

Lesson 3: Dilations with no Grid

Goals

- Create a dilation of a figure given a scale factor and center of dilation.
- Explain (orally) the effect of the scale factor on the size of the image of a polygon and its distance from the center of dilation.
- Identify the center, scale factor, and image of a dilation without a circular grid.

Learning Targets

- I can apply a dilation to a polygon using a ruler.

Lesson Narrative

In the previous lesson, students applied dilations on a circular grid. The circular grid provides two levels of scaffolding:

- The radial lines give rays from the center of the grid which help find the dilated image of points on those rays.
- The circles provide a way to measure the distance of points from the center of dilation.

In this lesson, students apply dilations to points with no grid. In order to perform a dilation, three pieces of information are still needed: a center of dilation, a scale factor, and a point which is dilated. Students practice identifying centers, scale factors, and images of dilation. They also use dilations to make perspective drawings.

Performing dilations without a grid engages students in MP1 as they think about the meaning of dilation in terms of the given information (center, scale factor, point being dilated).

Alignments

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Geometry toolkits

For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Ensure that rulers, index cards, and colored pencils are available in the geometry toolkits.

Student Learning Goals

Let's dilate figures not on grids.

3.1 Points on a Ray

Warm Up: 5 minutes

Students apply a dilation to points on a ray. The scaffold of the circular grid has been removed but the structure of dilations is the same.

Without the grid, students will need to come up with a way to measure in order to find the point twice as far from A as B and half as far from A as B . They can use a ruler or the edge of an index card.

Monitor for these methods:

- using a ruler to measure distances
- marking off distances on an index card (for problem 1)
- folding paper in half (for problem 2)

Select students who use these methods and invite them to present.

Addressing

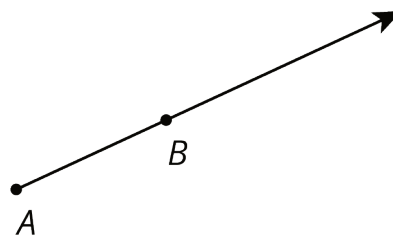
- 8.G.A

Launch

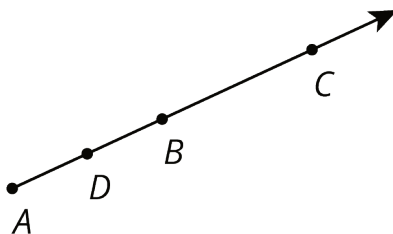
Provide access to geometry toolkits.

Student Task Statement

1. Find and label a point C on the ray whose distance from A is twice the distance from B to A .
2. Find and label a point D on the ray whose distance from A is half the distance from B to A .



Student Response



Activity Synthesis

Invite selected students to present their methods for finding the points which may include:

- using a ruler to measure distances
- marking off distances on an index card (for problem 1)
- folding paper in half (for problem 2)

Point out how this is similar to work with dilations on a circular grid (the points lie on the same ray at different distances) and how it is different (there are no marked distances). For the next activity, it is important for students to understand that C is the dilation of B with center A and scale factor 2. And D is the dilation of B with center A and scale factor $\frac{1}{2}$.

3.2 Dilation Obstacle Course

10 minutes (there is a digital version of this activity)

This activity investigates dilations with no grid. Students have seen these for the first time in the warm-up, which had a ray drawn between two points. That scaffold has been removed here so the teacher may need to provide guidance by suggesting that students draw appropriate rays.

Encourage students to measure distances carefully at first, since the problem statement does not state that the image of each point, after the dilation indicated, is one of the labeled points. After doing a few of the problems, the students should notice that the dilated point is always one of the labeled points and then use this observation to expedite the work. Also monitor for students who see the relationship between the scale factors used to send G to E and E to G (both with center H).

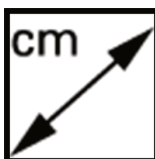
Addressing

- 8.G.A

Instructional Routines

- MLR8: Discussion Supports

Launch



Ask students to work on the first question and then pause. Demonstrate, or have a student demonstrate, drawing a ray from point A through point B (the ray goes through points H and I). Show that the length of AI is five times as long as the length of AB , either with a ruler or by marking intervals on the edge of a blank piece of paper. Or, if using the digital activity, use the measuring tool. (Click two points to measure the distance between them.)

