Lesson 1: Inputs and Outputs

Goals

- Describe (orally) how input-output diagrams represent rules.
- Identify a rule that describes the relationship between input-output pairs and explain (orally) a strategy used for figuring out the rule.

Learning Targets

- I can write rules when I know input-output pairs.
- I know how an input-output diagram represents a rule.

Lesson Narrative

This is the first of two lessons introducing students to functions, developing the idea of a function as a rule that assigns to each allowable input exactly one output. The word function is not introduced until the second lesson. In future lessons, students will expand on this definition as they work with different representations of functions.

In the first classroom activity students take turns guessing each other's rules from input-output pairs. In the second activity students use rules represented by input-output diagrams to fill out a table with inputs and associated outputs. In each table, the first input-output pair is identical, illustrating that a single pair is insufficient for determining a rule. The last table returns to the topic of the warm-up and introduces the idea that not all inputs are possible for a rule.

Alignments

Addressing

- 8.F.A.1:
  Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.

Building Towards

- 8.F.A.1:
  Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Function notation is not required in Grade 8.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

- Think Pair Share

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut up slips from the Guess my Rule blackline master. Prepare 1 copy for every 4 students. During the activity, you will want students to place the slips in a pile, face down, with rule A on top and rule D on the bottom.

Student Learning Goals

Let's make some rules.

1.1 Dividing by 0

Warm Up: 5 minutes
The purpose of this activity is for students to see why an expression that contains dividing by zero can't be evaluated. They use their understanding of related multiplication and division equations to make sense of this.

Building Towards

- 8.F.A.1

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Tell students to consider the statements and try to find a value for x that makes the second statement true.

Give students 1 minute of quiet think time followed by 1 minute to share their thinking with their partner. Finish with a whole-class discussion.

Student Task Statement

Study the statements carefully.

- $12 \div 3 = 4$ because $12 = 4 \cdot 3$
- $6 \div 0 = x$ because $6 = x \cdot 0$

What value can be used in place of x to create true statements? Explain your reasoning.

Student Response

None. Explanations vary. Sample explanation: There is no number that can be multiplied by zero to get something other than zero. Therefore, $x \cdot 0 = 6$ is never a true statement for any x.

Activity Synthesis

Select 2–3 groups to share their conclusions about x.

As a result of this discussion, we want students to understand that any expression where a number is divided by zero can't be evaluated. Therefore, we can state that there is no value for x that makes both equations true.

1.2 Guess My Rule

15 minutes (there is a digital version of this activity)
The purpose of this activity is to introduce the idea of *input-output rule*s. One student chooses inputs to tell a partner who uses a rule written on a card only they can see to respond with the corresponding output. The first student then guesses the rule on the card once they think they have enough input-output pairs to know what it is. Partners then reverse roles.

Monitor for students who:

- Appear to have a strategy for choosing which numbers to give, such as always starting with 0 or 1, or choosing a sequence of consecutive whole numbers.
- Notice the difference between the result for odd numbers and the result for even numbers for the card with a different rule for each.

Addressing

- 8.F.A.1

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2.

For students using the print version: Have a student act as your partner and demonstrate the game using a simple rule that does not match one of the cards (like “divide by 2” or “subtract 4”).

Ask groups to decide who will be Player 1 and who will be Player 2. Give each group the four rule cards, making sure that the simplest rules are at the top of the deck when face down. Tell students to be careful not to let their partner see what the rule is as they pick up the rule cards. If necessary, tell students that all numbers are allowed, including negative numbers.

For students using the digital version: Demonstrate how to use the applet by choosing one input value, clicking the appropriate button, and noting where the output is recorded. Prompt partners to discuss which input values to select for the rule and alternate who guesses the rule.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity by beginning with fewer cards. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of rules.
Supports accessibility for: Conceptual processing; Organization

Anticipated Misconceptions

Students may have difficulty with the last rule because it involves conditional statements. Encourage them to focus on a single value at a time to help them understand how this kind of rule works.

