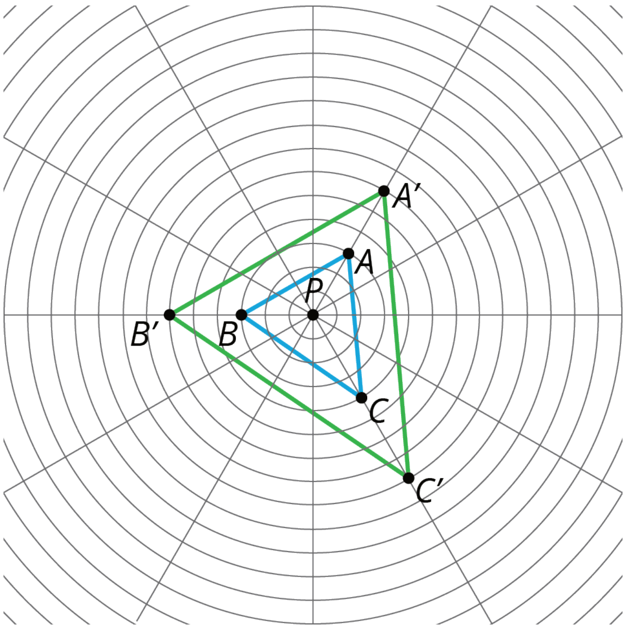
Activity Synthesis

Highlight the need to add line segments joining E, F, G, H to the center in order to find the image of those points under the dilation. Also highlight that the scale factor of $\frac{1}{3}$ resulted in an image that was smaller than the original figure instead of larger. You might ask students what scale factor would result in no change? That is, for what scale factor would the image land right on top of the original figure? They can likely name “1” as the scale factor that would accomplish this. So, scale factors that are greater than 1 result in an image larger than the original, and scale factors less than 1 result in an image smaller than the original.

Lesson Synthesis

- "What are some important properties of the circular grid?"
- "How does it help to perform dilations?"

Highlight the fact that the circular grid is mainly useful when the center of dilation is the center of the grid. When the scale factor is 3, for example, the circle with radius 1 grid unit maps to the circle with radius 3 grid units. More generally, each grid circle maps to a grid circle whose radius is three times as large.



To apply a dilation to a polygon, we can dilate the vertices and then add appropriate segments. For example, triangle $A'B'C'$ is the dilation of triangle ABC with scale factor 2 and center of dilation P:How does triangle $A'B'C'$ compare to triangle ABC? Make sure students see that it is a scaled copy with scale factor 2.

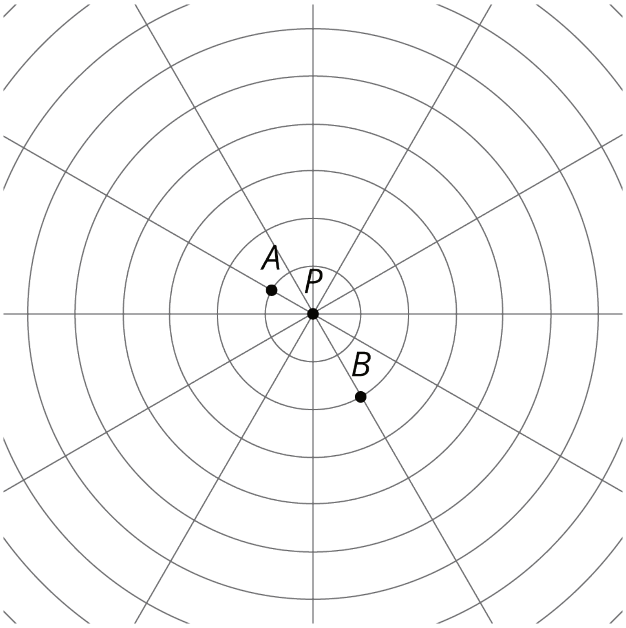
2.5 Dilating points on a circular grid

Cool Down: 5 minutes
Students apply dilations with scale factors larger than 1 to points on a circular grid that lie on radial lines.