If using the digital activity, it may be easiest for students to work with a partner, with one device used to manipulate the applet and the other device used to display the questions.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, allow time for students to work on the first question, then pause for a whole-class think aloud and discussion. Follow by presenting one question at a time and monitor students to ensure they are making progress throughout the activity.

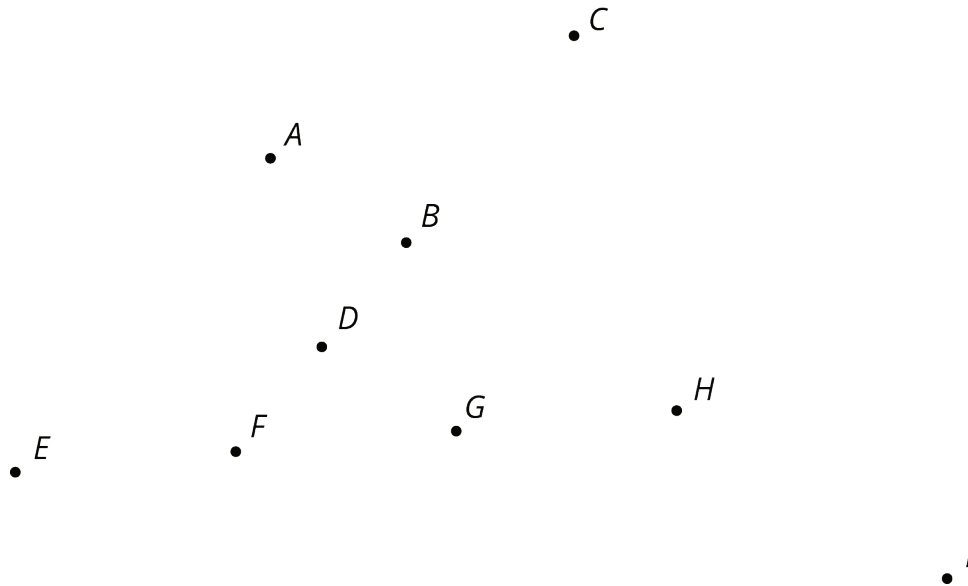
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Students might need to be reminded that the image of a point under dilation must lie on the same line as the point being dilated and the center of dilation. Students might think that for a point to be a dilation of itself, the scale factor is 0. If this happens, ask them to consider multiplying the distance of the point by 0. (If they want the distance to be the same, they actually need to multiply it by 1 instead.)

Student Task Statement

Here is a diagram that shows nine points.



1. Dilate B using a scale factor of 5 and A as the center of dilation. Which point is its image?
2. Using H as the center of dilation, dilate G so that its image is E . What scale factor did you use?
3. Using H as the center of dilation, dilate E so that its image is G . What scale factor did you use?
4. To dilate F so that its image is B , what point on the diagram can you use as a center?
5. Dilate H using A as the center and a scale factor of $\frac{1}{3}$. Which point is its image?
6. Describe a dilation that uses a labeled point as its center and that would take F to H .
7. Using B as the center of dilation, dilate H so that its image is itself. What scale factor did you use?

Student Response

1. I is on the same line as B and A and segment AI is 5 times the length of segment AB .
2. 3. Segment EH is 3 times as long as segment GH .
3. $\frac{1}{3}$. Segment GH is $\frac{1}{3}$ as long as segment EH .
4. C . It needs to be on the same line as F and B , but can't be between them (if restricted to positive scale factors).
5. B . It needs to be on the same line as A and H , but closer to A than H is.
6. Use E as the center and a scale factor of 3. The center must be on the same line as F and H and not between them, so the center is E . Segment EH is 3 times as long as segment EF , so the scale factor should be 3.

7. 1. To dilate H to itself using B as the center, the distance from B to H must stay the same. That means the scale factor must be exactly 1.