Student Task Statement

Keep the rule cards face down. Decide who will go first.

1. Player 1 picks up a card and silently reads the rule without showing it to Player 2.
2. Player 2 chooses an integer and asks Player 1 for the result of applying the rule to that number.
3. Player 1 gives the result, without saying how they got it.
4. Keep going until Player 2 correctly guesses the rule.

After each round, the players switch roles.

Student Response

$x + 7, 3x, 2x - 5, \frac{x}{2}, 3x + 1$

Are You Ready for More?

If you have a rule, you can apply it several times in a row and look for patterns. For example, if your rule was "add 1" and you started with the number 5, then by applying that rule over and over again you would get 6, then 7, then 8, etc., forming an obvious pattern.

Try this for the rules in this activity. That is, start with the number 5 and apply each of the rules a few times. Do you notice any patterns? What if you start with a different starting number?

Student Response

1. 5, 12, 19, 26, 33, . . .
2. 5, 15, 45, 135, . . .
3. 5, 5, 5, 5, . . .

4. 5, 16, 8, 4, 2, 1, 4, 2, 1, 4, 2, 1, 4, 2, 1

If you change the starting number from 5, the first two rules give very similar patterns, just adding 7 and tripling each time. The third pattern has a much harder pattern to recognize when you don't start with a 5. The fourth pattern turns out to be incredibly difficult! Most mathematicians suspect that no matter *what* value you start with, you will always eventually get back to 1, and starting cycling 1, 4, 2, 1, 4, 2, 1, ... Computers have checked that this pattern appears for any starting number up to one billion billion, but no one knows for sure that it will *always* appear! For more information, research the Collatz Conjecture.

Activity Synthesis

The goal of this discussion is for students to understand what an input-output rule is and share strategies they used for figuring out the rule. Tell students we start with a number, called the input, and apply a rule to that number which results in a number called the output. We say the output *corresponds* to that input.

To highlight these words, ask:

- “What is an example of an input from this activity?” (The input is the number the player without a card chose, or the number placed in the 'input' box in the applet.)
- “Where in the activity was the rule applied?” (The rule was applied when the player with the card applied the rule to the input their partner told them, or it's what happened in the black box in the applet.)
- “Where in the activity was the output?” (The output is the number the player said after applying the rule to the given input, or the number the black box in the applet gave.)

Select previously identified students who appeared to have a strategy for figuring out a rule to share it. Sequence students starting with the most common strategies to the least. Make connections between the successful aspects of each strategy (for example, if a student does not say it, stating clearly why using a sequence of integers or using 0 and 1 are helpful for determining the rule).

Students might think the last rule isn't allowable because there were two "different" rules. Explain that a rule can be anything that always produces an output for a given input. Consider the rule "flip" where the input is "coin." The output may be "heads" or "tails." We will consider several different types of rules in the following activities and lessons.

Access for English Language Learners

Listening, Speaking, Conversing: MLR7 Compare and Connect. Use this routine when students present their strategies for determining the rules for each card. Ask students to identify "What is similar and what is different" about each approach. Draw students' attention to the different ways the rules were represented in each approach. For instance, for the rule $4x + 1$, students may have expressed it as either $3 + 3 + 3 + 3 + 1$, or $2(3) + 2(3) + 1$, or $5(3) - (3) + 1$, or $4(3) + 1$. In this discussion, emphasize the mathematical language used to make sense of the different ways students generalized and represented the rule. These exchanges strengthen students' mathematical language use and reasoning when comparing strategies for identifying and representing functions.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

1.3 Making Tables

15 minutes
The purpose of this activity is for students to think of rules more broadly than simple arithmetic operations in preparation for the more abstract idea of a function, which is introduced in the next lesson.

Each problem begins with a diagram representing a rule followed by a table for students to complete with input-output pairs that follow the rule. Monitor for students who notice that even though the rules are different, each one starts out with the same input-output pair: $\frac{3}{4}$ and 7. An important conclusion is that different rules can determine the same input-output pair.