Addressing

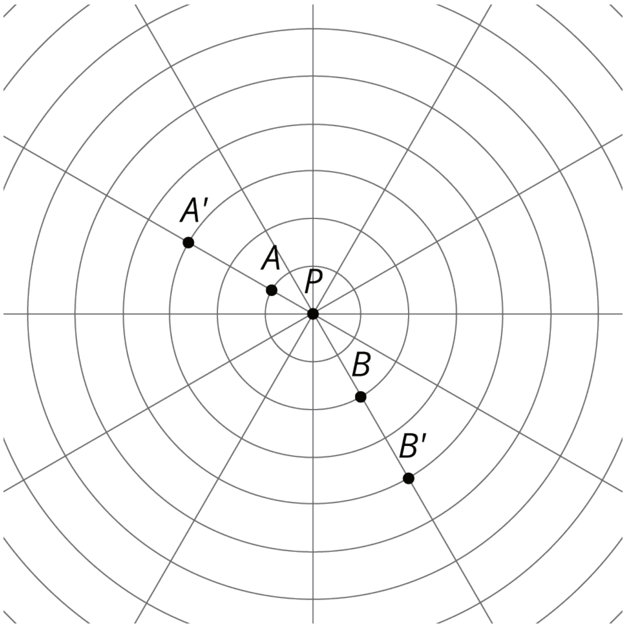
- 8.G.A

Student Task Statement



1. Dilate A using P as the center of dilation and a scale factor of 3. Label the new point A'.
2. Dilate B using P as the center of dilation and a scale factor of 2. Label the new point B'.

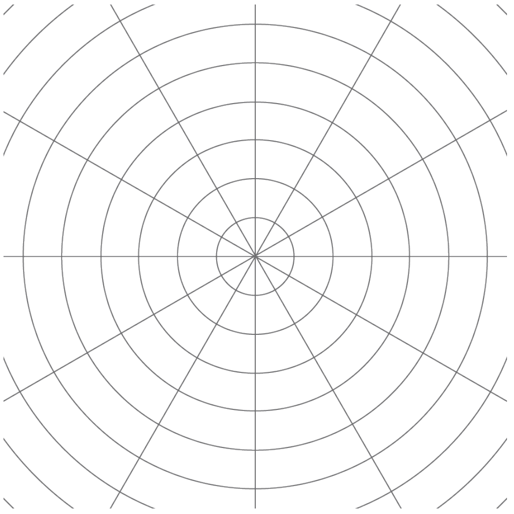
Student Response



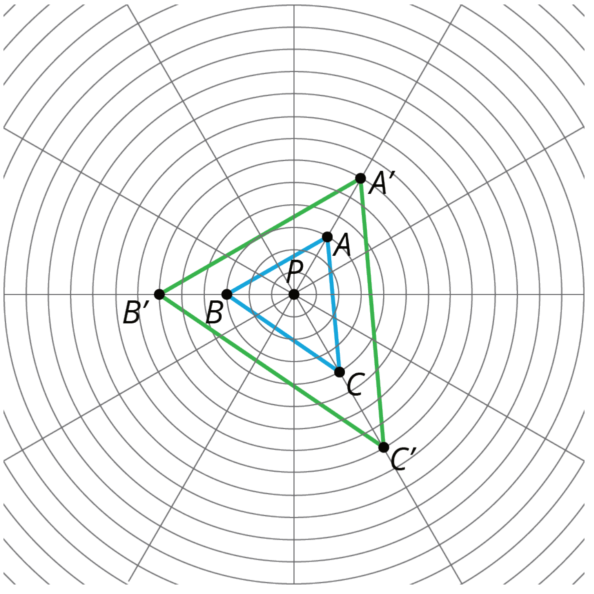
Student Lesson Summary

A circular grid like this one can be helpful for performing **dilations**.

The radius of the smallest circle is one unit, and the radius of each successive circle is one unit more than the previous one.



To perform a dilation, we need a **center of dilation**, a scale factor, and a point to dilate. In the picture, P is the center of dilation. With a scale factor of 2, each point stays on the same ray from P, but its distance from P doubles:



Since the circles on the grid are the same distance apart, segment PA' has twice the length of segment PA, and the same holds for the other points.