Activity Synthesis

Discuss any strategies used to solve the problems. Ask selected students who noticed that the answers to all of the questions were labeled points to share their observation and how it helped them answer the questions. Next ask selected students to share their observation about the scale factors for dilating G to E and dilating E back to G . One way to reverse or “undo” a dilation is to use the same center and reciprocal scale factor.

Other important ideas to bring out include:

- The center of dilation, the point being dilated, and the image of the point after dilation must all lie on the same line.
- A scale factor of 1 does not move any points. If the scale factor is not 1, only one point does not move (the center of dilation).

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. As students share their strategies with the class, encourage students to use full sentences and press for details in their explanations. Ask questions such as, “How do you know this is the image of the point?”, “How do you know this is the scale factor?”, “How do you know this is the center of dilation?”, and “Why can’t that point be located somewhere else?”. If necessary, rephrase “collinear” and explain that collinear means on the same line. Point out the Latin root “co,” which means “together.” For example, “collaborate” means to work together, and therefore “collinear” means on the same line together. This will support a rich and inclusive discussion about how to determine the scale factor, center, and image of a dilation.

Design Principle(s): Support sense-making

3.3 Getting Perspective

15 minutes (there is a digital version of this activity)

In this activity, students continue to apply dilations without a grid. Unlike in the previous activity, the dilated images of the points are not plotted. So rather than identifying the correct point, they will need to find an appropriate way to take measurements (MP5), most likely with the aid of a ruler or the edge of an index card. Different students will work with different scale factors and will produce perspective drawings of a box.

Watch for students who pick a point close to one vertex of the given rectangle. If the point is too close, it will be more difficult to visualize the box. Suggest that they move the point further away. Monitor for students who produce accurate drawings with different scale factors and invite them to share during the discussion.

Addressing

- 8.G.A

Instructional Routines

- MLR7: Compare and Connect

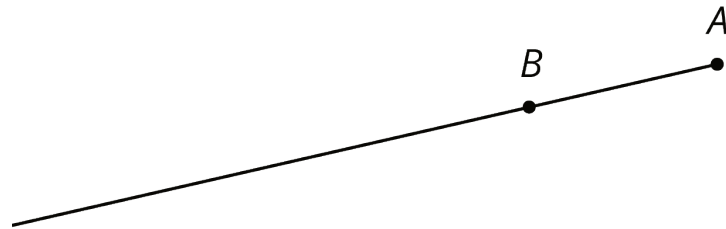
Launch

Provide access to geometry toolkits.

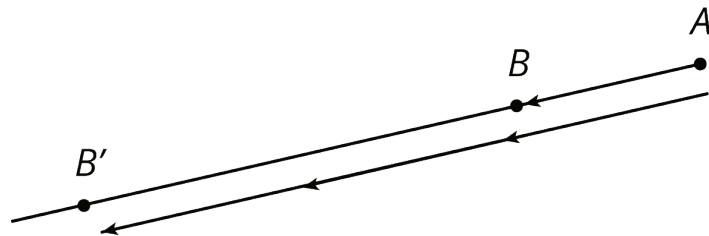
First, a demonstration about dilating a point on a plane with no grid.



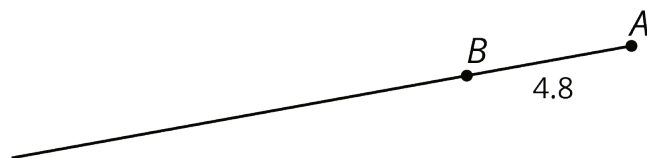
We want to dilate point B using A as the center of dilation and a scale factor of 3.



Use a straightedge to draw ray AB .

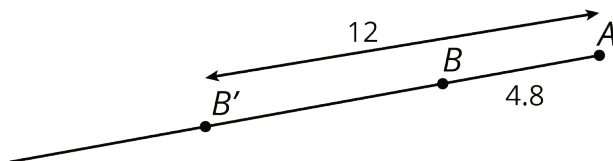


Measure the distance from B to A . Multiply the distance by 3. Draw B' so that it is 3 times as far away from A . For scale factors that are integers, an unmarked edge of an index card or a compass can also be used to transfer the distance along the ray.