If using digital activities, there is a rule generator as an extension. Students give an input and the generator gives an output, after a few inputs students can guess a potential rule, the generator indicates if the rule is "reasonable but not my rule", "correct! how did you know", or "does not match data."

Addressing

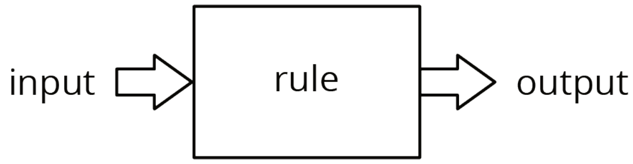
- 8.F.A.1

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Display the following diagram for all to see:



Tell students that this diagram is one way to think about input-output rules. For example, if the rule was "multiply by 2" and the input $\frac{3}{2}$, then the output would be 3. Tell students they will use diagrams like this one during the activity to complete tables of input and output values.

Give students 3–5 minutes of quiet work time to complete the first three tables. Then give them time to share their responses with their partner and to resolve any differences.

Give partners 1–2 minutes of quiet work time for the final rule followed by a whole-class discussion. Depending on time, have students add only one additional input-output pair instead of two.

Access for Students with Disabilities

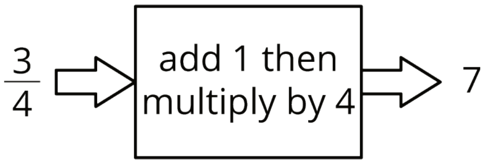
Representation: Internalize Comprehension. Activate or supply background knowledge about the similarities between this activity and the previous activity identifying input-output rules. Allow students to use calculators to ensure inclusive participation in the activity.
Supports accessibility for: Memory; Conceptual processing

Anticipated Misconceptions

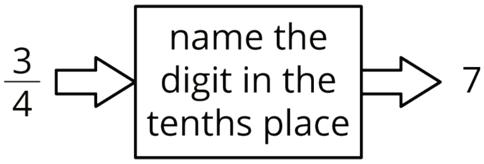
Students may have trouble thinking of "write 7" as a rule. Emphasize that a rule can be anything that produces a well-defined output, even if it ignores the value of the input. Students who know about infinite decimal expansions might wonder about the second rule, because $0.999\cdots = 1.0$, so the same number could have two outputs. If this comes up, discuss how you might refine the rule in this case.

Student Task Statement

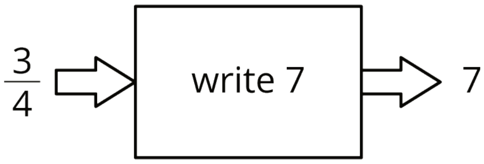
For each input-output rule, fill in the table with the outputs that go with a given input. Add two more input-output pairs to the table.

1. 

input	output
$\frac{3}{4}$	7
2.35	
42	

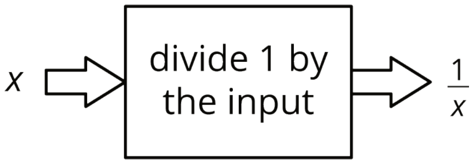
2. 

input	output
$\frac{3}{4}$	7
2.35	
42	

3. 

input	output
$\frac{3}{4}$	7
2.35	
42	

Pause here until your teacher directs you to the last rule.

4. 

input	output
$\frac{3}{7}$	$\frac{7}{3}$
1	
0	

Student Response

Answers vary for the last two rows in each table.

1.

input	output
$\frac{3}{4}$	7
2.35	13.4
42	172
9	40
1	8

2.

input	output
$\frac{3}{4}$	7
2.35	3
42	0
7.31	3
4.95	9

3.

input	output
$\frac{3}{4}$	7
2.35	7
42	7
511	7
91	7

4.

input	output
$\frac{3}{7}$	$\frac{7}{3}$
1	1
0	no output
2	$\frac{1}{2}$

Activity Synthesis

The purpose of this discussion is for students to see rules as more than arithmetic operations on numbers and consider how sometimes not all inputs are possible.