Glossary

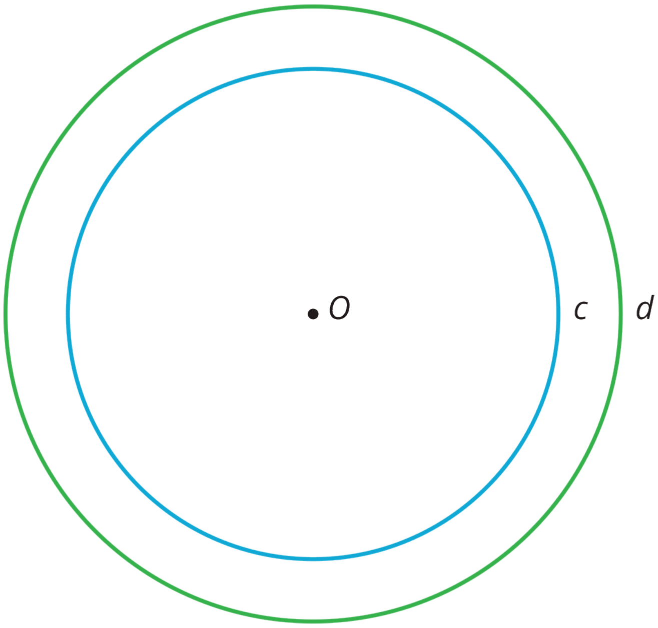
- center of a dilation
- dilation

Lesson 2 Practice Problems

Problem 1

Statement

Here are Circles c and d. Point O is the center of dilation, and the dilation takes Circle c to Circle d.



a. Plot a point on Circle c. Label the point P. Plot where P goes when the dilation is applied.

b. Plot a point on Circle d. Label the point Q. Plot a point that the dilation takes to Q.

Solution

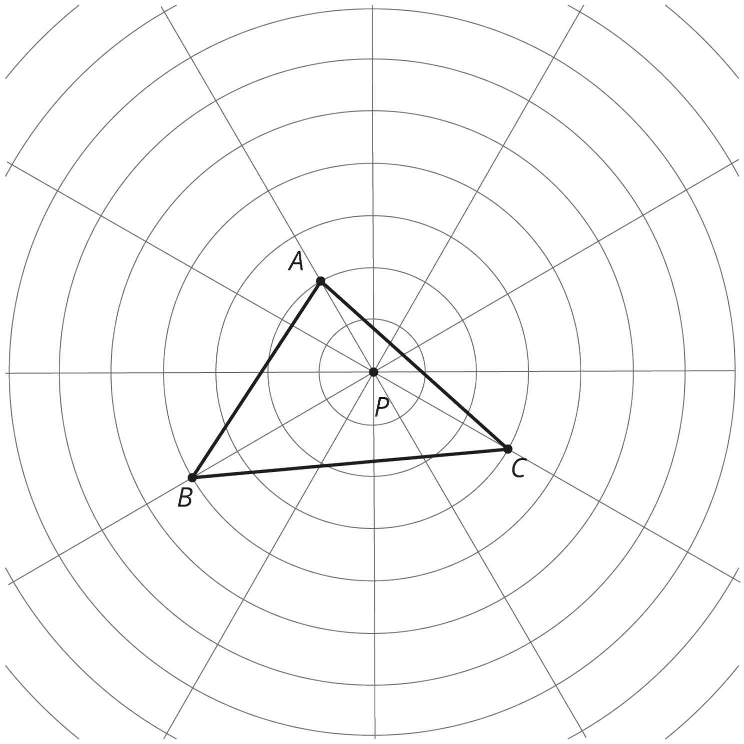
a. Plot any point P, then draw a ray from O through P. The point where this ray intersects circle d is P'.

b. Plot any point Q, then draw a ray from O through Q. The point where this ray intersects circle c is Q'.

Problem 2

Statement

Here is triangle ABC.