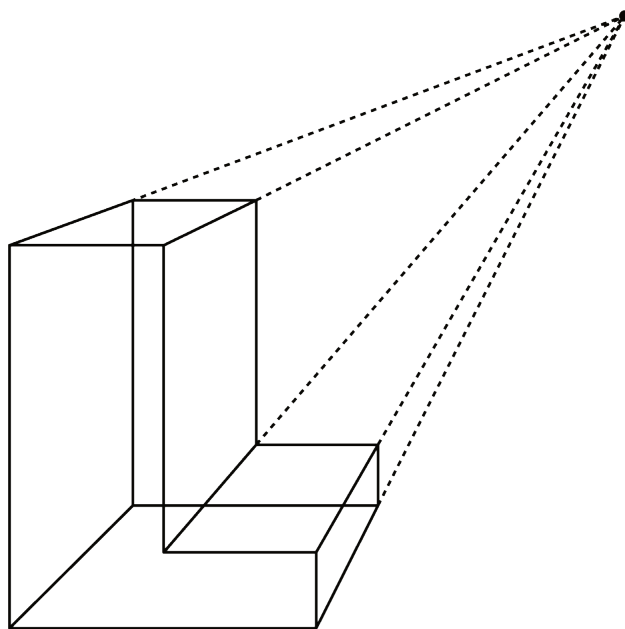
If we wanted a scale factor that is not an integer the procedure is the same. Measure the distance from A to B , multiply by the scale factor, and place B' at that new distance from A .

Let's say the distance from A to B is 4.8 cm.



If we wanted to use a scale factor of 2.5, the distance from A to the dilated point B' would be 12 cm, because $(4.8) \cdot (2.5) = 12$.

A perspective drawing is an optical illusion that makes an image printed on paper have a three-dimensional look. Display at least one example of a perspective drawing:



Students will practice some simple dilations of points, and then they will create a perspective drawing. Tell students to complete the first part of the activity dilating points P and Q . After you review their work, assign each student a scale factor to use for the second part. Appropriate scale factors include $\frac{1}{3}$, $\frac{1}{2}$, $1\frac{1}{2}$, and 2. It will work best if the center of the dilation is not too close to the rectangle the students are dilating.

Access for Students with Disabilities

Representation: Provide Access for Perception. Display or provide students with a physical copy of the Launch demonstration about dilating a point on a plane with no grid. Check for understanding by inviting students to rephrase directions for creating a dilation of points in their own words. Consider keeping the display of directions visible throughout the activity.

Supports accessibility for: Language; Memory

Anticipated Misconceptions

Students may all try to make their drawing match any example drawings shown in the launch. For example, if the center of dilation in an example is above and to the right, everyone might place their center of dilation above and to the right of the rectangle. Any point is fine as a dilation point, but the effect on what the picture looks like may vary.

Students may not recall that to dilate a polygon, they can first dilate the vertices and then connect them in the proper order. It may be necessary to show students how to dilate one of the vertices and allow them to perform the dilation on the other three vertices.

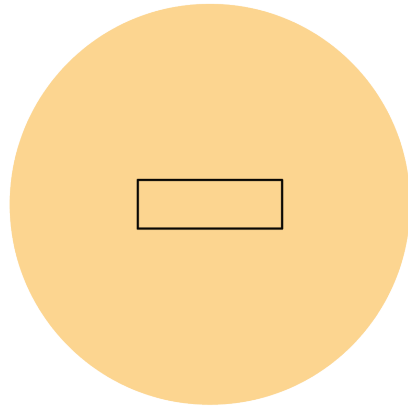
Student Task Statement

1. Using one colored pencil, draw the images of points P and Q using C as the center of dilation and a scale factor of 4. Label the new points P' and Q' .
2. Using a different color, draw the images of points P and Q using C as the center of dilation and a scale factor of $\frac{1}{2}$. Label the new points P'' and Q'' .



Pause here so your teacher can review your diagram. Your teacher will then give you a scale factor to use in the next part.