Display a rule diagram with input 2, output 6, and a blank space for the rule for all to see.

Select 2–3 previously identified students and ask what the rule for the input-output pair might be. Display these possibilities next to the diagram for all to see. For example, students may suggest the rules such as "add 4", "multiply by 3", or "add 1 then multiply by 2."

The last rule, "1 divided by the input," calls back to the warm-up. Explain to students that not all inputs are possible for a rule. To highlight this idea, ask:

- “Why was 0 not a valid input for the last rule?” (1 divided by 0 does not exist)
- “What are some other situations when a rule might not have a valid input?” (Any time an operation requires you to divide by 0, or when the input must be non-negative, such as a side length of a square when you know the area.)

- “How does using a variable x to denote the input and $\frac{1}{x}$ to denote the output help us understand the function rule?” (You can clearly see the relationship between the input and output.)

Access for English Language Learners

Writing: MLR3 Clarify, Critique, Correct. Present an incorrect statement that reflects a possible misunderstanding about what a rule can be. For example, Bryce said, “‘Write 7’ isn’t a rule because you just write the number 7.” Prompt students to critique the statement (e.g., ask students whether they agree, and why or why not), and then write feedback to the author that explains why “Write 7” is a rule. Improved statements should describe rules as more than arithmetic operations on numbers. This helps students evaluate, and improve on, the written mathematical arguments of others.
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

The main focus of this lesson are the parallel ideas of using sets of inputs and outputs to identify the rule describing the relationship between them and using a rule to create a set of inputs and outputs. When we have an input-output table that represents only some of the input-output pairs, we can guess a rule but we won't know the rule for sure until we see it. For some rules, there are some numbers that are not allowed as inputs because the rule does not make sense.

To highlight some things to remember about input-output rules, ask:

- "Which rule would you rather make an input output table for: 'divide 10 by the input' or 'write the current year'?" (I would rather make a table for the second rule since the outputs would all just be the current year. The table for the first rule will have fractions and you can't input 0 since 0 divided by 10 does not exist.)
- "What input can you not use with the rule 'divide 10 by the input added to 3'?" (We cannot use -3 since $-3+3=0$ and 0 divided by 10 does not exist.)

To conclude the discussion, poll the class to find out how many input-output pairs students think they would need to figure out a rule and record their answers for all to see. While much of the work students do in grade 8 is linear making "two" a sufficient number of pairs, if students think 2 is always enough point out that both the first and second rule in this activity share the pair $\frac{3}{4}$ and 7 and the pair -1 and 0. Depending on the context, 2 input-output pairs is not always sufficient to determine the rule.

1.4 What's the Rule?

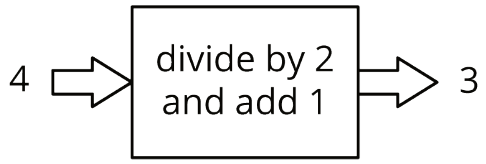
Cool Down: 5 minutes

Addressing

- 8.F.A.1

Student Task Statement

Fill in the table for this input-output rule:



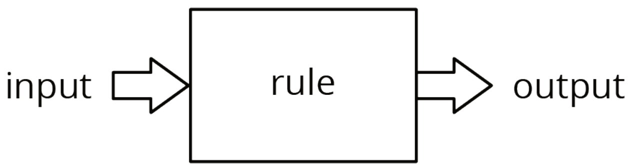
input	output
0	
2	
-8	
100	

Student Response

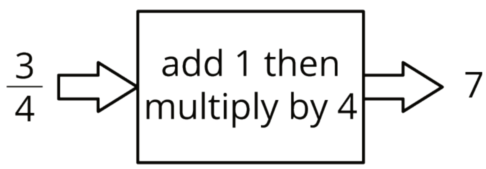
In each row, the output should be one more than half of the input.