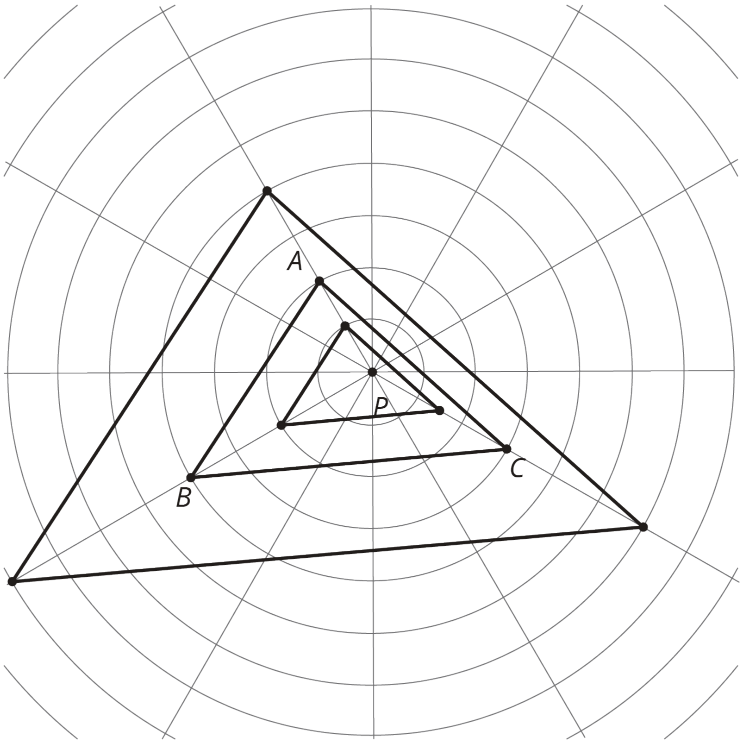
a. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of 2. Draw the triangle connecting the three new points.

b. Dilate each vertex of triangle ABC using P as the center of dilation and a scale factor of $\frac{1}{2}$. Draw the triangle connecting the three new points.

c. Measure the longest side of each of the three triangles. What do you notice?

d. Measure the angles of each triangle. What do you notice?

Solution



a. Triangle $A'B'C'$ has each respective point at the same ray. A' is 4 units from the origin, B' is 8 units from the origin, and C' is 6 units from the origin.

b. Triangle $A''B''C''$ has each respective point at the same ray. A'' is 1 unit from the origin, B'' is 2 units from the origin, and C'' is 1.5 units from the origin.

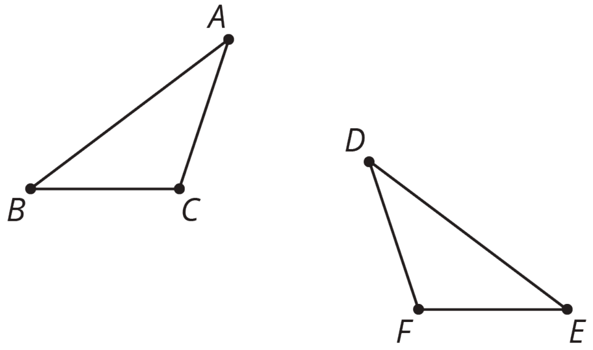
c. The longest side of the largest triangle is twice as long as the longest side of triangle ABC, which is twice as long as the smallest triangle.

d. The angles in all three triangles have the same measures.

Problem 3

Statement

Describe a rigid transformation that you could use to show the polygons are congruent.



Solution

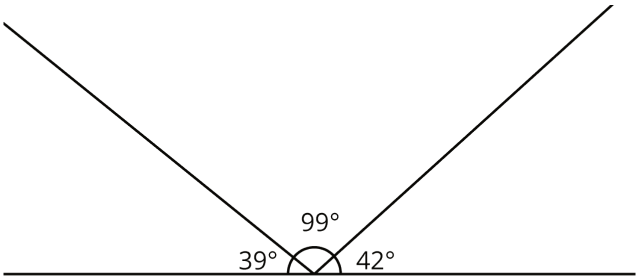
Reflect triangle ABC in a vertical line and translate so A meets D.

(From Unit 1, Lesson 12.)

Problem 4

Statement

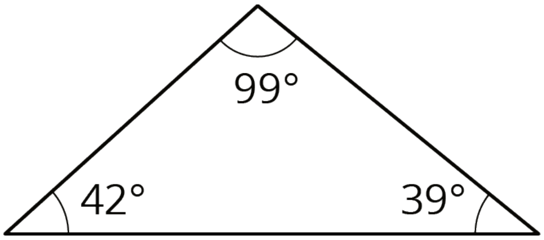
The line has been partitioned into three angles.



Is there a triangle with these three angle measures? Explain.

Solution

Yes



(From Unit 1, Lesson 15.)