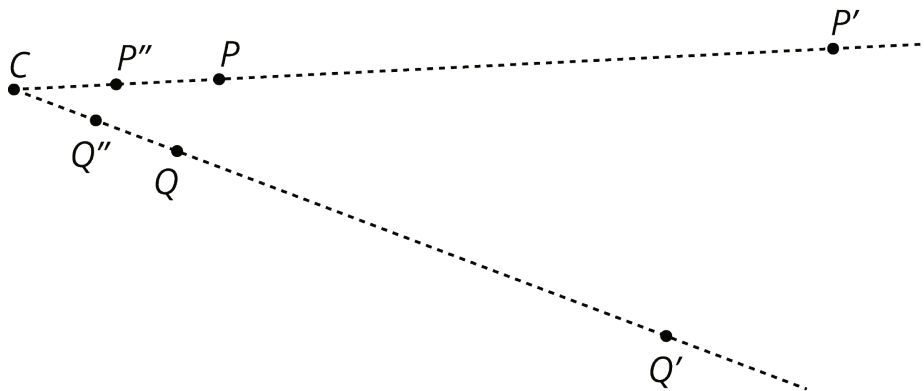
3. Now you'll make a perspective drawing. Here is a rectangle.



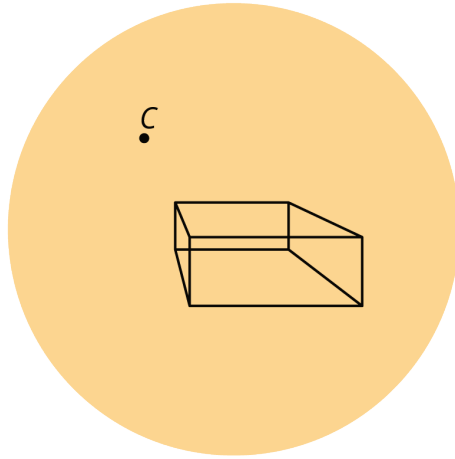
- Choose a point *inside the shaded circular region* but *outside the rectangle* to use as the center of dilation. Label it C .
- Using your center C and the scale factor you were given, draw the image under the dilation of each vertex of the rectangle, one at a time. Connect the dilated vertices to create the dilated rectangle.
- Draw a segment that connects each of the original vertices with its image. This will make your diagram look like a cool three-dimensional drawing of a box! If there's time, you can shade the sides of the box to make it look more realistic.
- Compare your drawing to other people's drawings. What is the same and what is different? How do the choices you made affect the final drawing? Was your dilated rectangle closer to C than to the original rectangle, or farther away? How is that decided?

Student Response

1,2

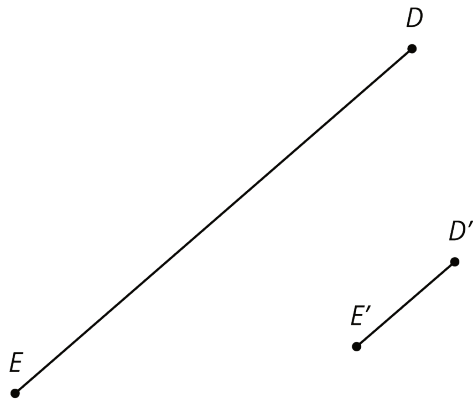


1. Answers vary. Possible response (scale factor 1.5):



Are You Ready for More?

Here is line segment DE and its image $D'E'$ under a dilation.



1. Use a ruler to find and draw the center of dilation. Label it F .
2. What is the scale factor of the dilation?

Student Response

1. Draw ray DD' and ray EE' . F is their intersection.
2. $\frac{1}{4}$

Activity Synthesis

Display the work of several students selected based on the different scale factors. Then ask students:

- “What are the effects of using a scale factor greater than 1?” (The image is larger than the original *and* farther away from the center of dilation than the original.)
- “What are the effects of using a scale factor less than 1?” (The image is smaller than the original *and* closer to the center of dilation than the original.)
- “What effect does the location of C , the center of dilation, have?” (It impacts the size and location of the dilated rectangle: if the scale factor is less than 1 then the dilated rectangle is closer to C than the original and if the scale factor is larger than 1 then the dilated rectangle is further away from C than the original.)