input	output
0	1
2	2
-8	-3
100	51

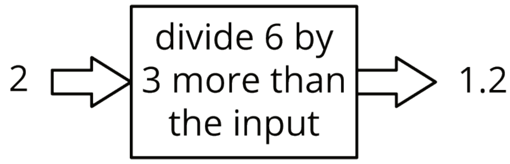
Student Lesson Summary



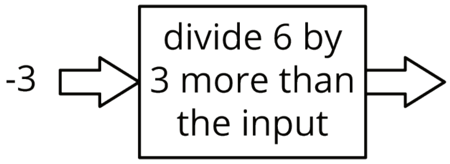
An *input-output rule* is a rule that takes an allowable input and uses it to determine an output. For example, the following diagram represents the rule that takes any number as an input, then adds 1, multiplies by 4, and gives the resulting number as an output.



In some cases, not all inputs are allowable, and the rule must specify which inputs will work. For example, this rule is fine when the input is 2:



But if the input is -3, we would need to evaluate $6 \div 0$ to get the output.



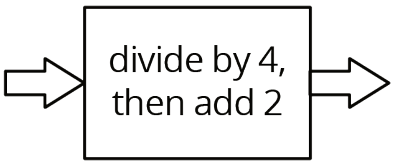
So, when we say that the rule is “divide 6 by 3 more than the input,” we also have to say that -3 is not allowed as an input.

Lesson 1 Practice Problems

Problem 1

Statement

Given the rule:



Complete the table for the function rule for the following input values:

input	0	2	4	6	8	10
output						

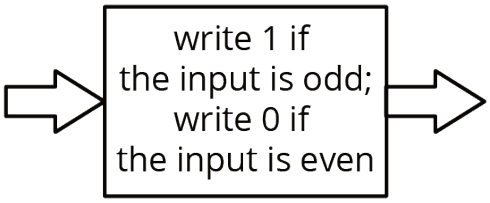
Solution

2, 2.5, 3, 3.5, 4, 4.5

Problem 2

Statement

Here is an input-output rule:



Complete the table for the input-output rule:

input	-3	-2	-1	0	1	2	3
output							

Solution

1, 0, 1, 0, 1, 0, 1, respectively

Problem 3

Statement

Andre’s school orders some new supplies for the chemistry lab. The online store shows a pack of 10 test tubes costs $4 less than a set of nested beakers. In order to fully equip the lab, the school orders 12 sets of beakers and 8 packs of test tubes.

a. Write an equation that shows the cost of a pack of test tubes, t, in terms of the cost of a set of beakers, b.

b. The school office receives a bill for the supplies in the amount of $348. Write an equation with t and b that describes this situation.

c. Since t is in terms of b from the first equation, this expression can be substituted into the second equation where t appears. Write an equation that shows this substitution.

d. Solve the equation for b.

e. How much did the school pay for a set of beakers? For a pack of test tubes?

Solution

a. $t = b - 4$

b. $8t + 12b = 348$

c. $8(b - 4) + 12b = 348$

d. $b = 19$

e. $19 and $15

(From Unit 4, Lesson 15.)

Problem 4

Statement

Solve: $\begin{cases} y = x - 4 \\ y = 6x - 10 \end{cases}$

Solution

$\left(\frac{6}{5}, \frac{\text{-}14}{5}\right)$. Substituting $x - 4$ for y into the second equation, we get $x - 4 = 6x - 10$. Solving this equation gives $x = \frac{6}{5}$. Substituting $x = \frac{6}{5}$ into $y = x - 4$, we get $y = \frac{\text{-}14}{5}$.

(From Unit 4, Lesson 14.)

Problem 5

Statement

For what value of x do the expressions $2x + 3$ and $3x - 6$ have the same value?

Solution

$x = 9$

(From Unit 4, Lesson 9.)