Time permitting, consider showing several student drawings with the same scale factor but a different location for the point C . How are they the same? How are they different? Two faces of these boxes (the original rectangle and the scaled copy) are congruent but the point of view or perspective on them is different.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students prepare their perspective drawings, identify the drawings with scale factors greater than 1 or less than 1. As students investigate each other’s work, ask them to share what is similar about the drawings with scale factor greater than 1 (or less than 1). Listen for and amplify statements such as “a scale factor greater than 1 results in an image larger than the original” and “a scale factor less than 1 results in an image smaller than the original.” Then encourage students to make connections between the value of the scale factor and the effect on the image. Listen for and amplify language students use to describe how the scale factor affects the size of the image and its distance from the center of dilation. This will foster students’ meta-awareness and support constructive conversations as they compare perspective drawings and make connections between the value of the scale factor and the image of the original polygon.

Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Ask students to think about how they would explain the steps for dilating a point, and either write them down or share them with a partner. Ask a few students to share their steps. Ensure that all of the important aspects are mentioned:

- “You need to know which point you want to dilate, which point is the center of dilation, and what scale factor to use.”
- “Use a straightedge to draw a ray from the center of dilation through the point you want to dilate.”

- “Measure the distance from the center of dilation through the point. Multiply this distance by the scale factor. Place the new point at this distance from the center of dilation and also on the ray you drew.”
- “If the scale factor is greater than 1, the new point will be farther from the center than the original point. If the scale factor is less than 1, the new point will be closer to the center than the original point.”

3.4 A Single Dilation of a Triangle

Cool Down: 5 minutes

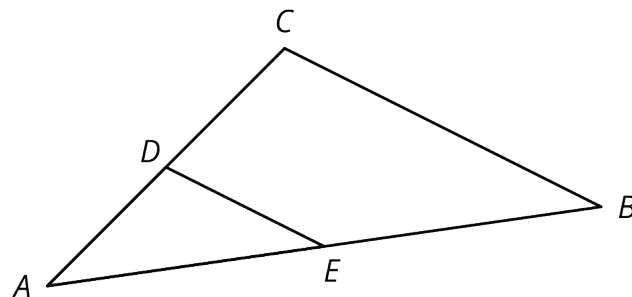
Students identify the center of a dilation given a figure, its dilation, and the scale factor.

Addressing

- 8.G.A

Student Task Statement

Lin drew a triangle and a dilation of the triangle with scale factor $\frac{1}{2}$:



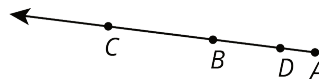
1. What is the center of the dilation? Explain how you know.
2. Which triangle is the original and which triangle is the dilation? Explain how you know.

Student Response

1. The center of dilation is A . Lines emanate from A and points lie along those lines.
2. Triangle ABC is the original and triangle AED is the dilation. Since the scale factor is less than 1, the dilation is smaller than the original figure.

Student Lesson Summary

If A is the center of dilation, how can we find which point is the dilation of B with scale factor 2?



Since the scale factor is larger than 1, the point must be farther away from A than B is, which makes C the point we are looking for. If we measure the distance between A and C , we would find that it is exactly twice the distance between A and B .

A dilation with scale factor less than 1 brings points closer. The point D is the dilation of B with center A and scale factor $\frac{1}{3}$.

Lesson 3 Practice Problems

Problem 1

Statement

Segment AB measures 3 cm. Point O is the center of dilation. How long is the image of AB after a dilation with . . .

- Scale factor 5?
- Scale factor 3.7?
- Scale factor $\frac{1}{5}$?
- Scale factor s ?

Solution

- 15 cm
- 11.1 cm
- $\frac{3}{5}$ cm
- $3s$ cm

Problem 2

Statement

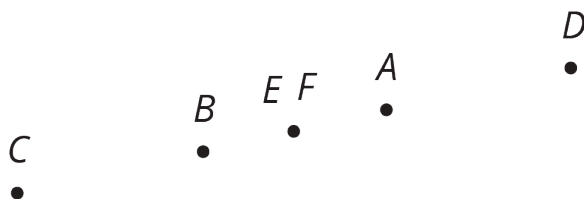
Here are points A and B . Plot the points for each dilation described.



- C is the image of B using A as the center of dilation and a scale factor of 2.
- D is the image of A using B as the center of dilation and a scale factor of 2.
- E is the image of B using A as the center of dilation and a scale factor of $\frac{1}{2}$.

d. F is the image of A using B as the center of dilation and a scale factor of $\frac{1}{2}$.

Solution



Problem 3

Statement

Make a perspective drawing. Include in your work the center of dilation, the shape you dilate, and the scale factor you use.

Solution

Answers vary.

Problem 4

Statement

Triangle ABC is a scaled copy of triangle DEF . Side AB measures 12 cm and is the longest side of ABC . Side DE measures 8 cm and is the longest side of DEF .

- Triangle ABC is a scaled copy of triangle DEF with what scale factor?
- Triangle DEF is a scaled copy of triangle ABC with what scale factor?

Solution

- $\frac{3}{2}$
- $\frac{2}{3}$

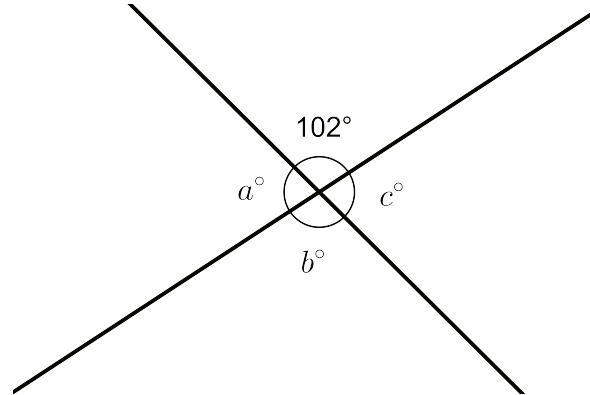
(From Unit 2, Lesson 1.)

Problem 5

Statement

The diagram shows two intersecting lines.

Find the missing angle measures.



Solution

$$a = 78$$

$$b = 102$$

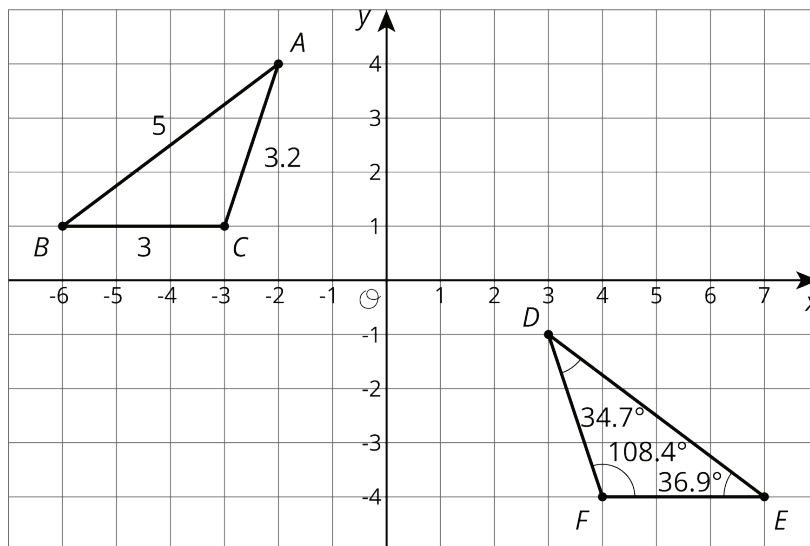
$$c = 78$$

(From Unit 1, Lesson 14.)

Problem 6

Statement

- Show that the two triangles are congruent.
- Find the side lengths of DEF and the angle measures of ABC .



Solution

- a. Reflect in the y -axis and translate until A meets D .
- b. Angle ABC is 36.9 degrees. Angle BCA is 108.4 degrees. Angle CAB is 34.7 degrees. $DE = 5$.
 $EF = 3$. $FD = 3.2$.

(From Unit 1, Lesson 12.)